Informational asymmetry between managers and investors in the optimal capital structure decision

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January 2005

JEL classification: G 32, G 13

Keywords: contingent claims valuation, capital structure decisions, incomplete information, asymmetric information, managerial entrenchment

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Abstract

In this article, we consider the impact of asymmetric information between managers and investors on the optimal capital structure decision. This is done within a continuous-time framework, where the relevant state variable is given by the EBIT value of the firm; an approach taken by Goldstein *et al.* (2001), Hackbarth *et al.* (2003), Dangl & Zechner (2004) amongst others. Our setup differs in that we assume the EBIT to follow an Arithmetic Brownian motion, i.e. it can assume negative values. More importantly, we extend this framework in two directions: (i) We introduce a separate management claim. (ii) We introduce asymmetric information between managers and investors by assuming that claimants receive noisy signals, which they process according to rational expectation principles.

Our results show, that managers always try to avoid debt, and that their optimal bankruptcy threshold is always lower, than the threshold set by equity holders. The introduction of asymmetric information changes the optimality conditions, and consequently the capital structure decision. It is shown, that the informational asymmetry can substantially lower the optimal leverage ratio, even without assuming that managers are entrenched.

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1 Introduction

Since Modigliani & Miller (1958), a large number of publications have contributed to the discussion about the optimal capital structure, which is a central topic of corporate finance. One fruitful approach has been provided by the contingent claims analysis, pioneered by Merton (1974), Black & Cox (1976), Brennan & Schwartz (1978) and Leland (1994). A recent contribution in this strand of literature has been provided by Goldstein *et al.* (2001), who take the EBIT-value of a firm to be the relevant state variable, upon which contingent claims can be valued. This kind of modelling has the significant advantage that it avoids the problem of possible arbitrage opportunities, that were inherent in earlier models that took the state variable to be the unlevered asset value.

In this article, we take up this approach and extend it in mainly two related directions. On the one hand, we consider agency problems by introducing management as a separate claim, while on the other hand we introduce asymmetric information by assuming that investors only have noisy signals about the state variable.

Considering agency costs in the optimal capital structure literature is definitely not new, and dates back at least to Jensen & Meckling (1976). An important contribution dealing with agency costs in the contingent claims framework is due to Leland (1998), who considers stockholder-bondholder conflicts, but excludes the conflicts between managers and investors.[1] However, entrenched managers, who pursue their own interests, can have a significant influence on leverage decisions. It is argued, that managers use their discretion to implement a leverage that is too low compared to equity holders optimum.

By allowing managers to have a separate claim on the underlying cash flow, our model shows how to reproduce this observations in the contingent claims framework. More concretely, it is shown that the value of the management claim is monotonically decreasing with increasing debt values. Thus, without any counteracting forces, managers always tend to prefer less debt.[2] Furthermore, it

[1]See Leland (1998), p.1215, fn.4

[2]This seems to be confirmed by empirical findings. See. e.g. Berger *et al.* (1997), Graham (2000) or de Jong & Veld (2001).

is shown, that the endogenously determined bankruptcy thresholds for equity holders and managers diverge; being significantly lower for the management claim, which implies that if investors cannot closely monitor, the management may have an incentive, in the absence of any counteracting forces, to delay bankruptcy.
The second extension of this contribution is given by the introduction of asymmetric information between managers and investors. We assume that investors, i.e. stock and bond holders only receive noisy signals about the EBIT value. It is further assumed, that investors will rationally process their given information, and thus form the best estimate of their observed signals. However, the fact that equity holders can only rely on estimates about the cash flow influences the optimality condition for the coupon level and the endogenous bankruptcy threshold. Not surprisingly, this will impact on the optimal capital structure decision. It is shown, that the informational asymmetry can account for a substantially lower leverage ratio.

Literature review

The related literature can be subdivided in broadly two different (but related) areas: (i) Optimal capital structure models (ii) Models of incomplete information and principal-agent conflicts.
Ad (i): Important factors influencing the optimal capital structure decision are the tax benefits provided by debt issuance on the one hand and the potential costs of financial distress, namely bankruptcy costs or agency costs on the other hand. While this has already been noted by Modigliani & Miller (1958), one of the first papers to make this explicit was Leland (1994). Using the framework introduced by Merton (1974) and Black & Cox (1976) he shows how optimal leverage ratios can be derived by considering tax benefits and bankruptcy costs. His analysis was extended among others by Leland & Toft (1996) who relax the assumption of infinite life debt, or Leland (1998) to account for asset substitution. However, in these models, the capital structure choice is essentially a static one, and does not account for the possibility of subsequent debt restructurings.
One of the earliest papers to address *dynamic* capital structure choices is Kane *et al.* (1984, 1985) and notably Fischer *et al.* (1989), who show that the optimal debt ratio of a firm can vary substantially over time. Common to both approaches - the static and the dynamic one - is the assumption, that the rel-

evant state variable is given by the (unlevered) firm value[3], which is modelled in the standard way as Geometric Brownian motion.

More recent contributions to this strand of literature, which avoid this assumption include Goldstein *et al.* (2001), Dangl & Zechner (2004), Ammann & Genser (2004), Christensen *et al.* (2000), and Hackbarth *et al.* (2003)

It is the paper by Goldstein *et al.* (2001) that first dispenses with the traditional assumption of the contingent claims analysis[4] to take the unlevered assets as state variable. For reasons to be explained later, they consider the Earnings Before Interest and Taxes (EBIT) as the state variable, which still is assumed to follow a Geometric Brownian motion. Within this setting, they consider the implications of the option to increase future debt levels. Compared to static capital structure models this implies that debt values decrease because of the rising bankruptcy level.

In the same spirit, but with another focus Hackbarth *et al.* (2003) make use of the EBIT-assumption to address the question of balancing bank and market debt in an optimal way.

Ad (ii): Besides the above mentioned papers, there exists a large body of literature that focuses on the impact of principal-agent conflicts and asymmetric information on the determination of optimal capital structure.[5] Jensen & Meckling (1976) are commonly credited as being the first contribution examining the impact of self-interested managers on capital structure (or ownership structure). A very recent paper in this direction is Morellec (2004), who uses a framework similar to Stulz (1990) and shows that managerial discretion can account for low leverage ratios.

There is also a large body of literature that deals with issues of asymmetric information and their impact on the financing choice. Pioneering work in this area is due to Leland & Pyle (1977), Ross (1977), Myers (1977) or Myers & Majluf (1984).[6] Within the class of models, that deals with asymmetric information, one has to distinguish further, between which parities the informational asymmetry exists. The contribution by Myers (1977) and Myers & Majluf (1984)

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[3]More precisely, the Fischer *et al.* (1989) model assumes the unlevered firm value plus its leverage potential as the state variable. We will come back to this point in a later section

[4]A notable exception is given by Mella-Barral & Perraudin (1997)

[5]Obviously, no sharp distinction between the two classes can be drawn, since the disadvantage of debt financing in the Leland (1994) framework stems from the bankruptcy costs, which are a form of agency costs.

[6]For reviews, see e.g. Stein (2003) or Harris & Raviv (1991).

looks at informational asymmetries between managers, who act in the best interest of existing equity holders, and outside investors. Within this framework, they conclude, that there is a *pecking order* of financing choices. New investments will be financed first with internal resources, then by debt, and finally with equity. Subsequent work by Brennan & Kraus (1987) tries to find conditions under which this adverse selection problem may be overcome.

Another natural source of asymmetric information is the manager-investor relationship, because, while managers can be assumed to have in-depth knowledge of the firm they are running, this may not be the case for its investors. The capital structure decision, taken by managers, may then serve as a *signalling* device to communicate insider information to outside investors. Early models within this approach are Leland & Pyle (1977), or Ross (1977).

Most models are static in the sense that they consider some random end-of-period firm or project value over which insiders are better informed than outsiders. In this paper we are interested in incorporating manager-investor asymmetric information in the continuous-time framework introduced in the previous section.

One notable contribution in this direction was recently made by Duffie & Lando (2001), who essentially extend the model by Leland & Toft (1996) to incorporate incomplete information on the part of investors. Being mainly concerned with the implication on credit spreads, they show that credit spreads in structural models can become non-negative for very short maturities, which remedies an important critique on structural credit risk models.

Giesecke (2003) builds upon Duffie & Lando (2001) to extend their analysis to consider not only incomplete information about the asset value but also imperfect information about the default barrier.[7]

A slightly different approach to incomplete information was provided by Çetin *et al.* (2003). They argue that in the Duffie & Lando (2001) model the market receives a noisy version of the asset value, i.e. the asset value plus a noise component added. The markets task is to 'filter' away the noise in the most efficient way, to obtain the best estimate of the asset value. This is essentially done by Bayesian updating. In contrast, in the Çetin *et al.* (2003) model the markets information set is not a noisy signal, but a *reduced* set of the information that is available to managers, i.e. the market has only a coarse partitioning of the information set generated by the state variable, which is in their setup the firm's

[7]See also Giesecke & Goldberg (2004) and Goldberg (2004)

cash flows. However, their model remains on an abstract mathematical level and does not lend itself to direct applications.

The remainder of the paper is organized as follows: Section 2 presents the general valuation framework, which is applied to the different claims. Section 3 discusses the optimality conditions under complete information, with and without entrenched managers. Section 4 introduces the asymmetric information and discusses the impact on the capital structure decision. Numerical examples are shown. Section 5 concludes.

2 The valuation framework

The model set-up is similar to Goldstein *et al.* (2001) in that we will take the EBIT value as the state variable. There are mainly two advantages related to this: On the one hand, it avoids the problem of possible arbitrage opportunities, that were inherent in earlier models that took the state variable to be the unlevered asset value.[8] On the other hand, it allows to treat all claims on this state variable in a consistent manner. The treatment of the tax claim in the earlier models was different from the equity and debt claims in that it was considered an inflow of funds, rather than a reduction in the outflow of funds.[9] In contrast to Goldstein *et al.* (2001) we will not assume the EBIT value to follow a Geometric Brownian motion, but to follow an Arithmetic Brownian motion.[10] This has the more realistic feature, that cash flows can become negative, which is by definition not possible with Geometric Brownian motion.

In this section, we give the derivation of the contingent claims analysis[11] under complete information, which will serve as the reference case.
As pointed out above, we take the state variable to be the EBIT, denoted by

[8] It is not possible to assume both the unlevered and the optimally levered assets to be traded assets, without introducing the possibility of arbitrage. Leland (1994) leaves this issue unresolved, while Fischer *et al.* (1989) considered the unlevered asset value plus its optimal leverage potential as the state variable.

[9] See Goldstein *et al.* (2001), p.487.

[10] Ammann & Genser (2004) also use Artithmetic BM, but since they deal with dynamic capital structure, they run into difficulties due to the loss of the homogeneity property.

[11] Brennan (2003) calls it the *certainty equivalent approach to valuation.*

X, and assume that it follows an arithmetic Brownian motion

$$\mathrm{d}X_t = \mu^P \mathrm{d}t + \sigma\, \mathrm{d}W_t^P \tag{1}$$

where μ^P and σ denote the drift and diffusion parameter which are assumed to be constant. W^P is a standard Wiener process. The superscript P emphasizes that the drift and the Wiener process are governed by the physical probability measure P.

In following standard arbitrage-free pricing principles, we further assume that there exists an equivalent risk-neutral probability measure Q, which ensures the absence of arbitrage. Under the Q-measure, we denote the EBIT-dynamics by

$$\mathrm{d}X_t = \mu\, \mathrm{d}t + \sigma\, \mathrm{d}W_t \tag{2}$$

where $\mu = \mu^P - \theta\, \sigma$ is now the risk-neutral drift, and W is a Q-Wiener process. θ denotes the endogenously obtained risk premium.

We can also write down the dynamics of an arbitrary claim $(C(X))$ on the payout flow by applying Itô's formula.

$$\begin{aligned} \mathrm{d}C(X_t) &= C_{(t)}\mathrm{d}t + C_{(x)}\mathrm{d}X + \frac{1}{2}C_{(xx)}(\mathrm{d}X)^2 \\ &= \big(C_{(t)} + \mu C_{(x)} + \frac{1}{2}\sigma^2 C_{(xx)}\big)\mathrm{d}t + \sigma C_{(x)}\mathrm{d}W_t \end{aligned} \tag{3}$$

where the subscripts in parentheses indicate the partial derivatives.

The absence of arbitrage opportunities implies that the total risk-neutral expected return on the claim has to equal the risk-free rate. If we consider general claims of the form $mX + c$, i.e. a claim that receives a fixed amount of c and a fraction m of the EBIT flow, we get the following relationship

$$\begin{aligned} rC\,\mathrm{d}t &= \mathrm{E}^Q[\mathrm{d}C + (mX + c)\,\mathrm{d}t] \\ &= \mathrm{E}^Q[(C_{(t)} + \mu C_{(x)} + \frac{1}{2}\sigma^2 C_{(xx)} + mX + c)\,\mathrm{d}t + \sigma C_{(x)}\mathrm{d}W_t] \\ &= (C_{(t)} + \mu C_{(x)} + \frac{1}{2}\sigma^2 C_{(xx)} + mX + c)\,\mathrm{d}t \end{aligned} \tag{4}$$

From which we get the partial differential equation that has to be fulfilled by any claim on the EBIT

$$C_{(t)} + \frac{1}{2}\sigma^2 C_{(xx)} + \mu C_{(x)} - rC + mX + c = 0 \tag{5}$$

In the following, we will only consider claims with no stated maturity, so equation (5) reduces to an ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2 C_{(xx)} + \mu C_{(x)} - rC + mX + c = 0 \tag{6}$$

which has the general solution

$$C(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{mX + c}{r} + \frac{\mu m}{r^2} \tag{7}$$

with A_1, A_2 being constants that have to be determined by appropriate boundary conditions, and β_1, β_2 are roots of the quadratic equation $Q(\beta) = \frac{1}{2}\sigma^2\beta^2 + \mu\beta - r = 0$ which are given by

$$\beta_1 = \frac{-\mu + \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2} \qquad \beta_2 = \frac{-\mu - \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}$$

Note, that $\beta_1 > 0$ and $\beta_2 < 0$.

