Trilateral Contracts and the Hold-up Problem☆

Regine Oexl*

Department of Economics, University of Padua, Italy

August 25, 2011

Abstract

We present a new approach to remedy the hold-up problem, applicable when more than one buyer is involved in the transaction. The case we consider is a company selling identical products to two buyers that have a common interest in inducing the seller to make a quality enhancing investment. We show that a trilateral contract may provide the correct incentives to restore efficiency. Because there is more than one buyer, the purchasing decision can be transformed from a contemporaneous to a sequential problem. This allows us to condition the purchasing decision on observed previous transactions. The contract induces a coalition proof Nash equilibrium and holds under complete as well as incomplete information. The extension to more than two buyers is straightforward.

Keywords: multilateral contract, trilateral contract, hold-up problem
JEL: D82, L14

1. Introduction

The hold-up problem has been extensively analyzed by the economic literature in the last decades. In its classical version, this problem applies when two parties, for instance a manufacturer and a customer, or, more generally, a seller and a buyer, are unable to extract all the surplus from their interaction. Typically, the party that should make a quality enhancing relation-specific investment is unable to receive all the benefits that accrue from this investment, as future (re)negotiation may confer parts of the benefit from the

☆This paper is part of my PhD thesis. I wish to thank to Antonio Nicolò for his help. Furthermore I am grateful for valuable comments to Stefano Comino, Stefano Galavotti, Luciano Greco, Cheng-Zhong Qin, Samuel Standaert, Christoph Wagner and the participants at the Annual meeting of the ASSET, 2010, University of Alicante, the SMYE 2011, University of Groningen, the 2011 ECORE Summer School on Market Failure and Market Design, Louvain-la-Neuve, and the 2^{nd} Workshop: IO: Theory, Empirics and Experiments 2011, University of Salento. I thank to the "Cassa di Risparmio di Padova e Rovigo" for financial assistance during my PhD. The usual disclaimer applies.

*Via del Santo 33, 35100 Padua - email: regine.oexl@unipd.it; phone: +39 049 827 3848; fax: +39 049 827 4211; web: www.sites.google.com/site/regineoexl/

customized investment to the party with higher bargaining power (Klein et al., 1978; Williamson, 1985). When neither the investment nor the induced quality can be verified by a third party, the contract cannot be contingent on them. Investment will be below the social optimum, since a contract with a fixed price - in which the seller receives a fixed payment for the product, independent on the level of quality - would give him no incentive to invest. Similarly, a contract in which the buyer has the option to buy the product depending on the quality gives no incentive to invest, since the seller anticipates that the buyer may renegotiate the terms of the contract once the investment is sunk (Hart and Moore, 1999).

In what follows, we present a new approach to remedy the hold-up problem, applicable in a setting where there is more than one buyer. We model a situation where a seller produces identical products for two noncompeting buyers that have a common interest in inducing the seller to make a quality enhancing investment. It can be shown that a trilateral contract may provide the correct incentives to lower the hold-up problem and restore efficiency.

The reason why the trilateral contract solves the hold-up problem is intuitively straightforward. If trade between the seller and each of the buyers is sequential, then it is possible to make payments contingent on observed previous payments. More specifically, before any investment is done the three parties sign a contract which stipulates that when the quality is high, the first buyer can purchase the product at market price. The second buyer on the other hand has to pay a premium in case the first buyer has bought the product. Thereafter, he can also buy the product at market price. The premium will induce the seller to invest more than he would have invested at market prices. Hence, even though an option contract with fixed prices is signed, the seller has an incentive to invest in quality enhancement and the induced level of investment may be as high as the social optimal. Moreover, the contract is self-enforcing, which means that neither party has incentive to renege (Baker et al., 2002a). It is coalition renegotiation proof, hence not even a subgroup of agents has incentive to renege jointly (Bernheim et al., 1987). By additionally specifying payments among the firms upon contracting, we ensure that all parties have incentive to participate in the contract.

Our trilateral contract constitutes a one shot cooperative project, in which investment is incurred a single time. In this sense, it differs from solving the problem by vertically integrating or restructuring firm boundaries and asset ownership, as suggested by Baker et al. (2002b) and Grossman and Hart (1986). Neither does the contract rely on repeated interaction with the same agent or within a group, where incentives arise based on reputational effects (Radner, 1981; Kandori, 1992; Dixit, 2003). Also, it does not require any additional agent like an intermediary or arbitrator, cases considered in Dixit (2004) and Laffont and Martimort (1997). Contrastingly, in our contract, all agents participating may benefit directly from the contract; the interaction among the agents *involved in the transaction* suffices to induce efficient incentives.

There are numerous examples of situations in which this kind of contract could be of use. Since we consider no competition among the buyers, the buyers might either be end-consumers, or firms operating in different markets or industries, or companies requiring basic research for processes and products to further developing different (end-) products. Hence, our contract may apply to any such one shot scenario regarding the hold-up problem. More specific examples include software provision for firms operating in different industries, research on cells/genes used for developing diverse medical cures for different pharmaceutical companies, or research on earthquake technology used by different countries.

Despite the direct application of our model, the method of solving can also be applied to other problems. The innovative part of this paper is the transformation of a contemporaneous setting to a sequential one. This way, we create the possibility to condition actions on previous occurrences. The sequentiality of the contract does not lead to an increase in information about quality, since such is observable at any time. Yet, the fact that the first buyer is buying gives the possibility to create a new, verifiable variable, which is perfectly correlated with quality.

The remaining part of the paper is organized as follows. Section 2 introduces the basic model of a trilateral contract, concentrating on the case with only two buyers. After presenting the benchmark and several verifiability issues in section 2.1, section 2.2 presents the model under complete information, taking into account the fact that the two buyers might have different valuations for the product. We show that efficiency can be restored. In section 3, we comment on joint renegotiation, showing that a modification of the contract is coalition renegotiation proof. Section 4 considers the case of asymmetric information: when the valuation of each buyer is private information, we show that there exists a modification of the multilateral contract that induces truthful revelation and restores the optimal level of investment. The extension of the model to more than two buyers is straightforward and exposed in section 5; section 6 concludes.

2. Model

Consider three players: one upstream firm A and two downstream firms P_i, $i \in \{1, 2\}$. The two downstream firms are not competing with each other. A produces two goods, and chooses the level of investment $e \geq 0$; investment is costly, with $c(e)$ an increasing convex function[1]. The two goods are of the same level of quality, which may be of high or low, depending on the level of investment and on chance. More precisely, the probability that the goods are of high quality is $\pi(e) \in [0, 1]$, with $\pi(e)$ an increasing quasiconcave function. The monetary value of the low quality good is normalized to zero[2]. The high quality

[1]Introducing an additional fixed marginal cost per good does not change the setting.

[2]Alternatively, one might think about the case in which the downstream firms value the low quality product ϵ, while the buyers on the market value it zero. In this case, the contract still works, as long as $\epsilon \leq m$; it is sufficient that equation (5) is fulfilled.

goods can be sold to the market at a price $m > 0$, each[3]. The downstream firms attach a value $v_i = \beta_i m$, $i = 1, 2$ to a high quality product, where $\beta_i \geq 1$.

