# Eliciting Probabilities, Means, Medians, Variances and Covariances without assuming Risk Neutrality

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**Abstract**

We wish to incentivise an agent to reveal a parameter of a distribution only known to the agent. The objective is to reward the agent for his or her reported value according to a realization of this random variable of interest. The risk preferences of the agent are not known. We first consider elicitation of probabilities and show that this is not possible with a deterministic reward function. However, there is a simple randomized reward function that causes the agent to report the true expected value. The same construction can be used to elicit the mean or the median of the underlying distribution. When rewarding conditional on two independent realizations of the random variable we are able to elicit the variance of one and the covariance of two unknown random variables.

## 1 Introduction

Recently there has been an increased interest in elicitation of beliefs. The standard method for eliciting a probability or a mean is to use the *quadratic scoring rule* (QSR, Brier, 1950). Yet this rule or reward function does not incentivize subjects to report events or expected values truthfully when the decision makers are not risk neutral (Holt, 1986). One response has been to investigate how reports differ from the truth when using QSR under various utility functions (Offerman *et al.* 2009). There are two alternative

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methods (Grether, 1981, Allen 1987) that elicit the probability of an event regardless of the underlying risk preferences. These methods use randomized reward functions that generate binary outcomes. There is also an earlier literature going back to Smith (1961) (see also Roth and Malouf, 1979) that shows how one can induce risk neutral preferences by rewarding using binary lottery tickets. However this literature has not been used explicitly to elicit parameters of a distribution. We want to connect these literatures.

Moreover, we answer some more general questions regarding elicitation of probabilities. Does one need a randomized scoring rule to elicit a subjective probability if risk preferences are not known? How do the different existing rules perform in terms of incentives to tell the truth and are there better rules? We also wish to extend our insights to the elicitation of the mean, the median, the variance or even the covariance.

Our results show indeed that randomization is necessary if one wishes to elicit a probability. A randomized version of the quadratic scoring rule elicits the probability of an event regardless of risk preferences. In fact, it yields the same payoff probabilities as under the rule of Allen (1987) and outperforms the rule suggested by Grether (1981). Moreover, it cannot be outperformed if these payoffs are quadratic in the announcement. This rule can also be used to elicit the mean, again regardless of risk preferences. Randomized rules for the elicitation of fractiles, variances and covariances are also presented.

# 2 Elicitation

In this paper we consider two people, an agent and an elicitor. The agent knows the distribution $F_X$ of a bounded random variable $X$ that yields outcomes belonging to $\Omega \subseteq [a, b]$. This agent is endowed with a Bernoulli utility function $u$ on $\mathbb{R}$ which is strictly increasing in its argument. The distribution $F_X$ need not be the true distribution but can be subjective beliefs of the agent about realizations of $X$. The elicitor would like to learn some parameter $\pi(X)$ of the distribution $F_X$ but does not know $F_X$ and only knows that $X$ yields outcomes belonging to $\Omega$. For instance, the elicitor may wish to learn the subjective probability of some event as perceived by the agent. Similarly, the elicitor may wish to learn the mean or the median. Instead of simply asking the agent to specify $\pi$ and to rely on the agent telling the truth the elicitor wants to reward the agent in a way that causes the agent to reveal $\pi$. The idea is to couple the announcement of the agent with a random realization of $X$. Specifically, the elicitor determines a reward function that specifies a reward to the agent conditional on the announcement of the agent and a single realization of $X$. The difficulty of determining the reward function is that the elicitor does not know the risk preferences of the agent. So the objective of the elicitor is to find a reward scheme that incentivizes the agent to reveal $\pi$ regardless of the specific risk preferences of the agent.

More specifically, each $X$ is associated to a distribution $F_X \in \Delta\Omega$ where $\Delta A$ is the set of all distributions over the set $A$. A *reward function* (or scoring rule) is a function $g : \Pi \times \Omega \to \mathbb{R}$ where $g(z, x)$ determines the monetary reward to the agent if $z$ is the value chosen (or announced) by the agent and $x$ is a realization of $X$. $\Pi$ is the set that contains the possible parameters, so $\Pi = \{z : \pi(X) = z \text{ for some } X \in \Omega\}$. So we wish to find $g$ such that $\{\pi(X)\} = \arg\max_z \int u(g(z, x))\, dF_X(x)$ holds for all strictly increasing functions $u$ and all $X \in \Omega$. In this case we say that $g$ elicits $\pi$. Sometimes we restrict the set of utility functions, for instance then say that $g$ elicits $\pi$ for all concave $u$. Of course one can also choose a probabilistic reward function, in this case $g(z, x) \in \Delta\mathbb{R}$. This is formulation is more general and includes the above deterministic reward as special case. Whether or not a reward function is deterministic will play a special role in this paper.