At first glance, it is tempting to think of the asset value (V) associated with the EBIT flow as the discounted expectation over all future cash flows, i.e. $V_t = \mathrm{E}^Q[\int_t^\infty e^{-r(s-t)} X_s\, \mathrm{d}s] = \frac{X_t}{r} + \frac{\mu}{r^2}$. While this is reasonable in the case when the EBIT flow follows a Geometric Brownian motion[12], more caution has to be applied in the arithmetic case. Here, cash flows can be negative, and at a sufficiently low EBIT-level it might become optimal to even shut down an *unlevered* firm. Therefore, in order to determine the asset value, we have to take into account the optimal shut down-level, which we denote by X_B. If we consider a claim on the entire payout flow, i.e. $m = 1$ and $c = 0$, we derive the specific solution by determining the constants A_1 and A_2 of equation (7) by appropriate boundary conditions. From the fact, that $\beta_1 > 0$, we immediately deduce, that $A_1 = 0$ since otherwise the solution diverges. On the other hand, the claim is worthless at the shut down-level X_B, i.e. $V(X_B) = 0$, and so we find $A_2 = -(\frac{X_B}{r} + \frac{\mu}{r^2})e^{-\beta_2 X_B}$, which gives

$$V(X) = \left(\frac{X}{r} + \frac{\mu}{r^2}\right) - \left(\frac{X_B}{r} + \frac{\mu}{r^2}\right)e^{\beta_2 (X - X_B)} \tag{8}$$

The first term in the solution coincides with the value of the discounted expectation over all future cash flows ($\mathrm{E}^Q[\int_t^\infty e^{-r(s-t)} X_s\, \mathrm{d}s]$); however, due to the possibility of a shut down we cannot pretend to have an claim on *all* future cash flows, but only on those that are realized as long as the EBIT value did not drop under the shut down level X_B.
If we denote by τ_{X_B} the first time, that the process X hits the level X_B[13] we can interpret $V(X_t)$ as $\mathrm{E}^Q[\int_t^{\tau_{X_B}} e^{-r(s-t)} X_s\, \mathrm{d}s]$, i.e. the expected discounted

[12] Cash flows cannot become negative, so "it is never optimal to shut-down an unlevered firm" (Hackbarth *et al.* (2003), p.7)

[13] More precisely, $\tau_{X_B} = \inf(t; X_t \leq X_B)$, which is a so-called stopping time.

cash flows from t until the stopping time τ_{X_B}.
Note further, that the term $e^{\beta_2(X-X_B)}$ can be interpreted as the present value of one unit of account conditional on the event, that the process X hits the boundary X_B. This can be seen by evaluating the expectation $\mathrm{E}[e^{-r\,\tau_{X_B}}]$ which turns out to be[14]

$$\mathrm{E}[e^{-r\tau_{X_B}}] = e^{\beta_2(X-X_B)} \tag{9}$$

2.1 All equity case

Now, while $V(X)$ has to be interpreted as the value of a claim on the entire cash flow, this is a hypothetical case, since even if we consider a firm that is entirely financed by equity, we must take into account, that not the whole cash flow accrues to shareholders, but that a substantial part has to be payed to the tax authority. So, even in the all equity case, we have two claimants: equity holders and the government.
Therefore, we introduce τ^c as the tax rate applying on the corporate level, and τ^d and τ^k as the investor's personal tax rate on dividends and coupon payments respectively. Then, the effective part of the cash flow, that goes to equity holders is $m = (1-\tau^c)(1-\tau^d)$, and we abbreviate this to $m = (1-\tau^e)$ by defining the effective tax rate τ^e.
With the same reasoning as above, the general solution for the valuation of the all equity claim (AE) is $AE(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{(1-\tau^e)X}{r} + \frac{(1-\tau^e)\mu}{r^2}$.
To simplify notation, make the following definitions

$$\hat{X} \equiv \frac{X}{r} + \frac{\mu}{r^2} \qquad\qquad \hat{X}_B \equiv \frac{X_B}{r} + \frac{\mu}{r^2} \tag{10}$$

Again, applying $A_1 = 0$ and $AE(X_B) = 0$, we find

$$AE(X) = (1-\tau^e)\Big(\hat{X} - \hat{X}_B\, e^{\beta_2(X-X_B)}\Big) \tag{11}$$

Clearly, in this case the claim of the government is simply the remaining part of the entire cash flow. Denoting the government claim in the all equity case by $T^{AE}(X)$, this is

$$T^{AE}(X) = \tau^e\Big(\hat{X} - \hat{X}_B\, e^{\beta_2(X-X_B)}\Big) \tag{12}$$

[14]See e.g. Karatzas & Shreve (1998). See Appendix A.1 for more details.

2.2 Capital structure and management

In this section, we consider a richer setting. On the one hand, staying in the tradition of the optimal capital structure literature, we analyze the claims on the cash flow, where we can distinguish four claimants: equity holders, debt holders, tax authority and bankruptcy costs.
On the other hand, we introduce the management as a separate party, thus forming the basis to analyze principal-agent conflicts in the sequel. The claim that is held by the managers is assumed to consist of a fixed payment as well as a performance-linked compensation, which is given by a part of the cash flow. Thus management can be viewed as having both an equity- and a debt-like claim. Furthermore, we assume that managers derive some non-pecuniary profits from their position.

External equity claim

To determine the equity claim, the cash flow (X) is reduced by the coupon payment to investors (k) as well as the fixed compensation payments to the management, which we denote by κ.[15] And after effective taxation, the entire equity claim receives $(1-\tau^e)(X-k-\kappa)$.
Furthermore, the entire equity claim is divided up between external equity holders and managers. If we denote the fraction of the equity claim that goes to the management by γ, the claim for external equity holders is $(1-\tau^e)(1-\gamma)(X-(k+\kappa))$, and the corresponding ODE for the external equity claim (E) is of the form: $\frac{1}{2}\sigma^2 E_{(xx)} + \mu E_{(x)} - rE + (1-\tau^e)(1-\gamma)(X-(k+\kappa)) = 0$.
In this section, we will consider the endogenous bankruptcy case, which means that we assume that equity holders can optimally choose the cash flow level where they will declare bankruptcy. We will denote this endogenous bankruptcy threshold by X_b. [16]
The boundary condition for the equity claim is therefore $E(X_b) = 0$, i.e. equity is worthless at the bankruptcy threshold.[17] Again, with $A_1 = 0$, we can

[15] In this context, one should not call X the EBIT, but rather EBITC - earnings before interest, taxes, and compensation payments. However, with this remark in mind we keep on calling X the EBIT or simply the 'cash flow', although this is admittedly not completely correct.

[16] The determination of X_b is deferred to a later section.

[17] While being a standard assumption, in practice, it can be observed that $E(X_b) > 0$, especially for firms declaring bankruptcy under the Chapter 11 provisions of the US bankruptcy law (see Milgrom & Roberts (1992), p.503). For a detailed analysis of the impact of differ-

determine A_2 to be $-(1-\tau^e)(1-\gamma)\big(\hat{X}_b - \frac{(k+\kappa)}{r}\big)e^{-\beta_2 X_b}$ and so the solution for equity is

$$E(X) = (1-\tau^e)(1-\gamma)\Big(\big(\hat{X} - \frac{(k+\kappa)}{r}\big) - \big(\hat{X}_b - \frac{(k+\kappa)}{r}\big)e^{\beta_2(X-X_b)}\Big) \quad (13)$$

where, again, we defined $\hat{X}_b \equiv \frac{X_b}{r} + \frac{\mu}{r^2}$.

Debt claim

The debt holders are entitled to the fixed coupon payment (k) as long as the firm is solvent. So, after accounting for personal taxes, investors receive $(1-\tau^k)\,k$. Therefore, the ODE for debt is $\frac{1}{2}\sigma^2 D_{(xx)} + \mu D_{(x)} - rD + (1-\tau^k)k = 0$ and the particular part of the solution is just $(1-\tau^k)\frac{k}{r}$.

In the case of bankruptcy, debt holders receive (part of) the remaining firm value according to their superior priority. More precisely, we assume, that the bankruptcy value of the firm is divided up between debt holders, tax authority and bankruptcy costs. Furthermore, we consider the bankruptcy value of the firm to be the value of an all equity claim on the cash flow at the bankruptcy level X_b. This is motivated by the fact, that the former debt holders are effectively the new owners of the EBIT-generating machine, and can thus be viewed as having an all equity claim on this cash flow.[18] Clearly, this claim carries the potential to optimally lever it again, and the new owners may decide to do so. However, since this touches on questions of reorganization which are not central to our analysis at the present stage, we will not pursue this issue further on.

Taking the bankruptcy value to be an all equity claim on the cash flow level X_b, and introducing α as the fraction, that is lost to bankruptcy costs, the value that is left to be distributed is $(1-\alpha)AE(X_b)$, which is then distributed between former debt holders and the tax authority.

For the debt claim we get the following boundary condition at the bankruptcy level: $D(X_b) = (1-\tau^e)(1-\alpha)AE(X_b)$.

From this, and from the fact that $A_1 = 0$, we can determine A_2 to be $\big(D(X_b) - (1-\tau^k)\frac{k}{r}\big)e^{-\beta_2 X_b}$, which gives the following solution for the debt claim

$$D(X) = \big(D(X_b) - (1-\tau^k)\frac{k}{r}\big)e^{\beta_2(X-X_b)} + (1-\tau^k)\frac{k}{r} \quad (14)$$

ent bankruptcy procedures on the valuation of corporate securities, see François & Morellec (2004).

[18]This is similar to the reasoning by Mello & Parsons (1992), or more recently Morellec (2004)

Management claim

Most of the optimal capital structure literature does not distinguish between management and equity holders. It is assumed that managers act in the best interest of equity holders, or that equity holders are able to enforce the management to do so. Here, we consider managers to be separate stakeholders, who can act in their own interest, which may not coincide with those of the other parties involved. In particular, we assume that managers receive a fixed compensation payment κ and a fraction γ of the cash flow. Furthermore, we assume managers to derive non-pecuniary benefits from their position. The consideration of non-pecuniary benefits (sometimes also called 'perquisites') dates at least back to Jensen & Meckling (1976), who subsumed such things as reputation, personal satisfaction or extraordinary perquisites (corporate jet, plush office, etc.) under this notion. In their model and subsequent work,[19] non-pecuniary benefits are important factors, that make managers objectives different from investors goals.

Here, we will denote these benefits by η. So, in sum the management claim is $(1-\tau^e)\gamma(X-(k+\kappa))+(1-\tau^k)\kappa+\eta$.

Solving the corresponding ODE, gives the general solution $A_1 e^{\beta_1 X}+A_2 e^{\beta_2 X}+(1-\tau^e)\gamma\hat{X}-\frac{(1-\tau^e)\gamma(k+\kappa)-(1-\tau^k)\kappa-\eta}{r}$.

At the bankruptcy threshold, managers not only loose their compensation payments but also their non-pecuniary benefits associated with their position. This gives us the boundary condition $M(X_b)=0$.[20] We deduce $A_2=\big(-(1-\tau^e)\gamma\hat{X}_b+\hat{k}\big)e^{-\beta_2 X_b}$. The solution for the management claim is then found to be

$$M(X)=\Big(-(1-\tau^e)\gamma\hat{X}_b+\hat{k}\Big)e^{\beta_2(X-X_b)}+(1-\tau^e)\gamma\hat{X}-\hat{k} \tag{15}$$

where $\hat{k}$ is defined as $\hat{k}\equiv\frac{(1-\tau^e)\gamma(k+\kappa)-(1-\tau^k)\kappa-\eta}{r}$.

[19] See e.g. Harris & Raviv (1991), Myers (2003), Bohlin (1997), or Hart (2001) for overview articles.

[20] Alternatively, we could assume that managers not only loose their benefits, but also suffer a punishment (in the spirit of Ross (1977), who introduces a 'bankruptcy penalty' for managers (see Ross (1977), p.28)), thus giving a negative boundary value. However, at least in a qualitative sense the same effect can be achieved by increasing the benefits that are lost in case of bankruptcy. So, while an explicit punishment is likely to be a slightly more realistic way of modelling, we do not pursue this here for reasons of parsimony.

Tax claim

The claim of the government on the EBIT flow consists of three parts. First, they receive $\tau^e(1-\gamma)\big(X-(k+\kappa)\big)$ from the effective taxation of equity holders. The second part is the tax income from debt holders which is $\tau^k\, k$, and the third part is given by the taxation of managers, which have to pay $\tau^e\gamma\big(X-(k+\kappa)\big)+\tau^k\kappa$.

In sum, this provides the tax claim (T) with $\tau^e X + (\tau^k - \tau^e)(k+\kappa)$ and we get the following ODE: $\frac{1}{2}\sigma^2 T_{(xx)} + \mu T_{(x)} - rT + \tau^e X + (\tau^k - \tau^e)(k+\kappa) = 0$

The boundary condition at the bankruptcy threshold according to the same reasoning as above is $T(X_b) = \tau^e(1-\alpha)AE(X_b)$, from which we get $A_2 = \big(T(X_b) - \tau^e\hat{X}_b - (\tau^k-\tau^e)\frac{(k+\kappa)}{r}\big)e^{-\beta_2 X_b}$, and the solution

$$\begin{aligned} T(X) = \Big(T(X_b) - \tau^e\hat{X}_b - (\tau^k-\tau^e)\frac{(k+\kappa)}{r}\Big)e^{\beta_2(X-X_b)} + \\ \tau^e\hat{X} + (\tau^k-\tau^e)\frac{(k+\kappa)}{r} \end{aligned} \tag{16}$$

Bankruptcy costs

Finally, we have a fraction of the remaining firm value, that is lost to bankruptcy costs. This claim vanishes for $X \gg X_b$, and is worth $\alpha AE(X_b)$ at the bankruptcy threshold. The solution to this claim is therefore easily found to be

$$BC(X) = \alpha AE(X_b)e^{\beta_2(X-X_b)} \tag{17}$$

As a consistency check we can sum up the five claims discussed above (i.e. eqs. (13), (14), (15), (16) and (17)). This sum has to equal the value of a claim on the entire cash flow (i.e. $m=1$) and the non-pecuniary benefits (η), together with the boundary condition $V(X_b) = AE(X_b)$. Indeed, we get from both calculations the solution

$$\Big(AE(X_b) - \hat{X}_b - \frac{\eta}{r}\Big)\,e^{\beta_2(X-X_b)} + \big(\hat{X} + \frac{\eta}{r}\big)$$

A detailed summary of the valuation of the various claims can be found in appendix A.2.

3 Capital structure decision under complete information

3.1 Optimal bankruptcy threshold

In this section we consider the determination of the optimal bankruptcy threshold of equity holders. The factors influencing their choice are on the one hand, the fact that the total firm value is maximized by a threshold as low as possible. On the other hand, the equity value must always be positive and equity holders won't accept a situation, where an increase in the cash flow level will decrease the value of their claim. Therefore, the lowest possible, i.e. the optimal bankruptcy threshold is found by determining the minimum of the equity value function that is non-negative. This is also known as the *smooth-pasting* condition, and ensures that the value of equity is maximized.[21]

We have to distinguish between the all equity case and the levered equity case. In the all equity case, the optimality condition is

$$\frac{\partial AE(X)}{\partial X}|_{X=X_B} = 0 \qquad \text{subject to } AE(X) \geq 0$$

while for the levered equity case, we have

$$\frac{\partial E(X)}{\partial X}|_{X=X_b} = 0 \qquad \text{subject to } E(X) \geq 0$$

Note the difference in the shut down or bankruptcy level.
Solving the above conditions (i.e. differentiating eqs. (11) and (13), evaluating at X_B and X_b respectively, setting them to zero, and solving for X_B^* and X_b^*) yields for the optimal bankruptcy thresholds[22]

$$X_B^* = \frac{1}{\beta_2} - \frac{\mu}{r} \tag{18}$$

$$X_b^* = \frac{1}{\beta_2} - \frac{\mu}{r} + k + \kappa \tag{19}$$

It is immediately obvious that $X_B^* < X_b^*$, which is reasonable from an economic interpretation. The bankruptcy level in the levered case is always greater than the shut down level in the all equity case. Note also, that $X_B^* < 0$, which indicates that equity holders are willing to keep the firm going even in the case of a negative EBIT level, but will choose to shut down the business if the EBIT is sufficiently negative.

[21] see e.g. Leland (1994), p.1222

[22] See appendix A.3 for details.

Another interesting question in this context is where managers would set *their* optimal bankruptcy level (X_{bm}), if they had the corresponding power of decision. Again, we have to find $\frac{\partial M(X)}{\partial X}|_{X=X_{bm}} = 0$ and solve for X^*_{bm}, which turns out to be

$$X^*_{bm} = \frac{1}{\beta_2} - \frac{\mu}{r} + k + a_\kappa\,\kappa + a_\eta\,\eta \tag{20}$$

where we have introduced the coefficients

$$a_\kappa = \frac{\gamma(1-\tau^e) - (1-\tau^k)}{\gamma(1-\tau^e)} \qquad a_\eta = -\frac{1}{\gamma(1-\tau^e)}$$

While it is obvious that $a_\eta < 0$, it depends on the actual parameter values which sign a_κ will assume. However, since we assume that $\tau^e > \tau^k$, i.e. a tax advantage to fixed payments, we get $a_\kappa < 0$.

If we consider for a moment the case where managers do not derive non-pecuniary benefits (i.e. $\eta = 0$), we see that X^*_{bm} is always smaller than X^*_b. Furthermore, the smaller the equity part of the management compensation, the lower their X^*_{bm}. Besides the fixed payment (κ), the non-pecuniary benefits (η) also work towards lowering the optimal bankruptcy threshold. In an extreme setting, with a very small fixed pay, a small equity part and very high non-pecuniary benefit, we might even get the case, that X^*_{bm} is lower than the all-equity shut down level X^*_B.