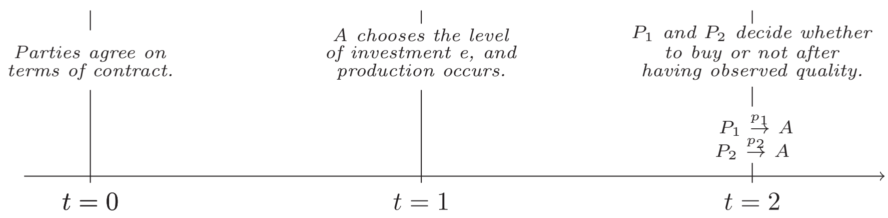

Figure 1: Timeline

In order to derive a closed-form solution for the model, we employ specific functional forms for the probability and cost functions. Namely, we assume that $\pi(e) = min\{\eta e, 1\}$, with $\eta > 0$, and $c(e) = \frac{\alpha}{2}e^2$, with $\alpha > 0$. Moreover, we assume that α is sufficiently high and η sufficiently small to prevent A to choose such a large investment level as to induce $\pi(e) = 1$. All players are risk neutral with standard utility functions. There are no transaction costs.

Timing is as follows (see figure 1): at time $t = 0$, partners decide upon the terms of the contract. After agreeing on the contract, at time $t = 1$, A chooses the level of investment, and production occurs. At $t = 2$, P_1 and P_2 decide whether to buy or not, after having observed the quality of the good.

2.1. Benchmark and Verifiability

To identify the efficient level of investment, we consider the case of a social planner that chooses the amount of investment e to maximize welfare:

$$\begin{aligned} & \max_e \pi(e)(\beta_1 + \beta_2)m - c(e) \\ = \quad & \max_e \eta e(\beta_1 + \beta_2)m - \frac{\alpha}{2}e^2. \end{aligned} \tag{1}$$

From the first order condition we can easily derive the optimal level of investment

$$e^{FB} \equiv \frac{\eta}{\alpha}(\beta_1 + \beta_2)m.$$

If *investment is verifiable, but quality is not*, this first best level of investment is still obtainable. A contract which specifies $e = e^{FB}$ and a fixed price $p_i \in [\frac{1}{2}c(e^{FB}), \beta_i m\pi(e^{FB})]$ for the product, independently of the realized quality, induces an efficient outcome and guarantees to each party profits at least as big as no trading.

[3] The contract works also for m = 0, setting the valuations of the downstream firms equal to $v_i = \beta_i$, with $\beta_i > 0$. Yet, then the exchange of the extra payment cannot any more be contingent on the exchange of monetary payments but must be made contingent on the exchange of the product, see also the comment in the conclusion.

Also if *investment is not verifiable, but quality is*, there exist incentive compatible contracts which induce efficient outcomes. Any contract which specifies a pair of prices (p_{h_i}, p_{l_i}) such that $p_{h_i} - p_{l_i} = \beta_i m$ and lump sum transfers $\tau \in R_+$ to distribute profits induces an efficient outcome. The most intuitive case is $p_i = \beta_i m$ for the high quality product and $p_i = 0$ for the low quality product, with $\tau = 0$.

Now suppose that *neither quality nor investment is verifiable*. On the one hand, a contract that specifies a fixed price for the good independently of the realized level of quality does not provide any incentive to invest to the upstream firm A. On the other hand, a contract where each downstream firm has the option to buy the good at time $t = 1$ at a price $p_i > m$ may be subject to renegotiation at time $t = 1$. If the downstream firm refuses to buy when the quality is high, the upstream firm can sell the products to the market just at a price m. Following Hart and Moore (1999), we assume that in the renegotiation stage the downstream firms have all bargaining power. Hence, anticipating the renegotiation, A invests

$$\begin{aligned} & \max_{e|p_i=m} U_A \\ = & \max_{e|p_i=m} \pi(e)(2m) - c(e) \\ = & \max_{e|p_i=m} \eta e 2m - \frac{\alpha}{2}e^2. \end{aligned} \tag{2}$$

From the first order conditions we can easily derive the induced investment level

$$e^{IC} \equiv \frac{\eta}{\alpha} 2m,$$

which is henceforth called the incentive compatible investment level. It is the highest level of investment that can be induced when neither quality nor investment is verifiable.

2.2. Multilateral contract

We now turn to the case of non-verifiable quality and non-verifiable investment. The following section shows that signing a complex contract involving all parties can lead to a more efficient solution than negotiating independent bilateral contracts. We focus on option contracts, i.e. contracts where P_1 and P_2 have the option to buy once they have observed the quality of the good. As we have shown, P_1 and P_2 will only exert the option if it costs less than the market price, m. Yet, parties may specify additional payments contingent on the fact that the other downstream firm buys the good. This way, parties can increase the surplus the upstream firm obtains in case of having produced a high-quality good. The contract specifies the following: P_1 has the option to buy the good at price m; if P_1 buys, then P_2 is required to pay ρ to A; once P_1 has taken its decision, P_2 has the option to buy the good at price m. The timing is summarized in figure 2 and 3.

When the goods are of high quality, then both P_1 and P_2 exert the option and firm A obtains a payoff of $(2m + \rho)$. By fixing $\rho = (\beta_1 + \beta_2 - 2)m$, the overall payment A obtains for two high quality goods is equal to $(\beta_1 + \beta_2)m$, the social optimum, and therefore A has incentives to choose the efficient level of investment. Given that P_2 is required to pay ρ contingent on P_1 having bought the good, P_2 needs to be compensated to fulfill the participation constraints. We denote x_0 and x_1 the payments that A and P_1 make in favor of P_2 upon signing the contract[4].

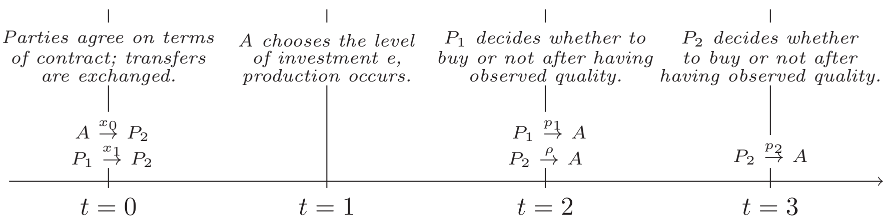

Figure 2: Timeline Trilateral Contract

While it may seem that some of the payments would cancel out -A pays x_0 to P_2, and P_2 pays ρ and m to A- this is not the case, seeing as the payments are conditional upon different events. The timing of the game is crucial: only by making payments conditional on previous payments is incentive compatibility assured. Note that the reason why sequential purchases work is not because it provides additional information -the firms observe the level of quality at all times- but because it generates a observable variable (i.e. whether P_1 bought the good or not), which is perfectly correlated with quality.

Proposition 1. *The trilateral contract* $\{p_1 = p_2 = m, \rho = (\beta_1 + \beta_2 - 2)m, x_0 = \frac{\eta^2}{\alpha} 2m^2(\beta_1 + \beta_2 - 2)$ *and* $x_1 = \frac{\eta^2}{\alpha} m^2(\beta_1 - 1)(\beta_1 + \beta_2 - 2)\}$ *is self-enforcing and induces the optimal level of investment.*

Proof Suppose the contract has been signed. Knowing that A gets the payments $p_1 = m$, $p_2 = m$ and ρ in case the product is of high quality, and since

[4]It can be shown that the contract works without monetary transfers from the upstream firm A, as well as without any transfers at all exchanged unconditional on the quality of the product. Yet, the range of valuations of the downstream firms that lead to a higher than the incentive compatible level of investment is smaller than in case the transfers are paid. The reason is that the unconditional transfers serve to relax the participation constraint of the downstream firm P_2. If A does not provide this transfer, the amount x_1 that P_2 receives is limited by the participation constraint of P_1.