# 3 Eliciting Probabilities

We first consider the situation where the elicitor wishes to learn about the probability of some event $E \subset \Omega$, so where $\pi(X) = \Pr(X \in E)$.

## 3.1 Quadratic Scoring Rule

The quadratic scoring rule (Brier, 1950) defined by $g(z, x) = -\frac{1}{2}(z - x)^2$ elicits the mean $EX'$ of the random variable $X'$ when it is known that the agent is risk neutral. Setting $X' = \mathbf{I}_{\{X \in E\}}$ and observing that $EX' = \Pr(X \in E)$ we find that $g\left(z, 1_{\{x \in E\}}\right)$ elicits $\Pr(X \in E)$ if the agent is risk neutral. Throughout the paper, we apply an affine transformation so that all rewards belong to $[0, 1]$. Accordingly, the reward function is given by $g(z, x) = 1 - \left(z - \mathbf{I}_{\{x \in E\}}\right)^2$. Specifically, the reward is given by $g_E(z) = 1 - (z - 1)^2$ if event $E$ occurs and by $g_N(z) = 1 - z^2$ if event $E$ does not occur.

More generally, if the elicitor knows the utility function $u$ of the agent then $g_u(z, x) = u(1) - \left(u(z) - u\left(1_{\{x \in E\}}\right)\right)^2$ elicits $\Pr(X \in E)$. This is because when rewards are described in terms of utility then it is as if the agent is risk neutral. However the quadratic scoring rule is not able to elicit $\Pr(X \in E)$ when the elicitor does not know $u$ (see Holt, 1986). This is not even possible if the elicitor knows that the agent is risk averse.

## 3.2 Limitations of Deterministic Rewards

Our first result states that the objective to elicit $\Pr(X \in E)$ for all risk preferences with a deterministic rule is too ambitious.

**Proposition 1** *It is not possible to elicit* $\Pr(X \in E)$ *with a deterministic reward function.*

The intuition is simple. The elicitor has only one parameter, the realization $x$ of $X$, to incentivize the agent to tell the truth. On the other hand, there are infinitely many unknown parameters as the elicitor not only does not know $\Pr(X \in E)$ but also does not know $u$.

**Proof.** We first show that $g(z,1)$ and $g(z,0)$ are differentiable almost everywhere. Once this is established the first order conditions reveal the impossibility.

Consider $X' \in \{0,1\}$ where $X' = \mathbf{I}_{\{X \in E\}}$. So $EX' = \Pr(X'=1) = \Pr(X \in E)$. Let $p = \Pr(X \in E)$. Assume that $g$ elicits $\Pr(X \in E)$ for all concave $u$. Then we have for all $z$

$$pu(g(p,1)) + (1-p)u(g(p,0)) > pu(g(z,1)) + (1-p)u(g(z,0)).$$

For $z < p$ and $u = Id$ we have

$$\begin{aligned} pg(p,1) + (1-p)g(p,0) &> pg(z,1) + (1-p)g(z,0) \\ &= zg(z,1) + (1-z)g(z,0) + (p-z)[g(z,1) - g(z,0)] \\ &\geq zg(p,1) + (1-z)g(p,0) + (p-z)[g(z,1) - g(z,0)] \end{aligned}$$

so

$$(p-z)[g(p,1) - g(p,0)] > (p-z)[g(z,1) - g(z,0)]$$

so

$$g(p,1) - g(p,0) > g(z,1) - g(z,0).$$

Hence we have shown that $g(z,1) - g(z,0)$ is strictly increasing in $z$.

Similarly, for $z > p$ we have

$$p(g(p,1) - g(z,1)) + (1-p)(g(p,0) - g(z,0)) \geq 0$$

and since

$$g(z,1) - g(z,0) > g(p,1) - g(p,0)$$

it follows that $g(p,0) > g(z,0)$. So $g(z,0)$ strictly decreasing in $z$ and hence $g(z,1)$ is strictly increasing.