This gives us a first interesting result. The mere fact that the management claim consists of three parts - namely a fixed payment, an equity-like part and a non-pecuniary benefit - implies a divergence between the goals of external equity holders and managers concerning the optimal liquidation strategy. Managers would in general choose to file for bankruptcy at a cash flow level that is too low compared to the optimal threshold for external equity holders, thus creating a principal-agent problem. In the case of complete information this might not pose a problem for equity holders, since given their perfect knowledge about the true cash flow level, they can always optimally liquidate the firm. However, the divergence of optimal liquidation policies already sets the scene for the principal-agent problems that will occur in the incomplete information case.

3.2 Optimal coupon without entrenched managers

So far, we have taken the coupon k as being exogenously determined. However, it is obvious that the coupon level needs to be treated as an endogenous variable

that determines the optimal capital structure.
We follow the common view (see e.g. Zwiebel (1996) or Novaes (2003)), that leverage decisions lie within the field of responsibility of managers.
First, to establish the reference case, we assume that managers act in the best interest of shareholder, i.e. their objective is to maximize shareholder wealth, and no agency problem occurs. From this, we will find the optimal coupon level as the solution to the following maximization problem

$$\max_k \big(D(x_t, k, X_b^*(k)) + E(x_t, k, X_b^*(k))\big) \tag{21}$$

The coupon level is chosen such that the equity value plus the proceeds from the debt issue are maximized. The debt and equity values are evaluated at the current EBIT level x_t. Further, they depend on the optimal bankruptcy level ($X_b^*(k)$) which itself depends on the coupon level k as can be seen from equation (19). Plugging in the optimal bankruptcy level of external equity holders is plausible, since in this perfect information case, given any coupon level they can always choose their corresponding optimal bankruptcy threshold.[23]
Unfortunately, after plugging in, differentiating equation (21) and setting it to zero, we cannot explicitly solve for k^*. Therefore, we cannot provide closed-form solutions for the optimal coupon and must resort to numerical analysis.[24]

3.3 Optimal coupon with entrenched managers

If managers cannot be forced to implement the equity value-maximizing coupon level, how would they choose *their* optimal coupon level? The first-best solution for managers would be to choose a coupon level that maximizes their claim. In their optimization problem they have to take account of the fact, that equity holders can observe the coupon level and adjust their bankruptcy level accordingly. So, in principle the optimal coupon level for management would be found by $k^* = \max_k\{M(x_t, k, X_b^*(k))\}$. If we compute the derivative $\frac{\partial M}{\partial k}$, we get

$$\frac{\partial M(x_t, k, X_b^*(k))}{\partial k} = \frac{\gamma(1-\tau^e)}{r}\big(e^{\beta_2(x_t - X_b^*(k))} - 1\big) + \big(\beta_2 \frac{\kappa(1-\tau^k)+\eta}{r}\big)e^{\beta_2(x_t - X_b^*(k))}$$

Note, that since $0 \leq e^{\beta_2(x_t - X_b)} \leq 1$ and $\beta_2 < 0$, both terms are negative so that we always have $M'(k) < 0$ which means that managers always prefer a coupon

[23] Note, that otherwise we had to treat X_b in the optimization as a constant, which in turn would yield a different result.

[24] See appendix A.4 for details.

level as low as possible. This result shows that in the absence of any counteracting forces, manager will try to avoid debt.[25,26] The driving force of this result is in the present context the fact that managers loose their compensation payments in the case of bankruptcy so that they can maximize their claim by minimizing the probability of default, which means taking on less debt.[27]

The argument taken to the extreme would mean that managers do not issue any debt at all. In a related framework, Noe & Rebello (1996) show that the first-best policy for managers is an all-equity financing. But obviously this is unlikely to happen, because equity holders will impose disciplining mechanisms - i.e. corporate government rules and control mechanisms - on managers to keep them from deviating from their desired optimal capital structure. The most convincing disciplining force is probably the threat of loosing their job through dismissal[28] or hostile takeover.[29] With the threat of being fired, the second-best solution for managers is then to implement the coupon level which is optimal for equity holders.

However, for various reasons this threat will often be mitigated. The different forms of mitigation are subsumed under the notion of 'managerial entrenchment', which - by following Berger *et al.* (1997) - may be defined as: "the extent to which managers fail to experience discipline from the full range of corporate governance and control mechanism".[30]

Various forms of entrenchment have been discussed in the literature: A long tenure in office, a low sensitivity of the compensation scheme to performance measures, a weak monitoring (e.g. a small board size, few outside directors, no major stock holders or weak voting powers of equity holders),[31] frictions

[25] This is in line with empirical results obtained by Jung *et al.* (1996) Berger *et al.* (1997), Graham (2000) and de Jong & Veld (2001).

[26] On the contrary, according to the 'control hypothesis' (e.g. Jensen (1986)) managers may choose voluntarily to issue debt because of its credible signal to avoid overinvestment.

[27] Other motives for the avoidance of debt have been put forward, like the protection of undiversified human capital (Fama (1980)), or the dislike of performance pressures (Jensen (1986)). Note, that Grossman & Hart (1982) put forward a different reasoning, by which debt creates an incentive for managers to work more efficiently, and thus to avoid bankruptcy.

[28] Huson *et al.* (2001) find an increased frequency for forced CEO turnover in the period from 1971 to 1994

[29] Note however, that Novaes (2003) argues that the takeover threat may not be enough in general to force managers to implement value-maximizing leverage levels, and depends crucially on the takeover costs.

[30] Berger *et al.* (1997), p.1411

[31] see e.g. Berger *et al.* (1997), de Jong & Veld (2001)

on the market for corporate control[32] (e.g. high costs of dismissal), or specific human capital.[33] Given that the managers are entrenched by one or more of these factors, they can act in their own interest, whereby the extent of the entrenchment determines how far they can deviate from equity holders optimum. Together with the above result, that managers always prefer to avoid debt, the leverage would ceteris paribus always be lower when managers are entrenched. As will be shown in a later section, leverage ratios may decline even without managerial entrenchment (with respect to the leverage decision) if the effects of asymmetric information are considered.

Before introducing incomplete information, the next section discusses numerical results in the complete information case.

3.4 Numerical example - Complete information

For illustrative purposes, we present in this section numerical results. Assume the following base case parameter values: The risk-free interest rate is flat at 5%, i.e. $r = 0.05$. Corporate taxes are $\tau^c = 0.35$, taxes on dividend payments are $\tau^d = 0.2$, which gives an effective tax rate of $\tau^e = 0.48$. Taxes on coupon payments are $\tau^k = 0.4$.[34]

Empirical evidence (see e.g. Givoly & Hayn (2000)) shows that the cash flow variability is broadly of the same order of magnitude than its level, so by assuming an initial cash flow level of $x_0 = 100$, we set $\sigma = 100$. Further, we assume a drift of $\mu = 2$.[35] Thus, we have $\frac{\mu}{r} = 40$ and $\frac{1}{\beta_2} = -296.86$. In following Goldstein *et al.* (2001), we set bankruptcy costs at 5%, i.e. $\alpha = 0.05$.[36]

With respect to parameters influencing the management compensation, we broadly follow results by Core *et al.* (1999), Core *et al.* (2003) and Palia (2001) and set the equity-linked compensation part to $\gamma = 0.02$. Given the initial cash flow level of $x_0 = 100$, we set the fixed pay to $\kappa = 1$ and the non-pecuniary benefits to $\eta = 0.2$

[32] see e.g. Zwiebel (1996)

[33] The incumbent manager disposes special skills by which he can add value to the firm, and which are lost by his dismissal. See Morellec (2004) for a recent model.

[34] see Goldstein *et al.* (2001)

[35] Recall, that μ is the drift under the risk-neutral measure Q.

[36] This is also in line with results by Weiss (1990)

Bankruptcy thresholds

Looking first at the optimal bankruptcy thresholds, we obtain the following values: (i) In the all-equity case, the optimal shut-down level is $X_B^* = -336.86$. (ii) In the levered case, the optimal bankruptcy threshold from the viewpoint of equity holders is $X_b^* = -199.11$, which turns out to be substantially higher. (iii) Also not surprisingly, the optimal bankruptcy threshold for the management is $X_{bm}^* = -276.03$, which - as discussed in the previous section - is lower than X_b^*. With respect to the comparative statics, all three thresholds decrease with an increasing drift value, i.e. : $\frac{\partial X_B^*}{\partial \mu} = \frac{\partial X_b^*}{\partial \mu} = \frac{\partial X_{bm}^*}{\partial \mu} = -\frac{1}{r} - \frac{1}{\beta_2 \sqrt{\mu^2+2r\sigma^2}} < 0$ Similarly, an increase in risk also lowers the thresholds: $\frac{\partial X_B^*}{\partial \sigma} = \frac{\partial X_b^*}{\partial \sigma} = \frac{\partial X_{bm}^*}{\partial \sigma} = -\frac{\sigma}{\sqrt{\mu^2+2r\sigma^2}} < 0$. However, in these derivatives k has been treated as a constant. Now, as it is clear from the preceding section, the optimal coupon k^* itself depends, amongst other things, on μ and σ. So, in general it is a priori not clear if the same signs still hold true. Unfortunately, k^* is not obtainable in closed-form, so we can only present numerical solutions. In the left panel of

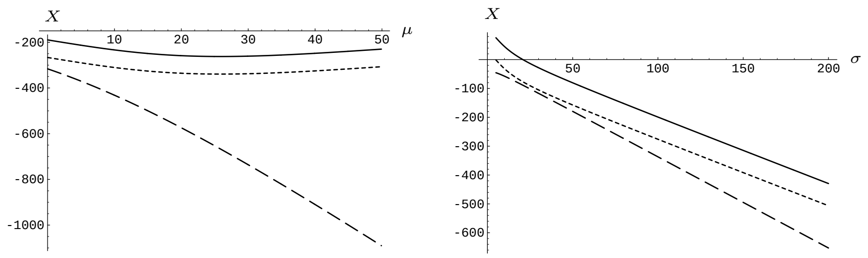

Figure 1: Numerical solutions for $X_b^*(\mu, \sigma)$ (solid line), $X_{bm}^*(\mu, \sigma)$ (dotted line), $X_B^*(\mu, \sigma)$ (dashed line)

figure 1, the thresholds are shown for increasing μ-values. It can be seen, that X_B^* is decreasing, which is obvious from the fact that in the all-equity case there is per definition no coupon payment. On the contrary, for X_{bm}^* and X_b^*, the fact that k^* itself depends on μ and σ changes their behavior. They only decrease marginally and can even increase for high μ-values. In the right panel, thresholds are shown as a function of σ. Although we still can observe different graphs for X_{bm}^* and X_b^*, it does not change the sign, so in this case, increasing the risk unambiguously lowers the three thresholds.

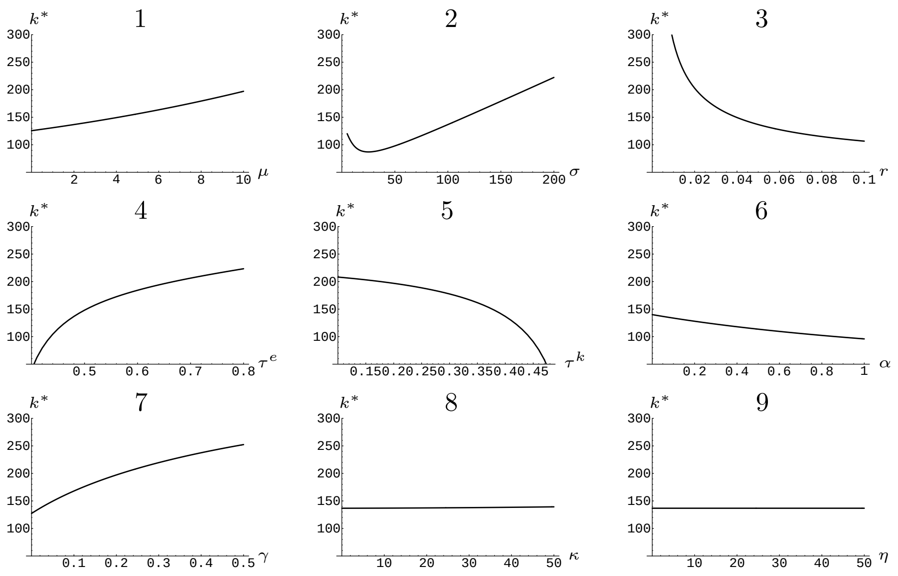

Figure 2: Comparative statics for the optimal coupon level k^*.

Optimal coupon

We already made use of the optimal coupon k^* in the preceding section without having discussed its comparative statics. First, we recall, that the optimal coupon is also depending on the current level of the cash flow X_0. By assuming an initial value of $X_0 = 100$, we obtain in the base case setting $k^* = 136.75$. Varying the different parameters, we obtain comparative static results as shown in figure 2. The upper three panels show the influence of the asset value process parameters and the risk-free rate. The middle three panels show the influence of the tax benefit/bankruptcy cost trade-off, and the lower three panels show the influence of the management parameter. Interpretations are more or less straightforward; with the exception of panel 2 (σ), we see monotonic relationships. We observe significant influences for μ, σ, r and the tax rates,[37] while α, γ and κ do only marginally influence the level of k^* and being independent of η.

[37]Note, that the coupon level depends on the *difference* between the effective taxation and the tax on coupon payments. When the difference goes down to zero there is no tax benefit to debt, and thus no incentive to issue debt.

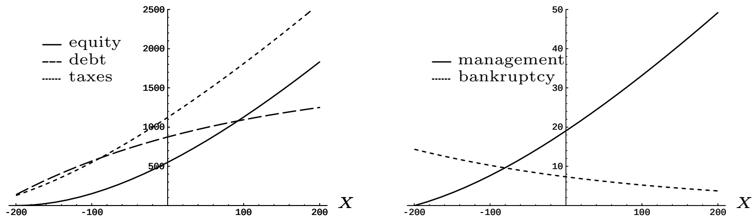

Figure 3: Claims as functions of X

Claims

Valuing the different claims in the base case setting, we arrive at the following results at the initial cash flow level $X_0 = 100$:

$$\begin{aligned} E(X_0) &= 1127.56 \quad D(X_0) = 1093.55 \quad M(X_0) = 33.17 \\ T(X_0) &= 1809.42 \quad BC(X_0) = 5.23 \quad V(X_0) = 4068.93 \end{aligned}$$

Treating the claims as functions of the cash flow, we get graphs as shown in figure 3, where the equity, debt and tax claims are shown in the left panel while the management and bankruptcy claim, which are of a much smaller order of magnitude, are shown in the right panel. We can observe, that the tax and equity claim are convex functions of the state variable, with equity smooth pasting to zero at the boundary X_b^*. The debt claim is concave and approaches $\frac{(1-\tau^k)k}{r}$ as $x \to \infty$.

With regard to the management claim we observe that at the base case parameter values it is a convex function of the cash flow, meaning that ceteris paribus an increase in risk raises the value of the claim. If the management claim is convex or concave depends on the proportion of fixed and variable compensations, i.e. on the values of γ, κ and η. The left panel of figure 4 shows the

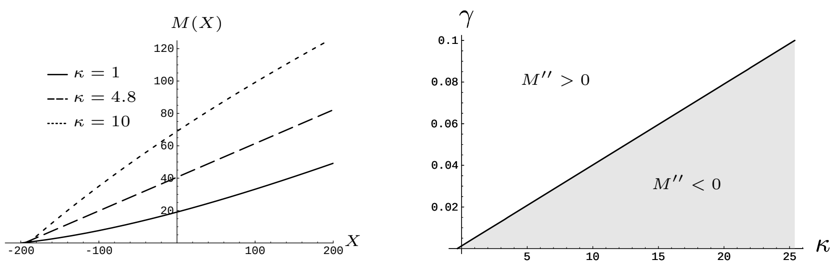

Figure 4: Management claim

management claim in three different value scenarios, giving a convex, nearly linear and concave function. The right panel discerns parameter constellations of γ and κ for which the second derivative ($M''(X_0)$) is positive or negative, respectively. The interpretation is straightforward: A high proportion of fixed compensation payments makes the management claim more debt-like, while a high proportion of variable payments makes the claim more equity-like. However, it is interesting to note that the dividing line (i.e. (γ-κ)-values for which $M'' = 0$) is nearly linear.[38]

Leverage

Given the numerical results for debt and equity, we can analyze the comparative static results for the leverage.