If instead there are no unconditional transfers exchanged at all, the participation constraint of P_2 is even more binding.

the payment x_0 is paid *before* the level of investment is chosen[5], A maximizes

$$\begin{aligned} & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1,2\} \\ \rho}\right\}} U_A & (3) \\ = & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1,2\} \\ \rho}\right\}} \pi(e)(\sum_{i=1}^{2} p_i + \rho) - c(e) \\ = & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1,2\} \\ \rho}\right\}} \eta e(2m + \rho) - \frac{\alpha}{2} e^2. \end{aligned}$$

From the first order conditions follows that

$$\tilde{e} \equiv \frac{\eta}{\alpha}(2m + \rho)$$

is the optimal level of investment for A, given the contract has been signed. It is strictly increasing in ρ. For any $\rho > 0$, $\tilde{e}$ is greater than the incentive compatible level of investment e^{ic}; for $\rho = (\beta_1 + \beta_2 - 2)m$, $\tilde{e}$ equals the optimal level of investment e^{FB}.

We now study the conditions under which this contract is renegotiation-proof. Suppose the quality of the good is high. Both downstream firms have the possibility to refuse buying from the upstream firm, which is then forced to sell the products to the market at a price m. Since we assumed zero transaction costs, P_1 and P_2 can then buy at a price m. Once the quality is known, P_i will not renegotiate if

$$\beta_i m - p_i \geq \beta_i m - m \quad \forall i \in \{1, 2\}. \tag{4}$$

It will not buy a good of low quality if

$$-p_i \leq 0 \quad \forall i \in \{1, 2\}. \tag{5}$$

[5]Even if x_0 was paid *after* choosing the level of investment, A would incur the same level of investment as specified below, as long as x_0 and ρ are specified as in (3).

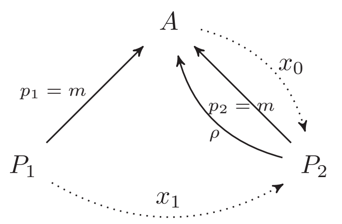

Figure 3: Trilateral contract

Both inequalities are clearly satisfied.

Is the contract individual rational? If the contract is not signed, A will invest e^{ic}, and the downstream firms pay m at the market. The expected value for the downstream firms computes to $\pi(e^{ic})(\beta_1 m - m)$. Therefore, having to pay x_1 upon signing the contract, P_1 is willing to participate in the contract if

$$-x_1 + \pi(\tilde{e})(\beta_1 m - p_1) \geq \pi(e^{ic})(\beta_1 m - m); \tag{6}$$

and P_2 will sign the contract if

$$x_0 + x_1 + \pi(\tilde{e})(\beta_2 m - p_2) - \pi(\tilde{e})\rho \geq \pi(e^{ic})(\beta_2 m - m). \tag{7}$$

A partakes when

$$\pi(\tilde{e})(2m + \rho) - c(\tilde{e}) - x_0 \geq \pi(e^{ic})(2m) - c(e^{ic}). \tag{8}$$

Assume the participation constraints (6) and (7) of the downstream firms to be binding. Replacing the resulting $x_0 = \pi(e^{ic})(\beta_1 + \beta_2 - 2)m - \pi(\tilde{e})(\beta_1 m + \beta_2 m - 2m - \rho)$, $\pi(\cdot)$, $c(\cdot)$ and the levels of investment e^{ic} and $\tilde{e}$ and choosing $\rho = (\beta_1 + \beta_2 - 2)m$, the participation constraints of A, P_1 and P_2 are fulfilled. The resulting x's are:

$$\begin{aligned} x_0 &= \frac{\eta^2}{\alpha} 2m^2(\beta_1 + \beta_2 - 2), \text{ and} \\ x_1 &= \frac{\eta^2}{\alpha} m^2(\beta_1 - 1)(\beta_1 + \beta_2 - 2). \square \end{aligned}$$

The trilateral contract that induces the efficient level of investment is not unique. However, they will all fix $p_1 = m, p_2 = m$ and $\rho = (\beta_1 + \beta_2 - 2)m$, and differ just in the payments at time $t = 0$, the x's. The contract that is specified above is such that the extra profits - compared to the incentive compatible case - are completely skimmed by the upstream firm.

Which of the two downstream firms P_1 and P_2 buys first is decided randomly. Since the expected payoff of the respective downstream firm is equal to the incentive compatible payoff they are indifferent to being the first or the second buyer[6].

3. Joint deviation under complete information

In the previous section we have shown that neither of the two downstream firms has incentive to renegotiate the contract. Yet, we have not ruled out the

[6] This makes sense as long as we assume that both transactions occur in a relatively short period of time. Obviously, it is also possible to change unconditional transfers such that the second downstream firm captures a higher share of the profit generated. Similarly, there exists a symmetric case in which both firms are paying and receiving the exact same transfers; see section Appendix A.4 in the appendix.

possibility that a subset of the participants coordinate its actions in a mutually beneficial way. Ignoring collusion would be an important oversight, especially since the participants can communicate at any stage of the contract.

For example, after signing the contract, P_1 and P_2 might agree not to exert the option and buy the good from the market at a price m. This would result in a joint payment for the two units of the good for P_1 and P_2 of $2m$ rather than $(2m+\rho)$. Hence, P_1 and P_2 have an incentive to deviate from the specified contract. In a similar vein, P_1 and A might want to defraud ρ from P_2 even in case the good is of low quality. In either case, to make deviation incentive compatible, parties need to agree upon the exchange of side payments. By including two additional clauses in the contract, the agents cannot credibly commit to fulfill the specifications of side-agreements. This way, we can make these deviations infeasible.

The two additional clauses we specify are the following. a) a clause inhibiting participating firms from making side contracts *conditional on the asserted quality*; and b) a clause specifying the exchange of a payment $0 < m_c < m$, payed from the first downstream firm P_1 to the second downstream firm P_2, in case P_1 claims low quality and P_2 claims high quality[7].

A contract is defined as coalition proof if it induces a Coalition-Proof Nash equilibrium. Being a Coalition-Proof Nash equilibrium means that no subcoalition of the agents taking part in the contract has incentive to deviate from the specified equilibrium (Bernheim, Peleg, and Whinston, 1987). Making use of the fact that a deviation is not self-enforcing, we show that

Proposition 2. *There exists a contract that is coalition deviation proof.*

There are three possible coalitions: $\{A, P_1\}$, $\{A, P_2\}$, and $\{P_1, P_2\}$. The coalition $\{A, P_2\}$ cannot gain anything by jointly deviating. Since the exchange of the payment ρ depends only on what the first downstream firm, P_1, reports, any possible joint deviation wanting to extract this amount has to include P_1. It remains to show that neither $\{A, P_1\}$ nor $\{P_1, P_2\}$ have incentive to deviate.

Proof See appendix, section Appendix A.1. □

The intuition behind the proof is the following. The coalition $\{A, P_1\}$ cannot gain anything in case the good is of high quality: when claiming low quality, P_1 would reduce the overall amount their coalition receives by ρ. If the good is of low quality, on the other hand, P_1 can increase the overall amount its coalition receives by ρ, claiming it to be of high quality. A and P_1 may agree upon a payment $\epsilon_{01} \in (m, \rho)$ as compensation for P_1 reporting falsely in such case. Yet, since the two parties are not allowed to make side contracts, the exchange of this payment is not incentive compatible. If ϵ_{01} is exchanged before P_1 reports the quality, P_1 does not have incentive to report falsely; if ϵ_{01} is supposed to

[7] Also with this clause the incentive compatibility constraint for P_2 when the good is of low quality is still satisfied: P_2 does not have incentive to buy a low quality product (claiming it to be high quality), since $-m + m_c < 0$.

be exchanged after P_1 has reported, A does not have incentive to pay, which in turn P_1 will anticipate.