From the above two strict monotonicity statements we obtain that $g(z,1)$ and $g(z,0)$ are differentiable almost everywhere. Let $A$ be the set where they are differentiable.

For $p \in A$ and differentiable $u$ we can calculate

$$\frac{d}{dz}\left(pu\left(g\left(z,1\right)\right) + (1-p)\,u\left(g\left(z,0\right)\right)\right) = pu'\left(g\left(p,1\right)\right)g'\left(z,1\right) + (1-p)\,u'\left(g\left(p,0\right)\right)g'\left(z,0\right)$$

and infer that

$$pu'\left(g\left(p,1\right)\right)g'\left(p,1\right) + (1-p)\,u'\left(g\left(p,0\right)\right)g'\left(p,0\right) = 0. \tag{1}$$

It is easy to argue with generalized version of the intermediate value theorem that there is $p \in A\backslash\{0\}$ such that $g'(p,1) > 0$. Consider $u$ that is differentiable with $u' > 0$. Then rewrite (1) as:

$$\frac{u'\left(g\left(p,0\right)\right)}{u'\left(g\left(p,1\right)\right)} = -\frac{(1-p)g'(p,0)}{pg'(p,1)}. \tag{2}$$

Since $g(z,1) - g(z,0)$ is strictly increasing in $z$ there is some $p_0 \in A \cap (0,1)$ such that $g(p_0,1) \neq g(p_0,0)$. So when $p = p_0$ the left hand side of (2) depends on $u$. Therefore, (2) cannot hold for all $u$. ■

## 3.3 Randomized Rewards

We now consider probabilistic or randomized reward functions. Hence $g(z,x) \in \Delta\mathbb{R}$ where $g(z,x)$ is now the payoff *distribution* awarded conditional on $(z,x)$. We show how adding such randomness enables to elicit $\Pr(X \in E)$ for all $u$. We then relate our findings to the existing literature.

In the following we demonstrate how one can elicit $\Pr(X \in E)$ with a probabilistic version of the quadratic scoring rule. We use the common technique (going back to Smith, 1961) of paying in lottery tickets instead of in money. Specifically, recall the *quadratic scoring rule* $\bar{g}(z,x) = 1 - \left(z - \mathbf{I}_{\{x \in E\}}\right)^2$ in the version that is affinely transformed to yield payoffs in $[0,1]$. Instead of giving reward $\bar{g}(z,x)$ we give reward 1 with probability $\bar{g}(z,x)$ and reward 0 with probability $1 - \bar{g}(z,x)$. Thus, the deterministic payoff $\bar{g} \in [0,1]$ is transformed into a probabilistic payoff $g_m \in \{0,1\}$. It is easy to verify that $Eg_m(z,x) =$

$\Pr(g_m(z,x)=1) = \bar{g}(z,x)$. Note that this rule is well defined as $\bar{g}(z,x) \in [0,1]$. For future reference we will call this transformation of deterministic into probabilistic payoffs the "randomization trick".

The reward function $g_m$ will be called the *randomized quadratic scoring rule* (rQSR). It is formally defined by $g_m(z,x) \in \{0,1\}$ and $\Pr(g_m(z,x)=1) = 1 - \left(z - \mathbf{I}_{\{x \in E\}}\right)^2$. rQSR can be implemented by drawing a random variable $y$ from $[0,1]$ according to a uniform distribution and then paying 1 if $y \leq \bar{g}(z,x)$ and paying 0 if $y > \bar{g}(z,x)$. By using the randomization trick the agent becomes risk neutral (at least in theory), since any subjective expected utility maximizer faced with different lotteries between the same payoffs $\{0,1\}$ will be interested only in the expected value. Thus, since the quadratic scoring rule elicits $\Pr(X \in E)$ for a risk neutral agent, the rQSR does so for any risk preferences. We present this statement formally.