In our base case setting we arrive at a leverage - defined as $L = \frac{D}{D+E}$ - of $L = 0.4923$, i.e. 49.23%.

This is significantly lower than in Leland (1994),[39] but it is well in line with results by Goldstein *et al.* (2001) for their static model. However, empirical results show that leverage levels in practice are still lower, being around 30%.[40] Goldstein *et al.* (2001) demonstrate that lower leverage levels can be obtained when the model is extended to a dynamic capital structure setup, while Morellec (2004) tries to explain low leverage levels through the empire building desire of managers. As we will argue in a later section, low leverage levels can also be the result of an informational asymmetry between managers and investors.

Figure 5 shows comparative static results for the leverage.

A higher drift rate increases the debt capacity and thus increases the leverage, which is more pronounced for lower risk levels. This is also reflected in the upper middle panel, where an increase in risk lowers the leverage (plotted for different levels of the bankruptcy cost). The dependence on the risk free rate is partly ambiguous and depends on the drift rate, but overall for reasonable interest rate levels, the influence on the leverage is rather small. This is also partly true for the bankruptcy costs, where only unreasonable high α-values do significantly change the leverage. The most important influence on the leverage level comes from the tax advantage. Either if τ^e is decreased to approach

[38] $M'''(X_0) \approx 0$

[39] An important criticism on the Leland (1994) model was that it could explain observed leverage levels only by assuming unreasonably high bankruptcy costs.

[40] See e.g. Eom *et al.* (2004)

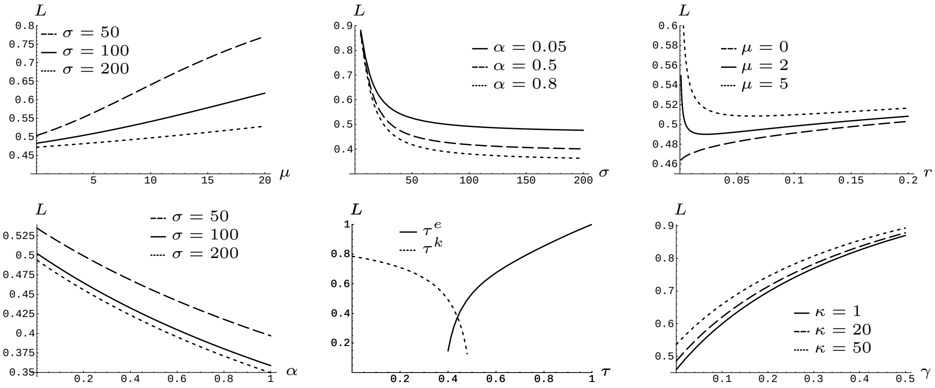

Figure 5: Comparative statics for the optimal leverage L.

$\tau^k = 0.4$, or if τ^k increases to the level of $\tau^e = 0.48$ the leverage declines rapidly. The last parameter shown is γ (lower right panel), and it can be seen that higher γ-values increase the leverage.[41] A higher γ means that a higher share of the cash flow is going to managers. This leaves the external equity holders with a smaller part of the cash flow, which gives them an incentive to increase the value of their claim by taking on more debt.[42]

Calculating the coupon rate as $K = \frac{k^*}{V(X_0)}$, we arrive at 3.36% in our base case scenario. This is broadly consistent with results by Goldstein *et al.* (2001) as well as with empirical results.[43]

As a further numerical plausibility check we calculate the recovery rate, which is given by: $R = \frac{D(X_b^*)}{D(X_0)}$. We find $R = 0.1293$, or 12.93% which is low compared to empirical findings where recovery ratios for unsecured senior debt varies around 45%.[44] However, as has been noted in the previous section, we

[41] This is in line with empirical results from Mehran (1992) who finds a positive relationship between leverage and the management compensation tied to performance.

[42] We do not consider the case when managers own more than 50% of the firm, since otherwise managers would have the majority of the voting power and would be able to implement *their* optimal leverage levels.

[43] See e.g. Eom *et al.* (2004)

[44] Altman *et al.* (2003) find a mean recovery rate for senior unsecured bonds of 42.3%. Acharya *et al.* (2004) and Renault & Scaillet (2004) find a mean of 41.9% and 46.7% respectively, but also show that recovery rates vary significantly in different industries. They find mean recovery rates of 27.3% and 24.7% respectively for the Telecom industry, while in Utilities recoveries are on average around 68%.

assumed the asset value at the bankruptcy threshold to be the value of an all equity claim on the remaining cash flow generator. In fact, this all equity claim carries the potential to optimally lever it again, and to be more in line with a 'realistic' economic scenario, we would have to calculate the recovery rate as the ratio of a restructured (i.e. optimally levered) claim at the bankruptcy threshold to the initial debt value. Taking $X_b^* = -199.11$ as the new initial cash flow level X_0' we get a new optimal coupon of $k'^* = 33.34$ and the value of the claim on the restructured assets is $V'(X_b^*) = 539.06$. Compared to the all equity claim at 286.44 this is an increase by the factor of 1.88 and thus the recovery rate under the assumption of a restructured claim increases to 24.33%.[45] While this is still at the lower boundary for empirical recovery rates, the model produces values within a reasonable order of magnitude.

Another interesting numerical result produced by the model relates to the one year probability of default (PD). In the given framework, this amounts to evaluate the probability that a Brownian motion with drift hits the boundary X_b from above within a one year time horizon. Applying results from first-passage theory, we arrive at a PD of 0.00261 or 0.261% in our base case setting.[46]

A last numerical result relates to equity volatility. Given the cash flow variability which we assumed to be $\sigma = 100$ at $X_0 = 100$ we can calculate the corresponding equity volatility.[47] Clearly, this would imply that all variability in equity prices is uniquely determined by changes in cash flows.
To determine the annual percentage equity standard deviation, we need to calculate $\frac{\sqrt{\mathrm{Var}[E(X_1)]}}{E(X_0)}$ with $\mathrm{Var}[E(X_1)] = \mathrm{E}_0[E(X_1)^2] - \mathrm{E}_0[E(X_1)]^2$. Thereby, the expectations have to be taken with respect to the probability law of X_1, which is given by the joint distribution of X, i.e. Brownian motion with drift, and the first-passage time distribution, i.e.

$$\mathrm{Prob}\{X_t \in \mathrm{d}x,\ \inf\{X_s;\ 0 < s \le t\} \ge X_b\}$$

In the base case scenario percentage equity standard deviation turns out to be

[45] More precisely, this value has been derived by assuming that the restructuring can only take place once, and that the recovery value of the restructured assets are given by an all equity claim. Therefore, by allowing for multiple restructurings, this would create additional recovery value. However, the increase is marginal. (With a restructuring at the restructured bankruptcy threshold the initial recovery rate is 24.89%)

[46] See appendix A.6 for a derivation.

[47] See appendix A.5 for details.

61.24% which is high, but still within a reasonable order of magnitude.

We conclude this section on numerical results by pointing out that while the model produces results that are broadly plausible, the usual caveat applies, and it is not our prime interest to formulate a model that closely fits empirical findings. We are well aware that the model as presented so far still lacks a number of features that are important factors in real-life capital structure decisions. Most notably, we consider a static setting while the possibility of changing the capital structure over time is known to have a significant influence.[48] Another limit is given by the fact that the cash flow *dynamics* are unaffected by either capital structure decisions or managerial discretion.[49]
While these are without doubt important factors, we will focus in the next section on another factor which is a significant determinant of capital structure decisions: the informational asymmetry between managers and investors about the true cash flow level.

4 Capital structure decision under incomplete information

In explaining why the irrelevance hypothesis of Modigliani & Miller (1958) does not hold, the literature has put forward broadly two classes of arguments. The first class is based upon agency costs, while the second focuses on informational asymmetries.[50] In this section, we will contribute to the second class in that we broadly follow the approach taken by Duffie & Lando (2001) who assume that the incomplete information on the part of investors is given by the fact that they only receive noisy signals about the relevant state variable. In forming rational estimates investors are faced with the problem of filtering away this noise. However, in contrast to considering a one-period problem, we provide a continuous-time formulation that fits the valuation framework introduced in the previous section. The solution to continuous-time filtering problems based upon linear Gaussian processes is given by the so-called Kalman-Bucy filter.
The main idea of this section is to assume that investors, i.e. debt holders as

[48] See e.g. Goldstein *et al.* (2001) and references therein

[49] See Leland (1998) or Ericsson (2000)

[50] One could also classify the models into explanations based on moral hazard behavior on the one hand, and adverse selection problems on the other hand. (See Bohlin (1997), p.200)

well as equity holders only receive noisy signals on which they have to rely when making their investment decisions. In particular, owners will have to determine their optimal bankruptcy threshold and their optimal coupon decision upon cash flow *estimates*. These estimates are made by applying the results of Kalman-Bucy filtering.

4.1 Impact on the optimal capital structure decision

To make things concrete, recall that the cash flow dynamics are given by $\mathrm{d}X_t = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$. However, investors only observe noisy signals of X_t. Denoting the observation at time t by O_t they can be assumed to be of the form

$$O_t = X_t + \delta w_t$$

where w_t denotes white noise (independent of W_t), and δ is a scaling parameter. To obtain a tractable stochastic integral representation,[51] we introduce $Z_t = \int_0^t O_s\,\mathrm{d}s$ and get the observation process

$$\mathrm{d}Z_t = X_t\,\mathrm{d}t + \delta\,\mathrm{d}V_t \tag{22}$$

where V_t is Brownian motion (independent of W_t).

The general *filtering problem* is to find the best estimate of the unobservable system, given the observed signals. In the present model, this means, investors have to form their best estimate of the cash flow which they cannot observe directly. Thereby, the notions of 'observations' and 'best estimate' need a precise interpretation. Denoting the estimate of X_t by ξ_t, saying that ξ_t is based on the observations $\{Z_t\}$ means that ξ_t is $\mathcal{F}_t$-measurable, where $\mathcal{F}_t$ is the σ-algebra generated by $\{Z_s, s \leq t\}$. The best estimate is defined to be the optimal mean square estimate, which satisfies $\mathrm{E}[|X_t - \xi_t|^2] = \inf\{\mathrm{E}[|X_t - Y_t|^2]; Y \in \mathcal{K}\}$, where $\mathcal{K}$ is the set of $\mathcal{F}_t$-measurable functions.

Now, a fundamental result of filtering theory[52] establishes that the solution of the above filtering problem is found by calculating the conditional expectation $\mathrm{E}[X_t|\mathcal{F}_t]$. In the case of linear Gaussian processes, the solution is known as the Kalman-Bucy filter.

In our context, it can be shown[53] that the solution $\xi_t = \mathrm{E}[X_t|\mathcal{F}_t]$ of the system

$$\begin{aligned} \mathrm{d}X_t &= \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t \\ \mathrm{d}Z_t &= X_t\,\mathrm{d}t + \delta\,\mathrm{d}V_t \end{aligned}$$

[51] see Øksendal (2000), p.80

[52] see Øksendal (2000), or Liptser & Shiryaev (2001)

[53] See appendix A.7 for the derivation.

satisfies the following stochastic differential equation

$$\mathrm{d}\xi_t = \mu\,\mathrm{d}t + \delta^{-2} S_t\,(\mathrm{d}Z_t - \xi_t\,\mathrm{d}t) \tag{23}$$

where $S_t = \mathrm{E}[(X_t - \xi_t)^2]$ is the mean square error which satisfies

$$\mathrm{d}S_t = (\sigma^2 - \delta^{-2} S_t^2)\,\mathrm{d}t \tag{24}$$

Obviously, for the initial condition $S_0 = \delta\sigma$ the solution to (24) is simply

$$S_t = \delta\sigma \tag{25}$$

Alternatively, for an initial condition $S_0 < \delta\sigma$ or $S_0 > \delta\sigma$, in both cases $S_t \to \delta\sigma$ as $t \to \infty$. So, we will assume that the mean square error started right away in its 'steady state', or - alternatively - that a sufficient amount of time already elapsed so that (25) can be justified.
Note, that by defining $U_t = \int_0^t \frac{\mathrm{d}Z_s - \xi_s \mathrm{d}s}{\delta}$, we can rewrite (23) as

$$\mathrm{d}\xi_t = \mu\,\mathrm{d}t + \delta^{-1} S_t\,\mathrm{d}U_t \tag{26}$$

Thereby, U_t is a standard Wiener process, which is also referred to as *innovation process*.[54]
S_t provides us with a measure of the (expected) noise that remains after the observer have formed their best estimates.
In the present context, this informational noise enters the model at two points (see figure 6).

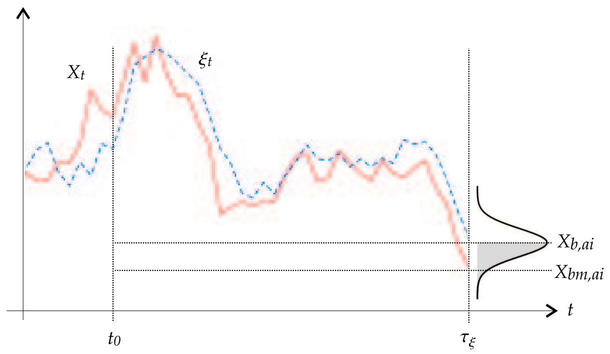

Figure 6: Informational asymmetry

i. Equity holders-owners will force the firm into bankruptcy when the level of their cash flow *estimate* hits their optimal bankruptcy threshold.

[54] see e.g. Liptser & Shiryaev (2001)

ii. At the time of the capital structure decision (t_0), equity holders-owners can determine their desired coupon level only based upon their cash flow *estimates* (ξ).

(ad i) Default event under asymmetric information

As shown in figure 6, the first role played by the informational asymmetry in our model is given by the determination of the default event.
Recall, that in the previous section the default triggering event was defined by the first time the cash flow process hits the bankruptcy threshold. This is plausible in the case when equity holders can perfectly observe the actual cash flow process and force the firm into bankruptcy when they consider this to be optimal. However, under asymmetric information they are only able to decide upon bankruptcy based on their estimations about the cash flows. Therefore, we assume the default triggering event to be the first time when the estimation process ξ_t hits a bankruptcy threshold under asymmetric information, denoted by $X_{b,ai}$. For the time being, $X_{b,ai}$ is undetermined and need not yet be the optimal choice.[55]

Now, rational investors should anticipate that, conditional on the default event having taken place, i.e. the estimate hits the threshold, the true cash flow can be higher or lower than the estimate. If the true cash flow actually turns out to be higher, the default is not justified and we assume that the bankruptcy procedure is stopped and the firm continues to operate as before. On the other hand, the actual cash flow may turn out to be lower than estimated. This presupposes that managers have an incentive to keep quiet about the fact that the true cash flow level already dropped below the bankruptcy threshold, i.e. to delay bankruptcy filing. Indeed, since the optimal bankruptcy threshold for the management is always lower than for equity holders, as discussed in the previous section, they do have this incentive. Ignoring any other factors, and given that equity holders haven't yet claimed bankruptcy, managers will delay bankruptcy filing until the true cash flow hits their optimal bankruptcy threshold ($X^*_{bm,ai}$). With respect to the decision about filing for bankruptcy, the informational asymmetry provides the manager with a form of entrenchment, which they can exploit for their self-interest.
This managerial discretion is harmful to the owners because bankruptcy does not occur at their desired optimal cash flow level, but is delayed to a lower,

[55] Therefore, it is not distinguished by a star.

suboptimal level. It is also harmful to debt holders since in the case when the true cash flow is lower than estimated and bankruptcy is declared the firm is liquidated and the former debt holders receive an all equity claim on the remaining (diminished) cash flows. Clearly, this makes the debt claim more risky and thus less valuable. In valuing the debt claim ex ante, investors have to rationally anticipate the managerial self-interest. I.e. investors have to take into account that managers will not file for bankruptcy at their optimal cash flow level, and that the liquidation value may correspondingly be based on a cash flow level that is lower than $X_{b,ai}$.