The coalition $\{P_1, P_2\}$ cannot gain anything when the good is of low quality, since by falsely claiming high quality they have to pay for products their valuation is actually zero. If however the good is of high quality, by falsely claiming low quality, P_1 and P_2 might gain ρ. In this case, both clauses are needed to prevent a profitable joint deviation. Assume P_1 and P_2 agree upon a payment ϵ_{21} when P_1 claims low quality. If ϵ_{21} is exchanged *before* P_1 reports the quality, P_1 does not have incentive to falsely claim low quality. If on the other hand ϵ_{21} is supposed to be exchanged *after* P_1 claims the good to be of low quality, then P_2 is strictly better of by not paying it and reporting high quality. Anticipating this, P_1 will claim the true quality.

In conclusion, appending the subclauses above to the contract will prevent any coalition of the firms from reneging on the contract or falsely reporting the quality of the good.

4. Asymmetric Information

Up to now we considered the value the downstream firms attach to a high quality good, β_i to be common knowledge. But does the contract hold as well under information asymmetries? In this section we show that the contract specified in section 2.2 can be extended to situations in which the valuation of the good to each buyer is private information.

Again, we consider the case of two downstream firms. Assume that, before the contract is made, each downstream firm privately observes its type $\beta^k, k \in \{H, L\}$, where, as before, $\beta^H \geq \beta^L \geq 1$. The types β^H and β^L are identically and independently distributed, with $\Pr\{\beta_i = \beta^H\} = p \in [0, 1]$, the distribution being common knowledge. Let $\hat{\beta}_i$ be the reported type.

Proposition 3. *By specifying the extra-payment ρ and the transfers conditional on the reported values $(\hat{\beta}_1, \hat{\beta}_2)$, there exists a contract that induces truthful revelation and the optimal level of investment.*

Proof See section Appendix A.2 in the appendix. □

The intuition behind the proof is the following. The parameters $\{\rho(\hat{\beta}_1, \hat{\beta}_2), x_0(\hat{\beta}_1, \hat{\beta}_2), x_1(\hat{\beta}_1, \hat{\beta}_2)\}$ can be specified for each possible state - both firms reporting high valuations, both firms reporting low, as well as the two cases when they report differently. With this set we can show that truthful revelation holds in dominant strategies: neither of the downstream firms has incentive to misreport its type, independently of being of high or low type, independently on what the other downstream firm reports. Assuming that each firm can decide whether to participate or not *after* each downstream firm has revealed its type, we show that the ex-post participation constraints are fulfilled[8] all agents have incentive to participate in the contract.

Unlike the case of complete information, the downstream firms now capture some of the payoff when being of high type, even when transfers are specified as to maximize the upstream firm A's payoff. This can be explained by the existence of informational rents for the downstream firms. If they are of the low type , their participation constraints are binding, while when they are of the high type, their constraints on truthful reporting are. Since the x_1 transferred in the case of reporting (β^L, β^H) is already the biggest P_1 can provide (the participation constraint is binding), to keep truthful reporting a (weakly) dominant strategy, also the x_1 exchanged in case of reporting (β^H, β^H) cannot be decreased. This results in informational rents for the downstream firms. A similar reasoning holds for the x_2's received by P_2.

Again, as in the case of complete information, the payoffs for the downstream firms do not depend on the order in which they are placed. Whether they are in the first or the second downstream firm, each firm receives the same amount depending on its type.

5. More than two downstream firms

When extending the model to more than two downstream firms, several modifications to the trilateral contract come to mind. We work out one possible non-symmetric case more than two downstream firms, assuming complete information. For a symmetric case, see section Appendix A.4 in the appendix. We show that the optimal level of investment can still be induced.

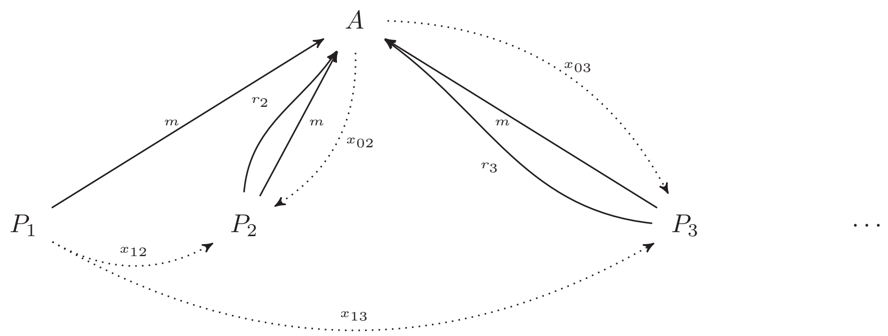

Figure 4: Multilateral contract with more than two downstream firms

[8]Since as ex-post constraints are stronger than interim and ex-ante participation constraints, the latter two will also be fulfilled.

The setting is very similar to the one considered in the previous sections. There are n downstream firms, each buying the product at a price equal to the market price $p_i = m$ in case the quality is high. Conditional on the first downstream firm, P_1, buying, the other downstream firms, P_i $i \in \{2, \ldots, k\}$, pay a share $r_i = \beta_i \frac{\sum_{i=1}^{k} \beta_i m - km}{\sum_{i=1}^{k} \beta_i}(1 + \frac{\beta_1}{\sum_{i=2}^{k} \beta_i})$ to the upstream firm.

Proposition 4. *A modification of the trilateral contract holds for $n \geq 2$ downstream firms. It is self-enforcing and induces the optimal level of investment.*

Proof See section Appendix A.3 in the appendix. □

The intuition of the proof follows the reasoning for the trilateral contract presented in section 2. When there are more than two downstream firms involved in the contract, one might rethink the distribution of bargaining power: now it seems plausible that the downstream firms receive a higher share of the generated profit. Yet, as mentioned above, through changes in the unconditional transfers, profit can be easily distributed in a different way.

6. Conclusion

In extending the literature on the hold-up problem, we have shown that such dilemma can be solved when there is more than one buyer involved in the transaction. By introducing sequentiality, we create the possibility to make transfers conditional on observed payments, thereby restoring efficiency. The result holds both under complete and under asymmetric information, and in the latter case induces truthful revelation of types. The contract is coalition renegotiation proof and extendable to more than two downstream firms.

The trilateral contract can induce the first best level of investment and satisfies the participation constraints of all agents. Yet, it is not the unique possible implementation of the contract. What is crucial is the exchange of payments *conditional* on another party buying the product. Depending on how the bargaining power is distributed, the surplus generated by the trilateral contract may be divided differently among the upstream firm and the downstream firms. While in the base-model we assumed the upstream firm to capture all the extra surplus, it is also possible to specify the transfers such that the surplus is divided differently. This might be of particular importance when considering more than two downstream firms.

Up to now, we have assumed that the high quality products can be sold at a price $m > 0$ to the market. Relaxing this assumption, the contract has to be changed slightly. If $m = 0$, we cannot condition the exchange of the extra payment upon the exchange of payments anymore. As before, the first downstream firm will only want to buy the good when it is of high quality. Yet, now it will not have to pay anything to get it. Hence, in this case the contract has to specify the exchange of an extra payment conditional on the *exchange of the product*. Apart from this modification, the reasoning on why the contract works stays the same.