**Proposition 2** *The randomized quadratic scoring rule elicits* $\Pr(X \in E)$.

**Proof.** Given the randomization trick we find that

$$\begin{aligned}\int_a^b Eu(g_m(z,x))\,dF_X(x) &= u(1)\int_a^b g_m(z,x)\,dF_X(x) + u(0)\int_a^b (1-g_m(z,x))\,dF_X(x) \\ &= u(0) + (u(1)-u(0))\int_a^b g_m(z,x)\,dF_X(x).\end{aligned}$$

It is as if the agent is risk neutral as the agent wishes to choose $z$ to maximize

$$\int_a^b g_m(z,x)\,dF_X(x).$$

Since the QSR elicits $\Pr(X \in E)$ for risk neutral decision makers the proof is complete. For later reference we verify this property of QSR explicitly and calculate the probability of obtaining the payoff 1 as

$$\begin{aligned}\int_a^b g_m(z,x)\,dF_X(x) &= 1 - \int_a^b \left(z - \mathbf{I}_{\{x\in E\}}\right)^2 dF_X(x) \\ &= 1 - p(1-z)^2 + (1-p)z^2.\end{aligned} \tag{3}$$

Taking the derivative of the right hand side with respect to $z$ we obtain

$$2(p-z). \tag{4}$$

Consequently, the unique utility maximizing choice is $z = p$ which completes the proof. ■

We now compare the above to the existing literature on probabilistic elicitiation.

Grether (1981) was the first to show how to elicit probabilities truthfully regardless of risk preferences. The idea is based on Becker et al. (1964) who show how to elicit a reservation price regardless of risk preferences. Instead of evaluating the reservation price in terms of money one measures it in terms of a probability of getting a prize. The subject is allowed to choose between receiving the prize with probability $y$ and receiving the prize when the event to be elicited occurs. For this the subject is asked to specify a cutoff point $z$ such that he or she prefers the artificial probability $y$ to the true event probability $p$ if $z \geq y$. More specfically, the reward function based on a draw $y$ from some distribution $F$ and is given by

$$g(z,x,y) = \begin{cases} l(p,1,0) & \text{if } y \leq z \\ l(y,1,0) & \text{if } z < y \end{cases}$$

where $l(q,1,0)$ is the lottery that assigns 1 with probability $q$ and 0 with probability $1-q$. The probability of receiving 1 when announcing $z$ is equal to $g_e(z,p) := p\int_0^z dF(x) + \int_z^1 x dF(x)$. It is easy to see that whenever $F$ has full support then the above rule elicts $p$. The above formulation without specifying the distribution $F$ of $y$ can be found in Möbius et al. (2007). Holt (2007, ch. 30; see also Karni, 2009) independently suggested the above rule with the additional specification that $y$ is drawn from the uniform distribution on $[0,1]$. The implementation of Grether (1981) is more complicated as it is based on two draws from a uniform distribution, however the probability of receiving 1 is identical as in the rule suggested by Holt (2007).[1]

We wish to investigate the incentives to tell the truth. Assume that $F$ has density $f$ and no point mass, then the derivative with respect to $z$ of the probability of receiving 1 is equal to $(p-z) f(z)$. In particular, under the rule of Grether (1981) we find

$$\frac{d}{dz} g_e(z,p) = p - z. \tag{5}$$

Given that

$$g_e(p,p) - g_e(z,p) = \int_z^p (p-x) f(x)\, dx$$

[1] The reward function suggested by Grether (1981) is based on

$$g(z,x,y_1,y_2) = \begin{cases} l(p,1,0) & \text{if } y_1 \leq z \\ 1 & \text{if } z < y_1 \text{ and } y_2 \leq y_1 \end{cases}$$

where $y_1$ and $y_2$ are independently drawn from a uniform distribution on $[0,1]$.

we see that the choice of $f \equiv 1$ that leads to the rule of Holt (2007) and hence of Grether (1981) equalizes the incentives to tell the truth across the range of possible probabilities.

Most importantly, comparing (4) to (5) we observe that the incentives to tell the truth are stronger under rQSR than they are under the rule of Grether (1981).

Allen (1987, see also McKelvey and Page, 1990) proposed the following alternative rule for eliciting probabilities truthfully regardless of risk preferences. Payoffs are either 1 or 0. If event $E$ occurs then payoff 1 is rewarded if the announcement $z$ is above the value $r_E$ of a random draw of random variable that has density $2(1-r_E)$ on $[0,1]$. If $E$ does not occur then payoff 1 is rewarded if $z$ lies below the random draw $r_N$ of a random variable that has density $2r_N$ on $[0,1]$. One easily verifies that the probabilities of receiving 1 conditional on $x \in E$ and conditional on $x \notin E$ are identical to those under rQSR. In other words, apart from the way the two rules are presented, the expected payoffs under rQSR are identical to those under the rule of Allen (1987). In fact, one can present a slightly different means to generate these probabilities which can be useful when implementing rQSR in experiments.