If we call the amount by which the cash flow is lower the *informational shortfall*, and denote it by $\mathcal{S} \leq 0$, then we can introduce $X_{\mathcal{S}}$ as the diminished liquidation cash flow given by

$$X_{\mathcal{S}} = X_{b,ai} + \mathcal{S}$$

The shortfall $\mathcal{S}$ itself consists of two parts: (i) As long as the actual cash flow is smaller than $X_{b,ai}$ but higher than $X_{bm,ai}$ which is the bankruptcy threshold under asymmetric information chosen by managers, the management has an incentive to delay bankruptcy filing as discussed above. Therefore, the first part consists of the expectation taken over the interval $(X_{bm,ai}, X_{b,ai})$, which we can write as $\mathrm{E}[(X - X_{b,ai})\, 1_{\{X_{bm,ai}<X<X_{b,ai}\}}]$. (ii) On the other hand, with the remaining probability that the actual cash flow is smaller than $X_{bm,ai}$, i.e. $Prob(X \leq X_{bm,ai})$, investors will get $X_{bm,ai}$, because managers have an endogenous motivation to file for bankruptcy since *their* optimal bankruptcy threshold has been hit. This can be written as $\mathrm{E}[(X_{bm,ai} - X_{b,ai})\, 1_{\{-\infty<X<X_{bm,ai}\}}]$. Taken together, the shortfall $\mathcal{S}$ can be written as

$$\begin{aligned} \mathcal{S} \quad = \quad & \int_{X_{bm,ai}}^{X_{b,ai}} (y - X_{b,ai})\, \phi(y)\, \mathrm{d}y + \\ & \int_{-\infty}^{X_{bm,ai}} (X_{bm,ai} - X_{b,ai})\, \phi(y)\, \mathrm{d}y \end{aligned} \tag{27}$$

where $\phi(\cdot)$ denotes the probability density function of the normal distribution with mean $X_{b,ai}$ and standard deviation $\sqrt{\delta\sigma}$, i.e. $\mathcal{N}(X_{b,ai}, \sqrt{\delta\sigma})$.
The $\mathcal{N}(X_{b,ai}, \sqrt{\delta\sigma})$ distribution is justified since we have $\mathrm{E}[X_t|\xi_t = X_{b,ai}] = X_{b,ai}$. Therefore, conditional on the bankruptcy event, which is given by $\xi_t = X_{b,ai}$, the mean of the true cash flow is given by $X_{b,ai}$. Furthermore, conditional on default, we have a variance of $\mathrm{Var}[X_t] = \mathrm{E}[X_t^2] - \mathrm{E}[X_t]^2 = \mathrm{E}[(X_t - \mathrm{E}[X_t])^2] = \mathrm{E}[(X_t - \xi_t)^2] = S_t = \delta\sigma$. The normal distribution is obvious from the fact that

the estimation process is driven by a Brownian motion.
This was illustrated in figure 6 as the distribution around the bankruptcy event at time τ_ξ.
Note again, that the thresholds $X_{b,ai}$, $X_{bm,ai}$ and $X_{\mathcal{S}}$ are not yet determined; an issue we will turn to in the next section.

(ad ii) Capital structure decision with incomplete information

Although, there is an informational asymmetry between managers and investors with regard to the true cash flow level, we assume no such asymmetry with respect to the capital structure, i.e. the amount of debt issued. Therefore, equity holders can observe the leverage ratio and enforce the management to implement their optimal debt level. In other words, we assume that managers do not enjoy any form of entrenchment with regard to the decision about the coupon level. As discussed in an earlier section, given that managers are entrenched, they will always have an incentive to lower the leverage ratio.
However, even without managerial entrenchment the informational asymmetry plays an important role in determining the optimal coupon to be issued since equity holders can now base their optimization problem only on their noisy estimates about the true cash flows.

In the first, but incomplete attempt to determine the optimal threshold for owners under asymmetric information, we can follow the same reasoning as in the previous section. Equity holders will determine the optimal (endogenous) bankruptcy threshold by maximizing their claim. However, this will differ from the previous X_b^* because it will be based upon the estimate ξ_t. Therefore, we write the smooth-pasting condition as follows

$$\frac{\partial \mathrm{E}[E(\xi)]}{\partial X}|_{X=X_{b,ai}} = 0 \qquad X_{b,ai}^* = \frac{e^{-\frac{1}{2}\beta_2^2\delta\sigma}}{\beta_2} - \frac{\mu}{r} + k + \kappa \tag{28}$$

Instead of differentiating the equity claim based upon the true cash flow, the optimality condition now involves the *expected* equity value based upon the *estimated* cash flow, i.e. the smooth-pasting condition refers to the expected equity value.[56]
This optimality condition assumes that equity holders only decide upon expected values, i.e. that they are risk-neutral with respect to the informational

[56]See appendix A.8 for a derivation.

asymmetry. Extending the optimality condition to account for risk averse behavior would mean introducing an additional utility function and the corresponding risk aversion parameters. With regard to the effects of risk aversion, we expect them to work in the same direction and to even aggravate the results under risk neutrality. Therefore, at this stage of the analysis we assume risk neutrality, and leave the incorporation of risk aversion for future research.

We can see from (28) that the solution to $X^*_{b,ai}$ has the same form as before (see eq. (19)), but that it differs in the numerator of the first term, where we have $e^{-\frac{1}{2}\beta_2^2\delta\sigma}$ instead of 1. Since the exponent is always negative for $\delta > 0$, this term is always smaller than one which implies that $X^*_{b,ai} > X^*_b$, i.e. the optimal bankruptcy threshold under asymmetric information is ceteris paribus always higher than under complete information.[57] However, this inequality holds only under the ceteris paribus condition since, as shown in the next section, the informational asymmetry will impact on the optimal debt level which no longer guarantees the above inequality. In fact, since the optimal coupon under asymmetric information will be smaller, this might actually reverse the inequality.

Contrary to the complete information case, this is however not the end of the story. If equity holders took $X^*_{b,ai}$ to be their optimal threshold, they would ignore the important reasoning from the previous section, namely that managers have an incentive to delay the bankruptcy filing which means that on average, bankruptcy filing occurs too late for investors. Therefore, let us call $X^*_{b,ai}$ the *naive* optimal threshold. In determining their 'sophisticated' optimal bankruptcy threshold, equity holders have to take the managerial self-interest and entrenchment into account.

As discussed above, they know that on average the bankruptcy threshold is diminished by the shortfall $\mathcal{S}$ so that the actual bankruptcy threshold is given by $X_{\mathcal{S}} = X_{b,ai} + \mathcal{S}$. So, they have to choose $X_{b,ai}$ such that $X_{\mathcal{S}}$ equals $X^*_{b,ai}$, i.e. such that the diminished threshold equals their naive optimal threshold. This implies that the sophisticated threshold, which we will denote by $X^{**}_{b,ai}$, should be increased by the shortfall $\mathcal{S}$, i.e.

$$X^{**}_{b,ai} = X^*_{b,ai} - \mathcal{S} = \frac{e^{-\frac{1}{2}\beta_2^2\delta\sigma}}{\beta_2} - \frac{\mu}{r} + k + \kappa - \mathcal{S} \tag{29}$$

For the management the optimal bankruptcy threshold is still given by (20), since they have complete information, i.e. they still can determine their optimal threshold based upon the true cash flow. However, given that the optimal

[57]Note, that for $\delta = 0$ (no noise), $e^{-\frac{1}{2}\beta_2^2\delta\sigma} = 1$ and we have the same solution as in (19)

coupon issued is affected by the asymmetric information as discussed in the next section, we obtain $X^*_{bm,ai} = X^*_{bm}(k^*_{ai})$.
In going back to equation (27), and after plugging in the optimal solutions for $X_{b,ai}$ and $X_{bm,ai}$, we see that $\mathcal{S}$ appears on both sides of the equation, but due to the fact, that the integral representation has no closed-form solution we cannot analytically solve for $\mathcal{S}$. However, numerical solutions can be obtained and will be presented in the next section.

Given the optimal threshold which, as before, can be interpreted as being a function of the coupon level: $X^{**}_{b,ai}(k)$, equity holders will maximize the sum of the ex post equity claim and the proceeds of the debt issue, i.e. if we denote the optimal coupon issued under asymmetric information by k^*_{ai}, the optimization problem is given by

$$k^*_{ai} = \max_k \left(\mathrm{E}[D(\xi_t, k, X^{**}_{b,ai}(k)) + E(\xi_t, k, X^{**}_{b,ai}(k))] \right) \tag{30}$$

Note, that in (30) the claims are now evaluated at the current estimate of the cash flow ξ_t. Furthermore, since the estimates are known to be random variables, equity holders care only about the *expected* values of the claims.[58]
As in the previous section, the solution to this optimization problem has no closed form, and we can only provide numerical examples in a later section.[59]

4.2 Numerical example - Asymmetric information

Bankruptcy thresholds, Informational shortfall and Optimal coupon

With the same base case parameters as in the previous section on complete information, we present numerical solutions for the bankruptcy thresholds $X^{**}_{bm,ai}$ and $X^*_{b,ai}$, for the informational shortfall $\mathcal{S}$ and for the optimal coupon k^*_{ai}. Thereby, three different scenarios for the volatility of the underlying process (σ) are plotted: Solid line: $\sigma = 100$, dotted line: $\sigma = 50$, dashed line: $\sigma = 150$. (see figure 7)
In the upper left panel of figure 7 the dependence of the informational shortfall $\mathcal{S}$ on the amount of noise, represented by δ, is shown. As expected, the higher the informational noise, the higher is the possibility for managers to delay bankruptcy filing, and thus the higher (in absolute terms) is our measure for

[58]The same remark made above with respect to risk-neutrality applies again.
[59]See appendix A.9 for more details.

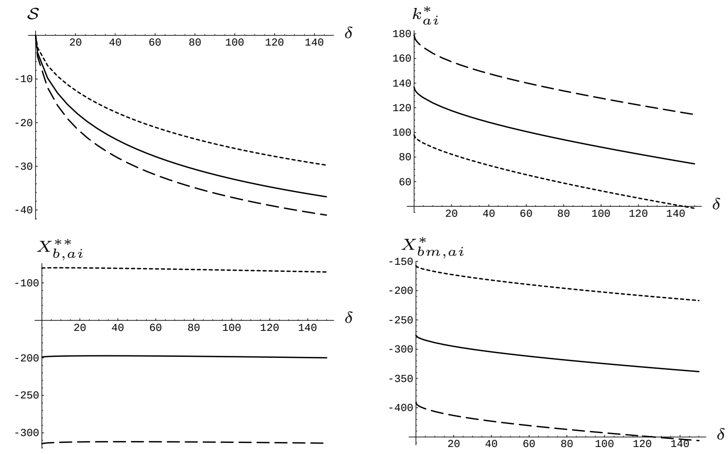

Figure 7: Bankruptcy thresholds, informational shortfall and optimal coupon under asymmetric information. (solid line: $\sigma = 100$, dotted line: $\sigma = 50$, dashed line: $\sigma = 150$)

this. Comparing the three graphs, it can be seen that the higher the volatility of the underlying cash flow process, the higher is c.p. the shortfall. Obviously, for $\delta = 0$, we have the complete information case, and thus the shortfall is zero. Taking $\delta = 50$ as our base case value for the noise, we obtain for the shortfall $\mathcal{S} = -25.9$.

The influence of noise on the optimal coupon k^*_{ai} is plotted in the upper right panel, and it can be observed that we have a decreasing relationship. Higher noise means that equity holders will want to issue less debt. Comparing the three graphs, we recognize the relationship already discussed in figure 2 that higher volatility increases the debt level. For $\delta = 50$ we find a coupon of $k^*_{ai} = 104.32$

In the lower left panel the influence of noise on the optimal bankruptcy threshold for owners is displayed, and all three graphs appear nearly constant at significantly different levels. In the base case scenario we find $X^{**}_{b,ai} = -197.33$. The apparent independence can be explained by the fact that the threshold is among others influenced by the shortfall $\mathcal{S}$ and the coupon k^*_{ai} which becomes obvious from equation (29). Since the two factors have the same dependence on noise, but enter with different signs in (29), they nearly cancel out each other. Given a fixed coupon, we have the unambiguous result, that noise always increases the bankruptcy threshold.[60]

The last panel (bottom-right) shows the optimal bankruptcy threshold chosen by managers, and we see a decreasing relationship. For higher noise levels, managers have a lower optimal threshold. The base case value is: $X^*_{bm,ai} = -308.46$ For all three parameters it can be verified, that for $\delta = 0$, we get the results already obtained in the numerical examples under complete information.

Valuation of the claims under asymmetric information

In valuing the different claims under asymmetric information, we now have to take into account that essentially three parameters have changed: (1) The optimal coupon issued, is now k^*_{ai}. (2) The endogenous bankruptcy threshold of the equity holders is $X^{**}_{b,ai}$. (3) The liquidation value is given by an all equity claim on the cash flow at the above bankruptcy threshold: $AE(X^{**}_{b,ai})$.

These changes have to be taken into account in equations (13) - (17).

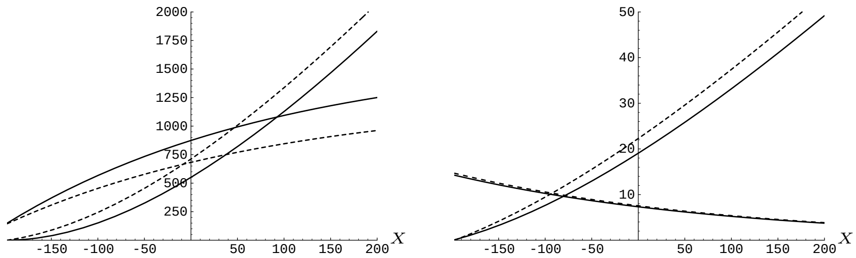

Figure 8: Valuation of the claims under asymmetric information (dotted line; $\delta = 50$) and under complete information (solid line).

The resulting graphs for the debt, equity, management and bankruptcy claim are shown in figure 8 as the dotted lines, where a noise level of $\delta = 50$ is chosen. The solid lines are the graphs for the corresponding claims under complete information.

The left panel of figure 8 shows the equity (convex) and debt claim (concave graph). Although one should be careful in comparing the claims under complete and incomplete information, since both are under the given conditions optima, it can be observed that debt looses value, while the equity claim increases.

[60] This can be seen from equation (29): A higher δ lowers the numerator in the first term, which is negative, and lowers $\mathcal{S}$ which enters with a minus sign.

This might be attributable to the fact that, although owners do suffer from the informational asymmetry as well, they are still able to optimally choose the bankruptcy threshold and the amount of debt to be issued.
In the right panel the graphs for the management and bankruptcy claims are plotted, and again, applying the same caution, we can observe that the management claim increases in value which was to be expected since they dispose of the complete information about the true cash flow.

Leverage ratio under asymmetric information

For our purposes, the most interesting influence of the informational asymmetry may be given by the change in the leverage ratio since this documents in how far the capital structure decision is dependent on informational asymmetries between management and investors. Figure 9 shows numerical results for the dependence of the leverage ratio (L) on the amount of noise (δ), displayed again for three different scenarios for the volatility of the underlying cash flow process: Solid line: $\sigma = 100$, dotted line: $\sigma = 50$, dashed line: $\sigma = 150$. First, observe that with no noise ($\delta = 0$) we obtain the leverage ratio already

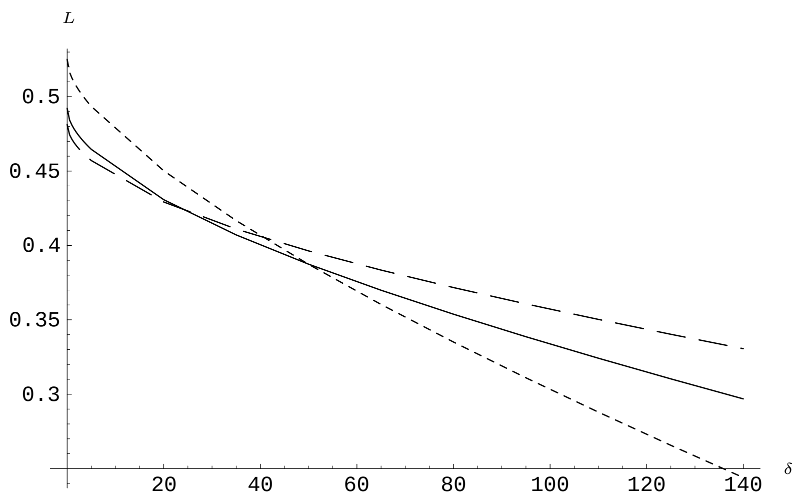

Figure 9: Leverage ratio under asymmetric information (Solid line: $\sigma = 100$, dotted line: $\sigma = 50$, dashed line: $\sigma = 150$).

known from the previous section where we treated the complete information case, i.e. $L = 0.492$. Furthermore, again without noise, the leverage decreases (increases) for a higher (lower) volatility of the cash flow process, as discussed and shown in figure 5.