Depending on the real cost-function of the investor, the gains from such a contract may be considerable. It is easily extendable to more than two downstream firms. For example, it could be used by the biggest players in the pharmaceutical industry to jointly invest in fundamental research, developing necessary basic processes and products to further develop different medicines. An interesting extension to our model would be to see what happens when we introduce competition among the downstream firms.

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Appendix A. Appendix

Appendix A.1. Proof of Proposition 2

Proof We will consider the possible deviations in turn.

a) Coalition $\{A, P_1\}$

Suppose the good is of high quality. A and P_1 cannot benefit when P_1 falsely reports low quality, since this reduces the amount the upstream firm A receives by ρ.

Now assume the good being of low quality. In this case, A and P_1 may benefit when P_1 announces it to be of good quality instead. While P_1 then pays $p_1 = m$ for the product that has no monetary value, A receives $\rho = (\beta_1 + \beta_2 - 2)m$ from P_2. In case $\beta_1 + \beta_2 \geq 3$, A may promise P_1 a payment of $\epsilon_{01} \in (m, \rho)$ for claiming the good to be of high quality, when instead it is low quality. This payment ϵ_{01} may be agreed upon after the quality is realized[9].

ϵ_{01} can be exchanged either before or after the downstream firm P_1 claims high quality. If A and P_1 agree upon exchanging it *after* P_1 has claimed high quality, since the payment is not legally enforceable, A is strictly better of by not paying ϵ_{01}, since

$$p_1 + \rho > p_1 + \rho - \epsilon_{01}.$$

Hence, P_1 wants to receive ϵ_{01} *before* claiming high quality. However, once P_1 has received ϵ_{01}, P_1 is strictly better of by not respecting the side-agreement with A, since

$$\epsilon_{01} - p_1 < \epsilon_{01}.$$

Therefore, this side-agreement is not deviation-proof, and hence not self-enforcing.

b) Coalition $\{P_2, P_1\}$

Assume the good is of low quality, or in other words had no monetary value for the two downstream firms. This means that they do not have incentive to jointly claim high quality, as they would have to pay an amount greater zero for receiving the good. Hence, at least one of the two firms is strictly worse of compared to reporting truthfully[10].

Now suppose the product is of high quality. When telling the truth, the downstream firms pay $(2m + \rho)$ for two high quality products. If instead they

[9]Before the quality is realized, the upstream firm would prefer not to agree to any side-payments, since it would gain strictly less when the quality is high after all. In any case, the reasoning also holds if the payment is agreed upon before quality is realized.

[10]The two downstream firms can both claim the good to being of high quality, in which case the first firm pays $-p_1 < 0$ and the second firm pays overall $-\rho - p_2 < 0$. Alternatively, they could agree upon P_1 claiming low and P_2 claiming high quality, in which case P_2 is strictly worse of $(-p_2 < 0)$ and P_1 is indifferent; or they can agree upon P_1 claiming high and P_2 claiming low quality, in which both P_1 and P_2 are worse of, with $-p_1 < 0$ and $-\rho < 0$ respectively.

deviate by agreeing on P_1 claiming low quality, they only pay $2m$ to the upstream firm. The "profit" ρ can be shared in such a way that both parties are strictly better of, specifying shares $\{\epsilon_{21}, \rho - \epsilon_{21}\}$ for P_1 and P_2 respectively, with $\epsilon_{21} \in (0, \rho)$. ϵ_{21} may be exchanged before or after P_1 claims low quality. If ϵ_{21} is exchanged *after* P_1 has claimed low quality, since the side-agreement is not legally enforceable, P_2 is strictly better of by deviating and not paying ϵ_{21}, since

$$\beta_2 m - p_2 + m_c > \beta_2 m - p_2 + m_c - \epsilon_{21}.$$

Now assume ϵ_{21} is exchanged *before* P_1 reports low quality[11]. Recall that m_c has to be paid by P_1 in case P_2 reports high quality *after* P_1 *has reported low quality*. Then, if P_1 has claimed the good to be of low quality, given that the quality is high, P_2 strictly prefers to claim high quality: since it already has paid ϵ_{21}, it can gain m_c when reporting a different level of quality than P_1. Hence, reporting high quality is preferred:

$$\beta_2 m - p_1 - \epsilon_{21} + m_c > \beta_2 m - m - \epsilon_{21}.$$

Anticipating this, P_1 will not claim low quality in the first place. He has already received ϵ_{21}, and reporting truthfully high quality, he does not have to pay m_c:

$$\beta_1 m - p_2 + \epsilon_{21} > \beta_1 m - m + \epsilon_{21} - m_c.$$

Therefore, also this deviation is not self-enforcing. It follows that our contract is coalition deviation proof.□

Appendix A.2. Proof of Proposition 3

Proof Incentive compatibility in (weakly) dominant strategies requires that there exists a strategy $\hat{\beta}_i = \beta_i^k$, $\forall\, i \in \{1, 2\}$, such that

$$U_i(\hat{\beta}_i, \hat{\beta}_{-i}|\beta_i) \;\; \geq U_i(\hat{\beta}'_i, \beta_{-i}|\beta_i) \; , \, \forall \hat{\beta}_i \text{ and all } \hat{\beta}'_i. \tag{A.1}$$

To find an equilibrium in dominant strategies, we need to specify the three functions $\{\rho(\hat{\beta}_1, \hat{\beta}_2), x_0(\hat{\beta}_1, \hat{\beta}_2), x_1(\hat{\beta}_1, \hat{\beta}_2)\}$ such that condition (A.1) is fulfilled. In addition, the participation constraints of the downstream firms (equations (A.16) - (A.23)) and of the upstream firm (equations (A.24) - (A.27)) have to be satisfied. Define $x_2(\hat{\beta}_1, \hat{\beta}_2) \equiv x_0(\hat{\beta}_1, \hat{\beta}_2) + x_1(\hat{\beta}_1, \hat{\beta}_2)$ and to simplify notation, replace $e(\hat{\beta}_H, \hat{\beta}_H)$ by e^{HH}, and similarly for e^{HL}, e^{LH}, and e^{LL}. Specify the x's

[11] W.r.t. specifying the exchanged payment before or after quality is realized, the same reasoning holds as with the previous coalition.

as follows[12]:

$$\begin{aligned}
\tilde{x}_1(\hat{\beta}_1^H, \hat{\beta}_2^H) &= \pi(e^{HH})(\beta^H - 1)m - \pi(e^{LH})(\beta^H - \beta^L)m & \text{(A.2)}\\
&\quad -\pi(e^{ic})(\beta^L - 1)m, &\\
\tilde{x}_2(\hat{\beta}_1^H, \hat{\beta}_2^H) &= \pi(e^{ic})(\beta^L - 1)m - \pi(e^{HH})(\beta^H m - m - \rho^{HH}) & \text{(A.3)}\\
&\quad +\pi(e^{HL})(\beta^H - \beta^L)m, &\\
\tilde{x}_1(\hat{\beta}_1^L, \hat{\beta}_2^H) &= [\pi(e^{LH}) - \pi(e^{ic})](\beta^L - 1)m, & \text{(A.4)}\\
\tilde{x}_2(\hat{\beta}_1^L, \hat{\beta}_2^H) &= \pi(e^{ic})(\beta^L - 1)m - \pi(e^{LH})(\beta^H m - m - \rho^{LH}) & \text{(A.5)}\\
&\quad +\pi(e^{LL})(\beta^H - \beta^L)m, &\\
\tilde{x}_1(\hat{\beta}_1^H, \hat{\beta}_2^L) &= \pi(e^{HL})(\beta^H - 1)m - \pi(e^{LL})(\beta^H - \beta^L)m & \text{(A.6)}\\
&\quad -\pi(e^{ic})(\beta^L - 1)m, &\\
\tilde{x}_2(\hat{\beta}_1^H, \hat{\beta}_2^L) &= \pi(e^{ic})(\beta^L - 1)m - \pi(e^{HL})(\beta^L m - m - \rho^{HL}), & \text{(A.7)}\\
\tilde{x}_1(\hat{\beta}_1^L, \hat{\beta}_2^L) &= [\pi(e^{LL}) - \pi(e^{ic})](\beta^L - 1)m, & \text{(A.8)}\\
\tilde{x}_2(\hat{\beta}_1^L, \hat{\beta}_2^L) &= \pi(e^{ic})(\beta^L - 1)m - \pi(e^{LL})(\beta^L m - m - \rho^{LL}). & \text{(A.9)}
\end{aligned}$$