The idea is that one has to compete against a robot in predicting from an expost perspective. The focus is on what does not happen. Specifically, the robot specifies a probability $y$ that the opposite event ocurrs and the subject is paid 1 if the robot specified a higher probability for what did not happen than the subject. So if $E$ occurs then the subject is paid if $y \geq 1-z$ and if $E$ does not occur then the subject is paid if $y \geq z$. If $y$ is drawn from a distribution on $[0,1]$ that has density $h$ the probability of being paid 1 is equal to

$$g_e(z,p) = p\int_{1-z}^{1} h(x)\,dx + (1-p)\int_{z}^{1} h(x)\,dx. \tag{6}$$

The marginal incentives are given by

$$\frac{d}{dz} g_e(z,p) = ph(1-z) - (1-p)\,h(z)\,.$$

The subject wishes to maximize this expression in (6). If $h$ is increasing then the solution is to announce $z = p$. If $h(x) = 2x$ for $x \in [0,1]$ then the above reduces to the payoff probabilities (3) of rQSR and hence of the rule of Allen (1987). Again the choice of $h = 2x$ equalizes the incentives to tell the truth across the range of parameters.

Given the above, the obvious question is whether there are other rules that are even better in incenitivizing to tell the truth. To limit attention to simple rules consider only rules where the reward is quadratic in the own announcement and where payments are in $\{0,1\}$. Let $g_E(z) = g(z,x)$ for $x \in E$ and $g_N(z) = g(z,x)$ for $x \notin E$. Assume that $g_E(z) = a_0 + a_1 z + a_2 z^2$ and $g_N(z) = b_0 + b_1 z + b_2 z^2$. The first order conditions require that $z g_E'(z) + (1-z) g_N'(z) = 0$ holds for all $z \in [0,1]$. Comparing coefficients of the polynomials we obtain that $b_2 = a_2$, $b_1 = 0$ and $a_2 = -a_1/2$. Hence $g_E(z) = a_0 + a_1 z - \frac{1}{2} a_1 z^2$ where $g_E(1) \leq 1$ implies that $a_0 + \frac{1}{2} a_1 \leq 1$ so $a_1 \leq 2 - a_0$. The fact that $g_E(0) \geq 0$ implies $a_0 \geq 0$. Hence $a_1 \leq 2$. We now look at the second derivative of the probability of receiving payoff 1, and find

$$p \frac{d}{dz}\frac{d}{dz}\left(a_0 + a_1 z + a_2 z^2\right) + (1-p)\frac{d}{dz}\frac{d}{dz}\left(b_0 + b_1 z + b_2 z^2\right) = -a_1.$$

Comparing this expression to (4) and knowing that $a_1 \leq 2$ if follows that one cannot improve over rQSR within the set of reward functions with success probabilities that are quadratic in the announcement.

# 4 Eliciting Other Parameters

As we have seen above, once there is a reward function that enables to elicit a parameter for a risk neutral agent, one can use the "randomization trick" to extend this to a randomized reward function that elicits this parameter for any risk preferences of the agent. Below we present some other applications.

## 4.1 Mean

Now assume that the elicitor is interested instead in learning about the mean $EX$. Again we combine the randomization trick with the fact that QSR $g(z,x) = -\frac{1}{2}(z-x)^2$ elicits the mean for risk neutral agents. The first step is to take an affine transformation so that outcomes are contained in $[0,1]$, so we define $\bar{g}(z,x) = 1 - \left(\frac{z-x}{b-a}\right)^2$. Next we apply the randomization trick and define $g_m(z,x) \in \{0,1\}$ such that $\Pr(g_m(z,x) = 1) = \bar{g}(z,x)$. Again, we call $g_m$ the randomized quadratic scoring rule, as in the setting where $\Omega = \{0,1\}$ and $X' = \mathbf{I}_{\{X \in E\}}$.

**Proposition 3** *The randomized quadratic scoring rule elicits $EX$.*

The proof is analogous to that of Proposition 2.

## 4.2 Median and Quantiles

The *fractile scoring rule* (Cervera and Muñoz, 1996) elicits the fractile or quantile $\alpha$ of the distribution $F_X$ of $X$ for any given $\alpha \in (0,1)$, its reward function is given by $g(z,x) = \alpha z - (z-x)\mathbf{I}_{z \geq x}$. We rescale this reward function with an affine transformation to yield values in $[0,1]$ for $X \in [a,b]$ and obtain

$$\bar{g}(z,x) = \frac{\alpha z - (z-x)\mathbf{I}_{z \geq x} + b(1-\alpha) - a}{b-a}.$$

The *randomized fractile scoring rule* $g_m$ is defined by $g_m(z,x) \in \{0,1\}$ with $\Pr(g_m(z,x) = 1) = \bar{g}(z,x)$.