L	$\delta = 0$	$\delta = 20$	$\delta = 50$	$\delta = 80$	$\delta = 120$
$\sigma = 100$	0.492	0.4307	0.3873	0.3537	0.3148
$\sigma = 50$	0.5251	0.4501	0.3872	0.335	0.2729
$\sigma = 150$	0.4813	0.4292	0.3963	0.3716	0.3435

Table 1: Numerical leverage ratios.

Now, if the noise increases, we can observe from figure 9 that the leverage ratio starts to decline rapidly. Table 1 presents leverage ratios for selected noise levels.

The decline in the leverage ratio is significantly more pronounced for a lower volatility of the cash flow process. For our base case scenario (i.e. $\sigma = 100$, $\delta = 50$) we have a decline in the leverage ratio from 49.2% down to 38.7% which is a decrease of roughly 20%. For a volatility of $\sigma = 50$ and high noise levels ($\delta = 120$) we can even observe that the leverage ratio drops by nearly 50%!

Thus, we conclude that noise can significantly impact on the optimal leverage ratio, and decrease it substantially.

As mentioned earlier, empirical leverage ratios vary on average around 30%.[61] The incorporation of informational asymmetry in our setup is able to produce results consistent with these empirical findings.

Another interesting finding is that the impact of a change in the cash flow volatility on the leverage ratio is reversed for high noise levels. While a negative relation can be observed for noise levels up to around $\delta = 50$, we get the reverse influence beyond this level. This is further illustrated in the left panel of figure 10 where the dependence of L on σ is plotted for tree different noise levels (Solid line: $\delta = 50$, dotted line: $\delta = 20$, dashed line: $\delta = 120$).

The right panel of figure 10 shows the dependence of the leverage ratio on the initial cash flow. The optimal coupon decision, and therefore the capital structure decision depends on the initial cash flow level which we assumed to be $X_0 = 100$ in the complete information case. In the incomplete information case the coupon and capital structure decision made by owners is now based upon their estimate about the cash flow ξ_0. With respect to the previous results in this section we assumed that equity holders have an initial estimate of $\xi_0 = 100$

[61] See e.g. Eom *et al.* (2004). However, it has to be stressed, that leverage ratios can heavily differ among individual corporations.

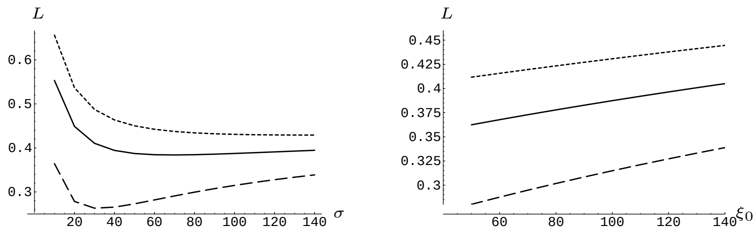

Figure 10: Leverage ratio under asymmetric information - Dependence on σ (left panel) and ξ_0 (right panel). (Solid line: $\delta = 50$, dotted line: $\delta = 20$, dashed line: $\delta = 120$).

(Clearly, without knowing that their estimate happens to be the true value). This assumption was made to make the results under asymmetric information comparable to complete information results.

The dependence shown in the right panel is straight-forward: For any noise level, the higher the initial estimate with respect to the true cash flow level the higher will be ceteris paribus the corresponding leverage ratio, whereby this relationship is more pronounced for high noise levels.

L	$\delta = 0$	$\delta = 20$	$\delta = 50$	$\delta = 80$	$\delta = 120$
recovery rate	0.1293	0.1531	0.1714	0.1873	0.2086
coupon rate	0.0336	0.0289	0.0256	0.0231	0.0202

Table 2: Numerical recovery and coupon rates.

Looking at the impact of noise on recovery and coupon rates (see Table 2), we observe that the recovery rates, defined as $R_{ai} = \frac{D(X^{**}_{b,ai})}{D(X_0)}$, increases which is due to the fact that while the debt value at the bankruptcy threshold is only slightly affected, it is significantly decreased at the current cash flow level (compare figure 8). Thus, the denominator decreases, making the ratio higher. Compared to empirical recovery rates,[62] the model with noise produces more realistic values.

With respect to the coupon rate ($K_{ai} = \frac{k^*_{ai}}{V(X_0)}$) it can be seen from table 2 that it decreases with higher noise which was to be expected from the fact that the optimal coupon decreases under noise (compare figure 7).

[62]See the references cited in footnote 44.

5 Conclusion

This paper deals with the impact of asymmetric information on the optimal capital structure decision. This is done within a framework similar to those proposed by Goldstein *et al.* (2001), Dangl & Zechner (2004) or Christensen *et al.* (2000), who take the state variable to be the EBIT value of a firm. In these papers, the cash flow dynamics are modelled as Geometric Brownian motion, which implies that cash flows never become negative. In contrast, in our setup, we assume the cash flow to follow an Arithmetic Brownian motion, which allows for negative values. In fact, as the numerical example shows, the endogenously obtained optimal bankruptcy threshold of equity holders will typically be negative, i.e. owners will tolerate negative cash flows as long as growth perspectives can outweigh the losses.

We further extend the above mentioned models by introducing managers as having a separate claim on the cash flow. By assuming that their claim consists of a performance-linked compensation and a fixed pay as well as non-pecuniary benefits it is shown that the optimal bankruptcy threshold chosen by managers is always lower than that chosen by equity holders. For the optimal coupon chosen by managers we find that they would always prefer a coupon as low as possible. This tendency to avoid debt is consistent with empirical findings.[63] The discretionary freedom to follow self-interested goals depends on the degree of entrenchment with which managers are endowed.

While being instructive, the introduction of a management claim was not a goal per se, but served as prerequisite to model informational asymmetries between managers and investors. Equity holders as well as debt holders were assumed to have only incomplete information with respect to the underlying cash flow process. They receive noisy signals about the EBIT and in rationally processing this information, they form their best estimates through Bayesian updating. Within the continuous-time framework this is done by applying Kalman-Bucy filtering results.

Thereby, the informational asymmetry enters the model at two points. On the one hand, investors must take into account that their estimate may overestimate the actual cash flow which may actually be already below their optimal bankruptcy threshold. This follows from the fact that self-interested managers do have an incentive to delay bankruptcy filing as long as their own optimal

[63] See the references in footnote 25.

threshold hasn't been hit. With respect to the decision to file for bankruptcy, the informational asymmetry does provide managers with a form of entrenchment. On the other hand, at the time of deciding over the optimal coupon, equity holders can base their decision only on their cash flow *estimates*. Therefore, their optimization problem involves expected values instead of realized values.

Taken together, the informational asymmetry works towards lowering the coupon to be issued, while keeping the endogenous bankruptcy threshold of owners nearly constant. More interestingly, it is shown that the leverage ratio can decline substantially if the informational asymmetry (i.e. the noise component) is increased. The decline being even more pronounced for a low volatility in the underlying cash flow process.

It has to be stressed, that this result follows even without assuming that managers do enjoy any entrenchment with respect to the coupon decision.[64] If managers were endowed with additional entrenchment, leverage ratios should be even lower.

Finally, some remarks on the limits of the model are in order. The capital structure decision considered here is a static one, i.e. we haven't considered the impact of the possibility to change the leverage in the future. Results by dynamic capital structure models indicate that this additional option works towards lowering the initial leverage ratio. A second limit concerns the possibility of *asset substitution*. Managers in our model cannot exert any influence on the dynamics of the underlying cash flow process. And as a last limit one should be aware that we haven't considered the impact of bond covenants or more generally of exogenously determined bankruptcy thresholds. These three issues are the subject of further research.

However, subject to the limits just mentioned, the model shows that, even without an explicit entrenchment of managers, the informational asymmetry can lead to substantially lower leverage ratios.

[64]This means, we assume that owners have the possibility to force managers to implement their optimal coupon level since the coupon decision is observable. However, owners cannot force managers to follow their optimal liquidation policy since they cannot observe the true cash flow. This provides managers with an entrenchment with respect to the bankruptcy decision as mentioned above.

References

Acharya, V.V., Bharath, S.T. & Srinivasan, A. (2004). Understanding the recovery rates on defaulted securities. *working paper*, London Business School and the Centre for Economic Policy Research (CEPR).

Altman, E.I., Resti, A. & Sironi, A. (2003). Default recovery rates in credit risk modelling: A review of the literature and empirical evidence. *working paper*, Stern School of Business, New York.

Ammann, M. & Genser, M. (2004). A testable ebit-based credit risk model. *working paper*, University of St.Gallen.

Berger, P.G., Ofek, E. & Yermack, D.L. (1997). Managerial entrenchment and capital structure decisions. *Journal of Finance*, **LII**, 1411–1438.

Black, F. & Cox, J.C. (1976). Valuing corporate securities: Some effects of bond indenture provisions. *Journal of Finance*, **XXXI**, 351–367.

Bohlin, E. (1997). A survey of managerial incentives and investment bias - common structure but differing assumptions. *Journal of Business Finance & Accounting*, **24**, 197–248.

Brennan, M. (2003). Corporate investment policy. In G.M. Constantinides, M. Harris & R.M. Stulz, eds., *Corporate Finance*, vol. 21, 1A of *Handbook of the Economics of Finance*, Elsevier, Amsterdam.

Brennan, M. & Kraus, A. (1987). Efficient financing under asymmetric information. *Journal of Finance*, **XLII**, 1225–1243.

Brennan, M. & Schwartz, E.S. (1978). Corporate income taxes, valuation and the problem of optimal capital structure. *Journal of Business*, **51**, 103–114.

Çetin, U., Jarrow, R.A., Protter, P. & Yildirim, Y. (2003). Modeling credit risk with partial information. *Annals of Applied Probability*, forthcoming.

Christensen, P.O., Flor, C.R., Lando, D. & Miltersen, K.R. (2000). Dynamic capital structure with callable debt and debt renegotiation. *working paper*, University of Copenhagen.

Core, J.E., Holthausen, R. & Larcker, D. (1999). Corporate governance, chief executive officer compensation, and firm performance. *Journal of Financial Economics*, **51**, 371–406.

Core, J.E., Guay, W.R. & Larcker, D.F. (2003). Executive equity compensation and incentives: A survey. *Economic Policy Review*, Federal Reserve Bank of New York, April 2003.

Dangl, T. & Zechner, J. (2004). Credit risk and dynamic capital structure choice. *Journal of Financial Intermediation*, **13**, 183–204.

de Jong, A. & Veld, C. (2001). An empirical analysis of incremental capital structure decisions under managerial entrenchment. *Journal of Banking & Finance*, **25**, 1857–1895.

Duffie, D. & Lando, D. (2001). Term structures of credit spreads with incomplete accounting information. *Econometrica*, **69**, 633–664.

Eom, Y.H., Helwege, J. & Huang, J.Z. (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies*, **17**, 499–544.

Ericsson, J. (2000). Asset substitution, debt pricing, optimal leverage and maturity. *working paper*, McGill University, Montreal.

Fama, E.F. (1980). Agency problems and the theory of the firm. *Journal of Political Economy*, **88**, 288–307.

Fischer, E.O., Heinkel, R. & Zechner, J. (1989). Dynamic capital structure choice: Theory and tests. *Journal of Finance*, **XLIV**, 19–40.

François, P. & Morellec, E. (2004). Capital structure and asset prices: Some effects of bankruptcy procedures. *Journal of Business*, **77**, 387–403.

Giesecke, K. (2003). Default and information. *working paper*, Cornell University.

Giesecke, K. & Goldberg, L. (2004). Forecasting default in the face of uncertainty. *working paper*, Cornell University.

Givoly, D. & Hayn, C. (2000). The changing time-series properties of earnings, cash flows and accruals: Has financial reporting become more conservative? *Journal of Accounting & Economics*, **29**, 287–320.

Goldberg, L. (2004). How good is your information? *Risk*, January 2004.

Goldstein, R., Ju, N. & Leland, H.E. (2001). An ebit-based model of dynamic capital structure. *Journal of Business*, **74**, 483–512.

Graham, J.R. (2000). How big are the tax benefits of debt? *Journal of Finance*, **LV**, 1901–1941.

Grossman, S.J. & Hart, O. (1982). Corporate financial structure and managerial incentives. In J.J. McCall, ed., *The Economics of Information and Uncertainty*, The University of Chicago Press, Chicago.

Hackbarth, D., Hennessy, C.A. & Leland, H.E. (2003). The optimal mix of bank and market debt: An asset pricing approach. *working paper*, University of California, Berkeley.

Harris, M. & Raviv, A. (1991). The theory of capital structure. *Journal of Finance*, **XLVI**, 297–355.

Harrison, J. (1985). *Brownian Motion and Stochastic Flow Systems*. John Wiley and Sons, New York.

Hart, O. (2001). Financial contracting. *Journal of Economic Literature*, **XXXIX**, 1079–1100.

Huson, M.R., Parrino, R. & Starks, L.T. (2001). Internal monitoring mechanisms and ceo turnover: A long-term perspective. *Journal of Finance*, **LVI**, 2265–2297.

Jensen, M.C. (1986). Agency costs of free cash flow, corporate finance, and takeovers. *American Economic Review*, **76**, 323–329.

Jensen, M.C. & Meckling, W.H. (1976). Theory of the firm: Mangerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, **3**, 305–360.

Jung, K., Kim, Y.C. & Stulz, R.M. (1996). Timing, investment opportunities, managerial discretion, and the security issue decision. *Journal of Financial Economics*, **42**, 159–185.

Kane, A., Marcus, A. & McDonald, R. (1984). How big is the tax advantage to debt? *Journal of Finance*, **XXXIX**, 841–853.

Kane, A., Marcus, A. & McDonald, R. (1985). Debt policy and the rate of return premium to leverage. *Journal of Financial And Quantitative Analysis*, **20**, 479–499.

Karatzas, I. & Shreve, S.E. (1998). *Methods of Mathematical Finance*, vol. 39 of *Applications of Mathematics. Stochastic Modelling and Applied Probability*. Springer, Berlin, 2nd edn.

Leland, H.E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, **XLIX**, 1213–1252.

Leland, H.E. (1998). Agency costs, risk management, and capital structure. *Journal of Finance*, **53**, 1213–1243.

Leland, H.E. & Pyle, D.H. (1977). Informational asymmetries, financial structure, and financial intermediation. *Journal of Finance*, **XXXII**, 371–387.

Leland, H.E. & Toft, K.B. (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, **LI**, 987–1019.

Liptser, R.S. & Shiryaev, A.N. (2001). *Statistics of Random Processes. General Theory*. Springer, Berlin.

Mehran, H. (1992). Executive incentive plans, corporate control, and capital structre. *Journal of Financial And Quantitative Analysis*, **27**, 539–560.

Mella-Barral, P. & Perraudin, W. (1997). Strategic debt service. *Journal of Finance*, **LII**, 531–556.

Mello, A.S. & Parsons, J.E. (1992). Measuring the agency costs of debt. *Journal of Finance*, **XLVII**, 1887–1904.

Merton, R.C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 449–470.

Milgrom, P. & Roberts, J. (1992). *Economics, Organization and Management*. Prentice Hall, New Jersey.

Modigliani, F. & Miller, M. (1958). The cost of capital, corporation finance, and the theory of investment. *American Economic Review*, **48**, 261–297.