First, we show that firms will prefer to report their evaluations truthfully, after which we show that the participation constraints of the downstream firms are satisfied and that the participation constraint of the upstream firm is fulfilled.

1. Truthful Revelation

The payoffs of the downstream firms are respectively:

$$\begin{aligned}
U_1(\hat{\beta}_1, \hat{\beta}_2|\beta_1) &= \pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))\,[\beta_1 m - p_1] - x_1(\hat{\beta}_1, \hat{\beta}_2), & \text{(A.10)}\\
U_2(\hat{\beta}_2, \hat{\beta}_1|\beta_2) &= \pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))\left[\beta_2 m - p_2 - \rho(\hat{\beta}_1, \hat{\beta}_2)\right] & \text{(A.11)}\\
&\quad +x_0(\hat{\beta}_1, \hat{\beta}_2) + x_1(\hat{\beta}_1, \hat{\beta}_2). &
\end{aligned}$$

Set $p_1 = p_2 = m$, and consider the downstream firms in turn.

a) Downstream firm P_1

We show that P_1 has no incentive to misreport its type, regardless of being of high or low type, and independently on what the second downstream firm P_2 reports. Suppose P_2 reports β^H, and suppose P_1 *is* of type β^H. Having specified the x's as above, when truthfully *reporting* being of type β^H, P_1 receives a payoff of

$$\pi(e^{LH})\left[\beta^H - \beta^L\right] m + \pi(e^{ic})\left[\beta^L - 1\right] m;$$

[12] The $x's$ are chosen according to the following strategy: to minimize the amount A has to provide, we choose x_1 such that P_1 pays the biggest amount possible satisfying its incentive compatibility and participation constraints. Similarly, x_2 is chosen such that P_2 receives the smallest amount possible such that its incentive compatibility and participation constraints are fulfilled.

when reporting to be of type β^L, it receives

$$\pi(e^{LH}) \left[\beta^H - \beta^L\right] m + \pi(e^{ic}) \left[\beta^L - 1\right] m.$$

Seeing as both payoffs are equal, reporting the true type weakly dominates non-truthful reporting. Now suppose the dowstream firm P_1 is of type β^L, while the downstream firm P_2 still reports being of high type. When P_1 reports to be of type β^H, it gets

$$\left[\pi(e^{LH}) - \pi(e^{HH})\right] \left[\beta^H - \beta^L\right] m + \pi(e^{ic}) \left[\beta^L - 1\right] m, \tag{A.12}$$

while when truthfully reporting β^L, it receives

$$\pi(e^{ic}) \left[\beta^L - 1\right] m. \tag{A.13}$$

x_0 is exchanged *before* the level of investment is incurred, hence, it can be shown that the payoff in (A.12) is smaller than the payoff in (A.13), since

$$\left[\pi(e^{LH}) - \pi(e^{HH})\right] \left[\beta^H m - \beta^L m\right] \leq 0.$$

Let us turn to the level of investment. Knowing that for a high quality good A receives an overall payment of $2m + \rho(\hat{\beta}_1, \hat{\beta}_2)$, it will try to maximize:

$$\begin{aligned}
& \max_{e|\left\{\substack{p_i = m, \forall i \in \{1,2\} \\ (\hat{\beta}_1, \hat{\beta}_2)}\right\}} U_A \\
= & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1,2\} \\ (\hat{\beta}_1, \hat{\beta}_2)}\right\}} \pi(e(\hat{\beta}_1, \hat{\beta}_2))[\sum_{i=1}^{2} p_i + \rho(\hat{\beta}_1, \hat{\beta}_2)] - c(e(\hat{\beta}_1, \hat{\beta}_2)) \\
= & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1,2\} \\ (\hat{\beta}_1, \hat{\beta}_2)}\right\}} \eta e(\hat{\beta}_1, \hat{\beta}_2)[2m + \rho(\hat{\beta}_1, \hat{\beta}_2)] - \frac{\alpha e(\hat{\beta}_1, \hat{\beta}_2)^2}{2}.
\end{aligned}$$

This results in $e(\hat{\beta}_1, \hat{\beta}_2) = \frac{\eta}{\alpha}(2m + \rho(\hat{\beta}_1, \hat{\beta}_2))$. Setting $\rho(\hat{\beta}_1, \hat{\beta}_2) = (\hat{\beta}_1 + \hat{\beta}_2 - 2)m$ will once more induce the optimal level of investment. Then, $e(\hat{\beta}_1, \hat{\beta}_2) = \frac{\eta}{\alpha}[\hat{\beta}_1 + \hat{\beta}_2]m$, and therefore, for $\beta^H \geq \beta^L \geq 1$, $e^{HH} \geq e^{HL}$. Hence, $\left[\pi(e^{LH}) - \pi(e^{HH})\right] \left[\beta^H - \beta^L\right] m \leq 0$.

Now suppose the downstream firm P_2 reports being of type β^L. Suppose P_1 is of type β^H. When reporting being of type β^H, it receives a payoff of

$$\pi(e^{LL}) \left[\beta^H - \beta^L\right] m + \pi(e^{ic}) \left[\beta^L - 1\right] m,$$

while when reporting being of type β^L, it receives

$$\pi(e^{LL}) \left[\beta^H - \beta^L\right] + \pi(e^{ic}) \left[\beta^L - 1\right] m.$$

Again, the two payoffs are equal, and reporting the true type weakly dominates non-truthfully reporting. Now suppose P_1 is of type β^L, with P_2 still reporting being of type β^L. When reporting being of type β^H, P_1 receives a payoff of

$$\left[\pi(e^{LL}) - \pi(e^{HL})\right] \left[\beta^H - \beta^L\right] m + \pi(e^{ic}) \left[\beta^L - 1\right] m,$$

while when truthfully reporting β^L, it receives

$$\pi(e^{ic})\left[\beta^L - 1\right]m.$$

Applying the reasoning above and taking into account that $e^{HH} \geq e^{LH}$, reporting the truth dominates non-truthful reporting. Since, in expected terms, final payoffs of the downstream firms are equal - given that they are of the same type - a symmetric reasoning holds for the truthful reporting of P_2.

In conclusion, with the $x's$ specified as in (A.2) - (A.9), the downstream firms have incentive to truthfully reveal their types. But do they also want to join the trilateral contract? In the following sectino, we show that the participation constraints are fulfilled.