**Proposition 4** *The randomized fractile scoring rule elicits the quantile $\alpha$.*

In particular, Proposition 4 shows how to elicit the median by setting $\alpha = 1/2$.

**Proof.** The proof follows immediately from the fact that the randomization trick makes the agent behave as if risk neutral and from the fact that the fractile scoring rule elicits the quantile $\alpha$ for risk neutral preferences. ■

## 4.3 Variance and Covariance

In order to elicit the variance of $F_X$ we incorporate two independent realizations $x_1$ and $x_2$ of $X$ when rewarding the agent. So we consider a reward function $g(z,x_1,x_2)$. If $X \in [a,b]$ then $VarX \leq \frac{1}{4}(b-a)^2$ so we can limit attention to $z \in \left[0, \frac{1}{4}(b-a)^2\right]$. Following Walsh (1962), $VarX = E\left(\frac{1}{2}(X_1 - X_2)^2\right)$ where $X_1$ and $X_2$ are independent copies of $X$. We combine this with the randomized quadratic scoring rule and define $g(z,x_1,x_2) = -\left(z - \frac{1}{2}(x_1-x_2)^2\right)^2$ which has range $\left[-\frac{1}{4}(b-a)^4, 0\right]$. Again, we rescale to generate rewards belonging to $[0,1]$ and let

$$\bar{g}(z,x_1,x_2) = \frac{g(z,x_1,x_2) + \frac{1}{4}(b-a)^4}{\frac{1}{4}(b-a)^4}$$

and then define the *randomized variance scoring rule* by $g_m(z,x_1,x_2) \in \{0,1\}$ with $\Pr(g_m(z,x_1,x_2) = 1) = \bar{g}(z,x_1,x_2)$.

**Proposition 5** *The randomized variance scoring rule elicits the variance of $X$.*

Similarly we can elicit the covariance when the agent knows the distributions of two random variables $X_1$ and $X_2$. We assume that $X_i \in [a_i, b_i]$ for $i = 1, 2$. Here we condition a realization $(x_1, x_2)$ drawn from $(X_1, X_2)$. Again following Walsh (1962), we use the fact that $Cov\,(X_1, X_2) = E\left(\frac{1}{2}(X_1 - X_2)^2\right)$ which has range $\left[-\frac{1}{4}(b_1 - a_1)(b_2 - a_2), \frac{1}{4}(b_1 - a_1)(b_2 - a_2)\right]$. Then define $g(z, x_1, x_2) = -\left(z - \frac{1}{2}(x_1 - x_2)^2\right)^2$ which has range $[-c, 0]$ where $\sqrt{c} = \frac{1}{4}(b_1 - a_1)(b_2 - a_2) + \frac{1}{2}\max\{b_2 - a_1, b_1 - a_2\}$. Normalizing we let

$$\bar{g}(z, x_1, x_2) = \frac{g(z, x_1, x_2) + c}{c}$$

and define the *randomized covariance scoring rule* $g_m$ by $g_m(z, x_1, x_2) \in \{0, 1\}$ with $\Pr(g_m(z, x_1, x_2) = 1) = \bar{g}(z, x_1, x_2)$.

**Proposition 6** *The randomized covariance scoring rule elicits the covariance of $X_1$ and $X_2$.*

# 5 Conclusion

In summary, we have presented a way to circumvent the limitations of deterministic rules to elicit subjective probabilities. By the use of the “randomization trick” we can transform a deterministic rule, such as the well-known QSR, into a randomized rule that incentivizes truthful reporting for all risk preferences. The incentives for truth-telling provided by existing mechanisms that use randomized rewards can be compared to the randomized QSR. It is found that the randomized QSR provides the maximum incentives within a class of ‘simple’ rules.

Randomized rules, and the randomized QSR in particular, thus provide a theoretically attractive elicitation method. It remains an open question whether these rules have the desired properties in actual experiments. Some doubts have been raised (see Selten et al. 1999) whether subjects rewarded using lotteries behaver as if risk neutral albeit such an investigation is not know for belief elicitation.

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