Morellec, E. (2004). Can managerial discretion explain observed leverage ratios? *Review of Financial Studies*, **17**, 257–294.

Myers, S.C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics*, **5**, 147–175.

Myers, S.C. (2003). Financing of corporations. In G.M. Constantinides, M. Harris & R.M. Stulz, eds., *Corporate Finance*, vol. 21, 1A of *Handbook of the Economics of Finance*, Elsevier, Amsterdam.

Myers, S.C. & Majluf, N.S. (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics*, **13**, 187–221.

Noe, T.H. & Rebello, M.J. (1996). Asymmetric information, managerial opportunism, financing, and payout policies. *Journal of Finance*, **LI**, 637–660.

Novaes, W. (2003). Capital structure choice when managers are in control: Entrenchment versus efficiency. *Journal of Business*, **76**, 49–81.

Øksendal, B. (2000). *Stochastic Differential Equations. An Introduction with Applications*. Springer, Berlin, 5th edn.

Palia, D. (2001). The endogeneity of managerial compensation in firm valuation: A solution. *Review of Financial Studies*, **14**, 735–764.

Renault, O. & Scaillet, O. (2004). On the way to recovery: A nonparametric bias free estimation of recovery rate densities. *Journal of Banking & Finance*, **28**, 2915–2931.

Ross, S.A. (1977). The determination of financial structure: The incentive-signalling approach. *Bell Journal of Economics*, **8**, 23–40.

Stein, J.C. (2003). Agency, information and corporate investment. In G.M. Constantinides, M. Harris & R.M. Stulz, eds., *Corporate Finance*, vol. 21, 1A of *Handbook of the Economics of Finance*, Elsevier, Amsterdam.

Stulz, R.M. (1990). Managerial discretion and optimal financing policies. *Journal of Financial Economics*, **26**, 3–27.

Weiss, L. (1990). Bankruptcy resolution: Direct costs and violation of priority of claims. *Journal of Financial Economics*, **27**, 285–314.

Zwiebel, J. (1996). Dynamic capital structure under managerial entrenchment. *American Economic Review*, **86**, 1197–1215.

A Appendix

A.1 Laplace transform of the hitting time

Denote τ as the first time a Brownian motion with drift ν hits or exceeds the level $y \geq 0$, i.e.

$$\tau = \inf\{t \geq 0; W_t + \nu t \geq y\}$$

The transformation formula for $\mathrm{E}[e^{-r\,\tau}]$ is then given by

$$\mathrm{E}[e^{-r\,\tau}] = \exp\{\nu y - y\sqrt{\nu^2 + 2r}\} \tag{A-1}$$

(See Karatzas & Shreve (1998), p.63)

We apply this result to the case, where τ_{X_B} denotes the first time the process X_t hits or falls below X_B, i.e.

$$\tau_{X_B} = \inf\{t \geq 0; X_t \leq X_B\}$$

Thereby, X_t is assumed to follow an Arithmetic Brownian motion. So, we have $X_t = X_0 + \mu t + \sigma W_t$, and we can write

$$\begin{aligned} \tau_{X_B} &= \inf\{t \geq 0; X_0 + \mu t + \sigma W_t \leq X_B\} \\ &= \inf\Big\{t \geq 0; W_t - \big(\frac{\mu}{\sigma}\big) \geq \frac{X_0 - X_B}{\sigma}\Big\} \end{aligned}$$

In order to apply the transformation formula (A-1), we identify $\nu = -\frac{\mu}{\sigma}$ and $y = \frac{X_0 - X_B}{\sigma}$, from which we get

$$\begin{aligned} \mathrm{E}[e^{-r\,\tau_{X_B}}] &= \exp\Big\{-\frac{\mu}{\sigma^2}(X_0 - X_B) - \frac{X_0 - X_B}{\sigma}\sqrt{\Big(\frac{\mu}{\sigma}\Big)^2 + 2r}\Big\} \\ &= \exp\Big\{\Big(-\frac{\mu}{\sigma^2} - \frac{\sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}\Big)(X_0 - X_B)\Big\} \\ &= \exp\{\beta_2(X_0 - X_B)\} \end{aligned} \tag{A-2}$$

A.2 Valuation of the various claims

The general solution to the ordinary differential equation

$$\frac{1}{2}\sigma^2 C_{(xx)} + \mu C_{(x)} - rC + mX + c = 0$$

has the form

$$C(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{mX + c}{r} + \frac{\mu m}{r^2}$$

with

$$\beta_1 = \frac{-\mu + \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2} \qquad \beta_2 = \frac{-\mu - \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}$$

As mentioned in the text, we will use the following shortcuts to ease notation

$$\hat{X} \equiv \frac{X}{r} + \frac{\mu}{r^2} \qquad \hat{X}_B \equiv \frac{X_B}{r} + \frac{\mu}{r^2} \qquad \hat{X}_b \equiv \frac{X_b}{r} + \frac{\mu}{r^2}$$

All equity claim $AE(X)$

For the all equity claim we have

$$m = (1-\tau^e) \qquad c = 0$$

Therefore, $AE(X)$ has to satisfy the ODE

$$\frac{1}{2}\sigma^2 AE_{(xx)} + \mu AE_{(x)} - rAE + (1-\tau^e)X = 0$$

which has the solution

$$AE(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{(1-\tau^e)X}{r} + \frac{(1-\tau^e)\mu}{r^2}$$

To determine the coefficients, for $X \gg X_B$ we deduce that $A_1 = 0$, since otherwise the solution diverges.
On the other hand we have $AE(X_B) = 0$, from which we can deduce A_2

$$\begin{aligned} & AE(X_B) = A_2\, e^{\beta_2 X_B} + \frac{(1-\tau^e)X_B}{r} + \frac{(1-\tau^e)\mu}{r^2} = 0 \\ \Leftrightarrow\quad & A_2\, e^{\beta_2 X_B} + (1-\tau^e)\hat{X}_B = 0 \\ \Leftrightarrow\quad & A_2 = -(1-\tau^e)\hat{X}_B\, e^{-\beta_2 X_B} \end{aligned}$$

Therefore, we have the solution

$$AE(X) = (1-\tau^e)\Big(\hat{X} - \hat{X}_B\, e^{\beta_2 (X - X_B)}\Big) \tag{A-3}$$

External equity claim $E(X)$

For the equity claim in the levered case, we have

$$m = (1-\tau^e)(1-\gamma) \qquad c = -(1-\tau^e)(1-\gamma)(k+\kappa)$$

Therefore, $E(X)$ has to satisfy the ODE

$$\frac{1}{2}\sigma^2 E_{(xx)} + \mu E_{(x)} - rE + (1-\tau^e)(1-\gamma)X - (1-\tau^e)(1-\gamma)(k+\kappa) = 0$$

which has the solution

$$E(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{(1-\tau^e)(1-\gamma)X - (1-\tau^e)(1-\gamma)(k+\kappa)}{r} + \frac{(1-\tau^e)(1-\gamma)\mu}{r^2}$$

To determine the coefficients, for $X \gg X_b$ we deduce that $A_1 = 0$, since otherwise the solution diverges.
On the other hand we have $E(X_b) = 0$, from which we can deduce A_2

$$\begin{aligned} & E(X_b) = A_2\, e^{\beta_2 X_b} + (1-\tau^e)(1-\gamma)\big(\frac{X_b}{r} + \frac{\mu}{r^2} - \frac{k+\kappa}{r}\big) = 0 \\ \Leftrightarrow\quad & A_2\, e^{\beta_2 X_b} + (1-\tau^e)(1-\gamma)\big(\hat{X}_b - \frac{k+\kappa}{r}\big) = 0 \\ \Leftrightarrow\quad & A_2 = -(1-\tau^e)(1-\gamma)\big(\hat{X}_b - \frac{k+\kappa}{r}\big)\, e^{-\beta_2 X_b} \end{aligned}$$

Therefore, we have the solution

$$E(X) = (1-\tau^e)(1-\gamma)\Big(\big(\hat{X} - \frac{(k+\kappa)}{r}\big) - \big(\hat{X}_b - \frac{(k+\kappa)}{r}\big)e^{\beta_2 (X - X_b)}\Big) \tag{A-4}$$

Debt claim $D(X)$

For the debt claim, we have

$$m = 0 \qquad c = (1-\tau^k)k$$

Therefore, $D(X)$ has to satisfy the ODE

$$\frac{1}{2}\sigma^2 D_{(xx)} + \mu D_{(x)} - rD + (1-\tau^k)k = 0$$

which has the solution

$$D(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + (1-\tau^k)\frac{k}{r}$$

To determine the coefficients, for $X \gg X_b$ we deduce that $A_1 = 0$, since otherwise the solution diverges.
On the other hand we have $D(X_b) = (1-\tau^e)(1-\alpha)AE(X_b)$, from which we can deduce A_2

$$\begin{aligned}
& D(X_b) = A_2\, e^{\beta_2 X_b} + \frac{(1-\tau^k)k}{r} = (1-\tau^e)(1-\alpha)AE(X_b) \\
\Leftrightarrow\quad & A_2\, e^{\beta_2 X_b} = (1-\tau^e)(1-\alpha)AE(X_b) - (1-\tau^k)\frac{k}{r} \\
\Leftrightarrow\quad & A_2 = \big((1-\tau^e)(1-\alpha)AE(X_b) - (1-\tau^k)\frac{k}{r}\big)\, e^{-\beta_2 X_b}
\end{aligned}$$

Therefore, we have the solution

$$D(X) = \big(D(X_b) - (1-\tau^k)\frac{k}{r}\big)e^{\beta_2(X-X_b)} + (1-\tau^k)\frac{k}{r} \tag{A-5}$$

Management claim $M(X)$

For the management claim, we have

$$m = (1-\tau^e)\gamma \qquad c = (1-\tau^k)\kappa - (1-\tau^e)\gamma(k+\kappa) + \eta$$

Therefore, $M(X)$ has to satisfy the ODE

$$\frac{1}{2}\sigma^2 M_{(xx)} + \mu M_{(x)} - rM + (1-\tau^e)\gamma X + (1-\tau^k)\kappa - (1-\tau^e)\gamma(k+\kappa) + \eta = 0$$

which has the solution

$$M(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{(1-\tau^e)\gamma X + (1-\tau^k)\kappa - (1-\tau^e)\gamma(k+\kappa) + \eta}{r} + \frac{\mu(1-\tau^e)\gamma}{r^2}$$

which we abbreviate to

$$M(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + (1-\tau^e)\gamma\hat{X} - \hat{k}$$

by introducing $\hat{k} \equiv \frac{(1-\tau^e)\gamma(k+\kappa)-(1-\tau^k)\kappa-\eta}{r}$.

To determine the coefficients, for $X \gg X_b$ we deduce that $A_1 = 0$, since otherwise the solution diverges.
On the other hand we have $M(X_b) = 0$, from which we can deduce A_2

$$\begin{aligned} & M(X_b) = A_2\, e^{\beta_2 X_b} + (1-\tau^e)\gamma \hat{X}_b - \hat{k} = 0 \\ \Leftrightarrow \quad & A_2 = \big(-(1-\tau^e)\gamma \hat{X}_b + \hat{k} \big)\, e^{-\beta_2 X_b} \end{aligned}$$

Therefore, we have the solution

$$M(X) = \Big(-(1-\tau^e)\gamma \hat{X}_b + \hat{k} \Big) e^{\beta_2 (X - X_b)} + (1-\tau^e)\gamma \hat{X} - \hat{k} \tag{A-6}$$

Tax claim $T(X)$

For the tax claim, we have

$$m = \tau^e \qquad c = (\tau^k - \tau^e)(k + \kappa)$$

Therefore, $T(X)$ has to satisfy the ODE

$$\frac{1}{2}\sigma^2 T_{(xx)} + \mu T_{(x)} - rT + \tau^e \gamma X + (\tau^k - \tau^e)(k+\kappa) = 0$$

which has the solution

$$T(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X} + \frac{\tau^e X + (\tau^k - \tau^e)(k+\kappa)}{r} + \frac{\mu \tau^e}{r^2}$$

To determine the coefficients, for $X \gg X_b$ we deduce that $A_1 = 0$, since otherwise the solution diverges.
On the other hand we have $T(X_b) = \tau^e (1-\alpha) AE(X_b)$, from which we can deduce A_2

$$\begin{aligned} & T(X_b) = A_2\, e^{\beta_2 X_b} + \frac{\tau^e X + (\tau^k - \tau^e)(k+\kappa)}{r} + \frac{\mu \tau^e}{r^2} = \tau^e (1-\alpha) AE(X_b) \\ \Leftrightarrow \quad & A_2\, e^{\beta_2 X_b} + \tau^e \hat{X}_b + (\tau^k - \tau^e)\frac{(k+\kappa)}{r} = \tau^e (1-\alpha) AE(X_b) \\ \Leftrightarrow \quad & A_2 = \big(\tau^e (1-\alpha) AE(X_b) - \tau^e \hat{X}_b - (\tau^k - \tau^e)\frac{(k+\kappa)}{r} \big) e^{-\beta_2 X_b} \end{aligned}$$

Therefore, we have the solution

$$\begin{aligned} T(X) = \Big(T(X_b) - \tau^e \hat{X}_b - (\tau^k - \tau^e)\frac{(k+\kappa)}{r} \Big) e^{\beta_2 (X - X_b)} + \\ \tau^e \hat{X} + (\tau^k - \tau^e)\frac{(k+\kappa)}{r} \end{aligned} \tag{A-7}$$

Bankruptcy claim $BC(X)$

For the bankruptcy claim, we have

$$m = 0 \qquad c = 0$$

Therefore, $BC(X)$ has to satisfy the ODE

$$\frac{1}{2}\sigma^2 BC_{(xx)} + \mu BC_{(x)} - rBC = 0$$

which has the solution

$$BC(X) = A_1\, e^{\beta_1 X} + A_2\, e^{\beta_2 X}$$

To determine the coefficients, for $X \gg X_b$ we deduce that $A_1 = 0$, since otherwise the solution diverges.
On the other hand we have $BC(X_b) = \alpha AE(X_b)$, from which we can deduce A_2

$$\begin{aligned} & BC(X_b) = A_2\, e^{\beta_2 X_b} = \alpha AE(X_b) \\ \Leftrightarrow \quad & A_2 = (\alpha AE(X_b))e^{-\beta_2 X_b} \end{aligned}$$

Therefore, we have the solution

$$BC(X) = (\alpha AE(X_b))e^{\beta_2 (X - X_b)} \tag{A-8}$$

A.3 Optimal X_B, X_b, X_{bm}

Differentiating the **all equity claim** ($AE(X)$) with respect to X gives

$$\frac{\partial AE(X)}{\partial X} = \frac{1-\tau^e}{r} - (1-\tau^e)\hat{X}_B\, \beta_2\, e^{\beta_2 (X - X_B)}$$

or at $X = X_B$

$$\frac{1-\tau^e}{r} - (1-\tau^e)\hat{X}_B\, \beta_2$$

Setting this to zero

$$\frac{1-\tau^e}{r} - (1-\tau^e)\hat{X}_B\, \beta_2\ = 0$$

and solving for X_B^* yields

$$X_B^* = \frac{1}{\beta_2} - \frac{\mu}{r} \tag{A-9}$$

Analogous, differentiating the **equity claim** ($E(X)$) with respect to X gives

$$\frac{\partial E(X)}{\partial X} = \frac{(1-\tau^e)(1-\gamma)}{r} - (1-\tau^e)(1-\gamma)\big(\hat{X}_b - \frac{(k+\kappa)}{r}\big)\, \beta_2\, e^{\beta_2 (X - X_b)}$$

At $X = X_b$ and set to zero yields

$$\frac{(1-\tau^e)(1-\gamma)}{r} - (1-\tau^e)(1-\gamma)\big(\hat{X}_b - \frac{(k+\kappa)}{r}\big)\, \beta_2 = 0$$

and finally

$$X_b^* = \frac{1}{\beta_2} - \frac{\mu}{r} + k + \kappa \tag{A-10}$$

Finally, differentiating the **management claim** ($M(X)$) with respect to X yields

$$\frac{\partial M(X)}{\partial X} = \frac{(1-\tau^e)\gamma}{r} + \big(-(1-\tau^e)\gamma\hat{X}_{bm} + \hat{k}\big)\, \beta_2\, e^{\beta_2 (X - X_{bm})}$$