2. Participation Constraints

a) Participation Constraints of Downstream Firms

The participation constraints of P_1 and P_2 are respectively:

$$\begin{aligned}
\pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))(\beta_1 m - p_1) - x_1(\hat{\beta}_1, \hat{\beta}_2) &\geq \pi(e^{ic})(\beta_1 m - m), \\
\pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))[\beta_2 m - p_2 - \rho(\hat{\beta}_1, \hat{\beta}_2)] + x_2(\hat{\beta}_1, \hat{\beta}_2) &\geq \pi(e^{ic})(\beta_2 m - m),
\end{aligned}$$

which is

$$\begin{aligned}
x_1(\hat{\beta}_1, \hat{\beta}_2) &\leq \pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))(\beta_1 m - m) - \pi(e^{ic})(\beta_1 m - m), &\text{(A.14)} \\
x_2(\hat{\beta}_1, \hat{\beta}_2) &\geq \pi(e^{ic})(\beta_2 m - m) &\text{(A.15)} \\
&\quad -\pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))[\beta_2 m - m - \rho(\hat{\beta}_1, \hat{\beta}_2)].
\end{aligned}$$

Assuming truthful reporting, for the respective values of $\hat{\beta}_i$ and β_i, A.14 and A.15 become

$$\begin{aligned}
x_1(\hat{\beta}_1^H, \hat{\beta}_2^H) &\leq [\pi(e^{HH}) - \pi(e^{ic})](\beta^H m - m), &\text{(A.16)} \\
x_2(\hat{\beta}_1^H, \hat{\beta}_2^H) &\geq \pi(e^{ic})(\beta^H m - m) - \pi(e^{HH})(\beta^H m - m - \rho^{HH}), &\text{(A.17)} \\
x_1(\hat{\beta}_1^L, \hat{\beta}_2^H) &\leq [\pi(e^{LH}) - \pi(e^{ic})](\beta^L m - m), &\text{(A.18)} \\
x_2(\hat{\beta}_1^L, \hat{\beta}_2^H) &\geq \pi(e^{ic})(\beta^H m - m) - \pi(e^{LH})(\beta^H m - m - \rho^{LH}), &\text{(A.19)} \\
x_1(\hat{\beta}_1^H, \hat{\beta}_2^L) &\leq [\pi(e^{HL}) - \pi(e^{ic})](\beta^H m - m), &\text{(A.20)} \\
x_2(\hat{\beta}_1^H, \hat{\beta}_2^L) &\geq \pi(e^{ic})(\beta^L m - m) - \pi(e^{HL})(\beta^L m - m - \rho^{HL}), &\text{(A.21)} \\
x_1(\hat{\beta}_1^L, \hat{\beta}_2^L) &\leq [\pi(e^{LL}) - \pi(e^{ic})](\beta^L m - m), &\text{(A.22)} \\
x_2(\hat{\beta}_1^L, \hat{\beta}_2^L) &\geq \pi(e^{ic})(\beta^L m - m) - \pi(e^{LL})(\beta^L m - m - \rho^{LL}). &\text{(A.23)}
\end{aligned}$$

By substituting $\tilde{x}_1(\hat{\beta}_1, \hat{\beta}_2)$ and $\tilde{x}_2(\hat{\beta}_1, \hat{\beta}_2)$ in the participation constraints (A.16) - (A.23), it can be seen that equations (A.18), (A.21), (A.22), and (A.23) are binding. Equations (A.16), (A.17), (A.19), and (A.20) can be simplified to, respectively

$$\begin{aligned}
(\pi(e^{LH}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0, \\
(\pi(e^{HL}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0, \\
(\pi(e^{LL}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0, \\
(\pi(e^{LL}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0,
\end{aligned}$$

which are all clearly satisfied for $\beta^H \geq \beta^L \geq 1$. So, the specified $\tilde{x}$'s also satisfy the participation constraints of P_1 and P_2 .

It remains to check if A can provide $x_0(\hat{\beta}_1, \hat{\beta}_2) = x_2(\hat{\beta}_1, \hat{\beta}_2) - x_1(\hat{\beta}_1, \hat{\beta}_2)$, which is in each case:

$$\begin{aligned}
\tilde{x}_0(\hat{\beta}_1^H, \hat{\beta}_2^H) &= \pi(e^{ic})(2\beta^L - 2)m - \pi(e^{HH})(2\beta^H m - 2m - \rho^{HH}) \\
&\quad +[\pi(e^{HL}) + \pi(e^{LH})](\beta^H - \beta^L)m \\
\tilde{x}_0(\hat{\beta}_1^L, \hat{\beta}_2^H) &= \pi(e^{ic})(2\beta^L - 2)m - \pi(e^{LH})(\beta^H m + \beta^L m - 2m - \rho^{LH}) \\
&\quad +\pi(e^{LL})(\beta^H - \beta^L)m \\
\tilde{x}_0(\hat{\beta}_1^H, \hat{\beta}_2^L) &= \pi(e^{ic})(2\beta^L - 2)m - \pi(e^{HL})(\beta^H m + \beta^L m - 2m - \rho^{HL}) \\
&\quad +\pi(e^{LL})(\beta^H - \beta^L)m \\
\tilde{x}_0(\hat{\beta}_1^L, \hat{\beta}_2^L) &= \pi(e^{ic})(2\beta^L - 2)m - \pi(e^{LL})(2\beta^L m - 2m - \rho^{LL}).
\end{aligned}$$

b) Participation Constraint of Upstream Firm

The ex-post participation constraint of A is

$$\pi(e(\hat{\beta}_1, \hat{\beta}_2))[2m + \rho(\hat{\beta}_1, \hat{\beta}_2)] - c(e(\hat{\beta}_1, \hat{\beta}_2)) - x_0(\hat{\beta}_1, \hat{\beta}_2) \geq \pi(e^{ic})(2m) - c(e^{ic}),$$

which for each case results in:

$$\begin{aligned}
x_0(\hat{\beta}_1^H, \hat{\beta}_2^H) &\leq \pi(e^{HH})(2m + \rho^{HH}) - \pi(e^{ic})(2m) - c(e^{HH}) \quad \text{(A.24)} \\
&\quad +c(e^{ic}) \\
x_0(\hat{\beta}_1^L, \hat{\beta}_2^H) &\leq \pi(e^{LH})(2m + \rho^{LH}) - \pi(e^{ic})(2m) - c(e^{LH}) \quad \text{(A.25)} \\
&\quad +c(e^{ic}) \\
x_0(\hat{\beta}_1^H, \hat{\beta}_2^L) &\leq \pi(e^{HL})(2m + \rho^{HL}) - \pi(e^{ic})(2m) - c(e^{HL}) \quad \text{(A.26)} \\
&\quad +c(e^{ic}) \\
x_0(\hat{\beta}_1^L, \hat{\beta}_2^L) &\leq \pi(e^{LL})(2m + \rho^{LL}) - \pi(e^{ic})(2m) - c(e^{LL}) \quad \text{(A.27)} \\
&\quad +c(e^{ic})
\end{aligned}$$

Inserting the respective values for $\rho(\cdot)$, $e(\cdot)$, $\pi(\cdot)$, $c(\cdot)$, and $\tilde{x}_0(\cdot)$, equations (A.24) - (A.27) become

$$2(\beta^L)^2 - 4\beta^L + 2 \geq 0,$$
$$\frac{1}{2}(\beta^H)^2 + \frac{5}{2}(\beta^L)^2 - \beta^H\beta^L - 4\beta^L + 2 \geq 0,$$
$$\frac{1}{2}(\beta^H)^2 + \frac{5}{2}(\beta^L)^2 - \beta^H\beta^L - 4\beta^L + 2 \geq 0,$$
$$2(\beta^L)^2 - 4\beta^L + 2 \geq 0.$$

It can be checked that all four equations hold for $\beta^H \geq \beta^L \geq 1$. So we have shown that there exist $\tilde{x}$'s that induce truthful revelation and fulfill the participation constraints of each firm. □

Appendix A.3. Proof of Proposition 4

Proof When there are k downstream firms, A receives an overall payment of $\sum_{i=1}^{k} p_i + \sum_{i=2}^{k} r_i$. It will try to maximize:

$$\begin{aligned} & \max_{e|\left\{\substack{p_i \,=\, m,\, \forall i \,\in\, \{1,\ldots k\} \\ \rho}\right\}} U_A \\ = & \max_{e|\left\{\substack{p_i \,=\, m,\, \forall i \,\in\, \{1,\ldots k\} \\ \rho}\right\}} \pi(e)(\sum_{i=1}^{k} p_i + \sum_{i=2}^{k} r_i) - c(e) \\ = & \max_{e|\left\{\substack{p_i \,=\, m,\, \forall i \,\in\, \{1,\ldots k\} \\ \rho}\right\}} \eta e(km + \rho) - \frac{\alpha e^2}{2}, \end{aligned}$$

which results in

$$\bar{e} = \frac{\eta}{\alpha}(km + \rho), \quad \text{with} \;\; \rho = \sum_{i=2}^{k} r_i.$$

Summing the payments r_i over the $(k-1)$ downstream firms, it can be seen that ρ is equal to the amount required to induce the optimal level of investment: $\sum_{i=2}^{k} r_i = \rho = \sum_{i=1}^{k} \beta_i m - km$. The incentive compatible level of investment on the other hand equals $e^{ic} = \frac{\eta}{\alpha}(km)$, which comes from maximizing A's $\pi(e)(km) - c(e)$. The incentives for renegotiation are similar to the trilateral contract - since no downstream firm pays more *for the product* than the market price, no one has incentive to renegotiate once the contract has been signed. Additionally, none of the downstream firms has an incentive to buy a low quality product. The participation constraint of P_1 is

$$\pi(\bar{e})(\beta_1 - 1)m - \sum_{i=2}^{k} x_{1i} \geq \pi(e^{ic})(\beta_1 - 1)m.$$

For the remaining $(k-1)$ downstream firms $P_i, \;\forall i \in \{2, \ldots, k\}$, it equals

$$\pi(\bar{e})(\beta_i m - m - r_i) + x_{0i} + x_{1i} \geq \pi(e^{ic})(\beta_i - 1)m.$$

Summing up over all downstream firms, this results in

$$\sum_{i=2}^{k} x_{0i} + \sum_{i=2}^{k} x_{1i} \geq \pi(e^{ic})(\sum_{i=2}^{k} \beta_i - (k-1))m - \pi(\bar{e})(\sum_{i=2}^{k} \beta_i m - (k-1)m - \rho).$$

The participation constraint of the upstream firm is

$$\pi(\bar{e})(km + \rho) - c(\bar{e}) - \sum_{i=2}^{k} x_{0i} \geq \pi(e^{ic})(km) - c(e^{ic}). \qquad \text{(A.28)}$$

Taking the participation constraints of all downstream firms as binding and inserting the respective values for $\sum_{i=2}^{k} x_{0i}$ and $\sum_{i=2}^{k} x_{1i}$, equation (A.28) becomes $\sum_{i=1}^{k} \beta_i \geq k$. For $\beta_i \geq 1, \forall i \in \{1, \ldots, k\}$, this condition is fulfilled. The contract is implementable. □

Appendix A.4. More Than 2 Downstream Firms: Symmetric Case

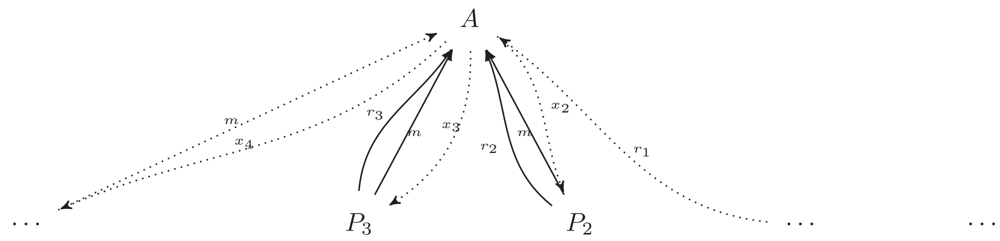

Figure A.5: Multilateral contract with more than two downstream firms, symmetric case

Proposition 5. *There exists a self-enforcing multilateral contract $\tilde{c}$ that induces the optimal level of investment, and therefore increases overall welfare.*

Each downstream firm, when buying the product from A, pays a price p_i equal to the market price m. Conditional on buying, another downstream firm pays a transfer r_i to the upstream firm (see figure A.5).

Proof Knowing that for producing high quality products, A receives a payment of $km + \rho$, it will try to maximize

$$\begin{aligned} & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1, \ldots k\} \\ \rho}\right\}} U_A \\ = & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1, \ldots k\} \\ \rho}\right\}} \pi(e)(\sum_{i=1}^{k} p_i + \sum_{i=1}^{k} r_i) - c(e) \\ = & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1, \ldots k\} \\ \rho}\right\}} \eta e(km + \rho) - \frac{\alpha e^2}{2}, \end{aligned}$$

which results in

$$\bar{e} = \frac{\eta}{\alpha}(km + \rho), \ \text{with} \rho = \sum_{i=1}^{k} r_i.$$

The incentive compatible level of investment with k downstream firms is $e^{ic} = \frac{\eta}{\alpha}(km)$, resulting from A's maximization of $\pi(e)(km) - c(e)$. The ρ required to induce the optimal level of investment is $\rho = \sum_{i=1}^{k} \beta_i m - km$. The incentives for renegotiation are similar to the trilateral contract - since no downstream firm pays more *for the product* than the market price, no one has incentive to renegotiate once the contract has been signed. As well, no downstream firm has incentive to buy a low quality product. The participation constraints of the k downstream firms P_i, $i \in \{1, \dots, k\}$, are respectively

$$\pi(\bar{e})(\beta_i m - m - r_i) + x_i \geq \pi(e^{ic})(\beta_i m - m), \tag{A.29}$$

which results, for all downstream firms together, in

$$\sum_{i=1}^{k} x_i \geq \pi(e^{ic})(\sum_{i=1}^{k} \beta_i - k)m - \pi(\bar{e})(\sum_{i=1}^{k} \beta_i m - km - \rho).$$

The participation constraint of the upstream firm A is

$$\pi(\bar{e})(km + \rho) - c(\bar{e}) - \sum_{i=1}^{k} x_i \geq \pi(e^{ic})(km) - c(e^{ic}), \tag{A.30}$$

which, taking (A.29) as binding and replacing $\sum_{i=1}^{k} x_i$ can be simplified to

$$[\pi(\bar{e}) - \pi(e^{ic})] \sum_{i=1}^{k} \beta_i m \geq c(\bar{e}) - c(e^{ic}). \tag{A.31}$$

Replacing the functional forms, this results in $\sum_{i=1}^{k} \beta_i \geq k$. This condition is fulfilled for $\beta_i \geq 1 \ \forall i \in \{1, \dots, k\}$, and hence, the participation constraints are fulfilled. The contract is implementable. $\square$