At $X = X_{bm}$ and set to zero yields

$$\frac{(1-\tau^e)\gamma}{r} + \big(-(1-\tau^e)\gamma \hat{X}_{bm} + \hat{k}\big)\,\beta_2 = 0$$

Solving for X^*_{bm} gives

$$X^*_{bm} = \frac{1}{\beta_2} - \frac{\mu}{r} + \frac{r}{\gamma(1-\tau^e)}\hat{k}$$

Expanding the last term and rearranging terms gives

$$X^*_{bm} = \frac{1}{\beta_2} - \frac{\mu}{r} + k + \frac{\gamma(1-\tau^e)-(1-\tau^k)}{\gamma(1-\tau^e)}\,\kappa - \frac{1}{\gamma(1-\tau^e)}\,\eta \tag{A-11}$$

A.4 Optimal coupon

For equity holders the optimal coupon (at time t) is found by solving the maximization problem

$$\max_k \big(D(X_t, k, X^*_b(k)) + E(X_t, k, X^*_b(k))\big)$$

In principle, the solution is found by differentiating the sum of the equity and debt claim with respect to k, setting it to zero and solving for k^*. Thereby, both claims make use of the optimal X^*_b, which is $X^*_b = \frac{1}{\beta_2} - \frac{\mu}{r} + k + \kappa$. Using the same abbreviations as before (ref), we write the sum as

$$\begin{aligned}
\Sigma(k) \;&=\; t\Big(\hat{X} - \frac{k+\kappa}{r} - \hat{X}_B\, e^{\beta_2(X-X_B-k-\kappa)}\Big) \\
&+\; \Big(a_1\big(\hat{X}_B + \frac{k+\kappa}{r} - \hat{X}_B\, e^{\beta_2(k+\kappa)}\big) - a_2\, k\Big)e^{\beta_2(X-X_B-k-\kappa)} + a_2\, k
\end{aligned}$$

with the additional definitions

$$t = (1-\gamma)(1-\tau^e) \quad a_1 = (1-\alpha)(1-\tau^e)^2 \quad a_2 = \frac{1-\tau^k}{r}$$

Differentiating Σ with respect to k yields

$$\begin{aligned}
\frac{\partial \Sigma}{\partial k} \;&=\; t\Big(-\frac{1}{r} + \beta_2\, \hat{X}_B\, e^{\beta_2(X-X_B-k-\kappa)}\Big) \\
&+\; \Big(a_1\big(\frac{1}{r} - \beta_2\, \hat{X}_B\, e^{\beta_2(k+\kappa)}\big) - a_2\Big)e^{\beta_2(X-X_B-k-\kappa)} \\
&-\; \Big(a_1\big(\hat{X}_B + \frac{k+\kappa}{r} - \hat{X}_B\, e^{\beta_2(k+\kappa)}\big) - a_2\, k\Big)\,\beta_2\, e^{\beta_2(X-X_B-k-\kappa)} + a_2
\end{aligned}$$

Setting this to zero, there exists no closed-form solution for k^*, but it can be solved numerically.

A.5 Equity volatility

To determine the annual percentage equity standard deviation, we need to calculate

$$\frac{\sqrt{\mathrm{Var}[E(X_1)]}}{E(X_0)}$$

with

$$\mathrm{Var}[E(X_1)] = \mathrm{E}_0[E(X_1)^2] - \mathrm{E}_0[E(X_1)]^2$$

Thereby, the expectations have to be taken with respect to the probability law of X_1, which is given by the joint distribution of X, i.e. Brownian motion with drift, and the first-passage time distribution, i.e.

$$\mathrm{Prob}\{X_t \in \mathrm{d}x,\ \inf\{X_s;\ 0 < s \leq t\} \geq X_b\}$$

From general results of first-passage time distributions of Brownian motion with drift (See e.g. Harrison (1985)), we know that

$$\begin{aligned} f(x,y,t) &= \mathrm{Prob}\{X_t \in \mathrm{d}x, M_t \geq y\} \\ &= \frac{1}{\sigma\sqrt{t}}\left(\phi\Big(\frac{x-\mu t}{\sigma\sqrt{t}}\Big) - e^{\frac{2\mu y}{\sigma^2}}\phi\Big(\frac{x-2y-\mu t}{\sigma\sqrt{t}}\Big)\right) \end{aligned}$$

where X_t is a general Brownian motion with drift μ and dispersion σ, M_t is defined as $\inf\{X_t;\ 0 < s \leq t\}$, and $X_0 = 0$. That is, $f(x,y,t)$ is the joint density function for a Brownian motion with drift starting at the origin and hitting the threshold $y < 0$.
In our context, y is given by $X_b - X_0$, so that the expectation can be evaluated as

$$\mathrm{E}_0[E(X_1)] = \int_{X_b - X_0}^{\infty} E(X + X_0)\, f(X, (X_b - X_0), 1)\, \mathrm{d}X \tag{A-12}$$

A.6 Probability of default

With the same reasoning as above, we can determine the probability that the firm hits the bankruptcy threshold within a one-year horizon, which is given by the probability $\mathrm{Prob}\{\inf\{X_t;\ 0 < s \leq t\} \leq X_b\}$.
From e.g. Harrison (1985), the first passage time distribution is given by

$$F(t) = \mathrm{Prob}\{M_t > y\} = N\Big(\frac{-y+\mu t}{\sigma\sqrt{t}}\Big) - e^{\frac{2y\mu}{\sigma^2}}\, N\Big(\frac{y+\mu t}{\sigma\sqrt{t}}\Big)$$

where, again, y is in our context $X_b - X_0$, and $N(\cdot)$ is the cumulative distribution function of the standard normal distribution.
The one-year probability of default is then

$$PD_1 = 1 - F(1)$$

This is plotted in figure 11 for different values of the underlying cash flow volatility (σ).

A.7 Kalman filter equation

The extension of the Kalman-Bucy filter can be found in Liptser & Shiryaev (2001), p.392ff. They establish the follwing result.
Given the system

$$\begin{aligned} \mathrm{d}X_t &= (a_0(t) + a_1(t)X_t + a_2(t)Z_t)\mathrm{d}t + b_1\mathrm{d}W_t^1 \\ \mathrm{d}Z_t &= (A_0(t) + A_1(t)X_t + A_2(t)Z_t)\mathrm{d}t + B_1\mathrm{d}W_t^2 \end{aligned}$$

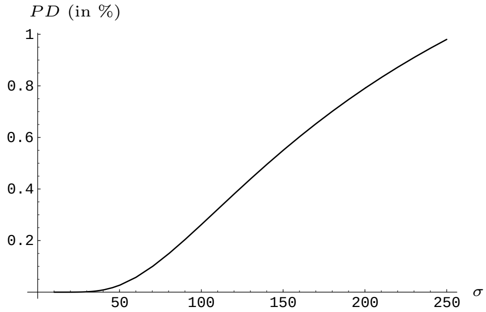

Figure 11: 1-year probability of default

where W_t^1 and W_t^2 are independent Wiener processes.
Then the optimal estimate ξ_t and the mean square error S_t satisfy the following equations

$$\begin{aligned}
\mathrm{d}\xi_t &= (a_0(t) + a_1(t)\xi_t + a_2(t)Z_t)\mathrm{d}t + \frac{A_1(t)S_t}{B_1(t)^2}\big(\mathrm{d}Z_t - (A_0(t) + A_1(t)\xi_t + A_2(t)Z_t)\mathrm{d}t\big) \\
\frac{\mathrm{d}S_t}{\mathrm{d}t} &= 2a_1(t)S_t + b_1(t)^2 - \frac{A_1(t)^2 S_t^2}{B_1(t)^2}
\end{aligned}$$

In our context, we have

$$\begin{aligned}
a_0(t) &= \mu, \quad b_1(t) = \sigma \quad A_1(t) = 1 \quad B_1(t) = \delta \\
a_1 &= a_2 = A_0 = A_2 = 0
\end{aligned}$$

From this it follows, that

$$\mathrm{d}\xi_t = \mu\,\mathrm{d}t + \delta^{-2}S_t\,(\mathrm{d}Z_t - \xi_t\,\mathrm{d}t) \tag{A-13}$$

$$\frac{\mathrm{d}S_t}{\mathrm{d}t} = \sigma^2 - \delta^{-2}S_t^2 \tag{A-14}$$

For the initial condition $S_0 = \delta\sigma$, it is obvious that $\mathrm{d}S_t = 0$ and $S_t = \delta\sigma$. In general, the solution for S_t is derived as follows

$$\begin{aligned}
& \mathrm{d}S_t = (\sigma^2 - \delta^{-2}S_t^2)\,\mathrm{d}t \\
\Leftrightarrow\quad & \Big(\frac{\delta^2}{\sigma^2\delta^2 - S_t^2}\Big)\,\mathrm{d}S_t = \mathrm{d}t \\
\Leftrightarrow\quad & S_t = \delta\sigma\Big(\frac{S_0\,\cosh\big(\frac{\sigma t}{\delta}\big) + \delta\sigma\,\sinh\big(\frac{\sigma t}{\delta}\big)}{\delta\sigma\cosh\big(\frac{\sigma t}{\delta}\big) + S_0\sinh\big(\frac{\sigma t}{\delta}\big)}\Big) && \text{(A-15)}
\end{aligned}$$

where

$$\Big(\frac{S_0\,\cosh\big(\frac{\sigma t}{\delta}\big) + \delta\sigma\,\sinh\big(\frac{\sigma t}{\delta}\big)}{\delta\sigma\cosh\big(\frac{\sigma t}{\delta}\big) + S_0\sinh\big(\frac{\sigma t}{\delta}\big)}\Big) \to 1 \quad \text{as} \quad t \to \infty$$

A.8 Optimal X_b under incomplete information

By introducing the following definitions

$$t = (1-\tau^e)(1-\gamma) \quad c_1 = \big(\frac{\mu}{r^2} - \frac{k+\kappa}{r}\big)t \quad c_2 = \Big(\big(\hat{X}_b - \frac{k+\kappa}{r}\big)e^{-\beta_2 X_b}\Big)t$$

we write the equity claim based on the optimal estimate as

$$E(\xi_t) = \frac{t}{r}\,\xi_t + c_1 - c_2\,e^{\beta_2 \xi_t}$$

Taking the expectation gives

$$\begin{aligned}
\mathrm{E}[E(\xi_t)] &= \frac{t}{r}\,\mathrm{E}[\xi_t] + c_1 - c_2\,e^{\beta_2 \mathrm{E}[\xi_t] + \frac{1}{2}\beta_2^2 \mathrm{Var}[\xi_t]} \\
&= \frac{t}{r}\,X_t + c_1 - c_2\,e^{\beta_2 X_t + \frac{1}{2}\beta_2^2\,S_t}
\end{aligned}$$

Differentiating with respect to X at $X = X_b$ gives

$$\frac{\partial \mathrm{E}[E(\xi)]}{\partial X}|_{X=X_b} = \frac{t}{r} - c_2\,\beta_2\,e^{\beta_2 X_b + \frac{1}{2}\beta_2^2\,S_t}$$

Setting the result to zero, re-substituting c_2 and solving for X_b yields

$$\begin{aligned}
&\frac{1}{r} - \Big((\hat{X}_b - \frac{k+\kappa}{r}) e^{-\beta_2 X_b} \Big)\,\beta_2\,e^{\beta_2 X_b + \frac{1}{2}\beta_2^2\,S_t} = 0 \\
&\frac{1}{r} - \Big((\frac{X_b}{r} + \frac{\mu}{r^2} - \frac{k+\kappa}{r}) \Big)\,\beta_2\,e^{\frac{1}{2}\beta_2^2\,S_t} = 0 \\
&\frac{e^{-\frac{1}{2}\beta_2^2\,S_t}}{\beta_2} = \big(X_b + \frac{\mu}{r} - (k+\kappa) \big) \\
&X_b = \frac{e^{-\frac{1}{2}\beta_2^2\,S_t}}{\beta_2} - \frac{\mu}{r} + (k+\kappa)
\end{aligned} \qquad \text{(A-16)}$$

and $S_t = \delta\sigma$.

A.9 Optimal coupon under incomplete information

The reasoning to determine the optimal coupon under incomplete information is analogous to appendix A.4. The maximization problem is given by

$$k_{ai}^* = \max_k \big(\mathrm{E}[D(\xi_t, k, X_{b,ai}^{**}(k)) + E(\xi_t, k, X_{b,ai}^{**}(k))] \big)$$

Using the same abbreviations as before, the sum can be written as

$$\begin{aligned}
\Sigma(\xi, k) &= t\Big(\frac{\xi}{r} + \frac{\mu}{r^2} - \frac{k+\kappa}{r} - \big(\hat{X'}_b - \frac{k}{r}\big)\,e^{\beta_2(\xi - X_b' - k)} \Big) \\
&+ \Big(a_1\big(\hat{X'}_b + \frac{k}{r} - \hat{X}_B\,e^{\beta_2(X_b' + k - X_B)}\big) - a_2\,k \Big) e^{\beta_2(\xi - X_b' - k)} + a_2\,k
\end{aligned}$$

where $X_b' = \frac{e^{-\frac{1}{2}\beta_2^2\,\delta\sigma}}{\beta_2} - \frac{\mu}{r} + \kappa$ and $\hat{X'}_b = \frac{X_b'}{r} + \frac{\mu}{r^2}$. Taking the expectation gives

$$\begin{aligned}
\mathrm{E}[\Sigma(\xi)] &= t\Big(\frac{\mathrm{E}[\xi]}{r} + \frac{\mu}{r^2} - \frac{k+\kappa}{r} - \big(\hat{X'}_b - \frac{k}{r}\big)\,e^{\beta_2((\mathrm{E}[\xi] + \frac{1}{2}\beta_2 \mathrm{Var}[\xi]) - X_b' - k)} \Big) \\
&+ \Big(a_1\big(\hat{X'}_b + \frac{k}{r} - \hat{X}_B\,e^{\beta_2(X_b' + k - X_B)}\big) - a_2\,k \Big) e^{\beta_2((\mathrm{E}[\xi] + \frac{1}{2}\beta_2 \mathrm{Var}[\xi]) - X_b' - k)} + a_2\,k \\
&= t\Big(\frac{X_t}{r} + \frac{\mu}{r^2} - \frac{k+\kappa}{r} - \big(\hat{X'}_b - \frac{k}{r}\big)\,e^{\beta_2((X_t + \frac{1}{2}\beta_2 S_t) - X_b' - k)} \Big) \\
&+ \Big(a_1\big(\hat{X'}_b + \frac{k}{r} - \hat{X}_B\,e^{\beta_2(X_b' + k - X_B)}\big) - a_2\,k \Big) e^{\beta_2((X_t + \frac{1}{2}\beta_2 S_t) - X_b' - k)} + a_2\,k
\end{aligned}$$

Next, differentiating with respect to k yields

$$
\begin{aligned}
\frac{\partial \mathrm{E}[\Sigma]}{\partial k} &= t\Big(-\frac{1}{r} + \big(\beta_2\big(\hat{X}'_b - \frac{k}{r}\big) + \frac{1}{r}\big)\, e^{\beta_2((X_t+\frac{1}{2}\beta_2 S_t)-X'_b-k)}\Big) \\
&+ \Big(a_1\big(\frac{1}{r} - \beta_2\,\hat{X}_B\, e^{\beta_2(X'_b+k-X_B)}\big) - a_2\Big)e^{\beta_2((X_t+\frac{1}{2}\beta_2 S_t)-X'_b-k)} \\
&- \Big(a_1\big(\hat{X}'_b + \frac{k}{r} - \hat{X}_B\, e^{\beta_2(X'_b+k-X_B)}\big) - a_2\, k\Big)\,\beta_2\, e^{\beta_2((X_t+\frac{1}{2}\beta_2 S_t)-X'_b-k)} + a_2
\end{aligned}
$$

Again, setting this to zero, this cannot explicitly be solved for k, but numerical results are available.