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# Horizontal Product Differentiation: Disclosure and Competition\*

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## Abstract

The unraveling argument says that when a firm may produce different qualities and quality is unknown to consumers, the firm has an incentive to disclose the private information as in any pool of firms there is a best quality firm and this firm has an incentive to disclose. Recent literature has established that this argument does not carry over to an environment where the product is not vertically, but horizontally differentiated. This paper argues that with horizontally differentiated products, competition restores the unraveling argument. In a duopoly market we show that all equilibria of the disclosure game have firms fully disclosing the variety they produce.

JEL Classification: D43, D82, D83, M37

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# 1 Introduction

In a large number of markets, sellers have important information about product attributes that are not publicly observable. In many instances, however, firms have the option of voluntarily disclosing this information in a credible and verifiable manner through a variety of means such as independent third party certification, labeling, rating by industry associations (or government agencies) and through informative advertising.

There is a large literature dealing with the question whether firms have appropriate incentives to disclose information about the product they produce. Most of this literature deals with this issue in the context of vertical product differentiation, where different firms sell different qualities. In this context, the well-known unraveling argument,<sup>1</sup> establishes that a firm whose product is actually better than the average has a positive incentive to voluntarily disclose the quality of its product to buyers. This then induces every firm whose quality is above the average *undisclosed* quality to also disclose. The unraveling argument results in a situation where all private information about quality should be revealed through voluntary disclosure. Observed nondisclosure is then explained in terms of "disclosure frictions", such as disclosure costs, consumers not understanding the information that is disclosed, etc. (see, e.g., Grossman and Hart (1980), Jovanovic (1982) and Fishman and Hagerty (2003)). Alternatively, Janssen and Roy (2010) show that nondisclosure can also be explained by a combination of *market competition* and the availability of *signaling* as an alternative means (to disclosure) of communicating private information.

Recently, Sun (2011) and Celik (2011) have analyzed the incentives for firms to disclose their product characteristics when horizontal product differentiation is the only or main dimension of differentiation. Both papers are set in a monopoly context. Sun (2011) shows that seller types with unfavorable horizontal attributes (towards the extreme points of the product line) do not have an incentive to disclose. In combination with vertical differentiation, her results imply that if either full disclosure of both attributes or no disclosure at all are the only possible reporting strategies, a seller with private information about both horizontal and vertical attributes may not want to disclose quality even if it is high. Celik (2011) shows that the amount of information disclosure critically depends on the strength of the buyer's preference for her ideal attribute. If buyers have very strong preferences for particular product varieties, then there exists an equilibrium in which the seller fully reveals variety. Otherwise, the seller only partially reveals the variety he produces. Moreover, the set of fully revealed locations monotonically shrinks from all to (almost) none as the buyer's preference

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<sup>1</sup>See, Viscusi (1978), Grossman (1981), Milgrom (1981), Jovanovic (1982).

for her ideal taste becomes weaker.

In this paper, we show that the findings of possible non-disclosure of horizontal product attributes do not extend to a competitive environment. In particular, we show in a duopoly set-up that full disclosure is always an equilibrium, and moreover, that there does not exist an equilibrium where firms do not fully disclose their product information. As there can be many messages with which firms fully disclose their information, the equilibrium strategies are not unique, but the equilibrium outcome of full disclosure is.

The model we consider has two firms located on a Hotelling line, where each particular location represents the variety of the product. Location is known to both firms, but not to consumers.<sup>2</sup> The case where rival firms know each others' *vertical* characteristic is studied by, e.g., Board (2009) and Hotz and Xiao (2011). This type of literature, and thus our paper, is relevant for markets where firms have been active for some time and have the ability (and due to the frequent interaction also the incentives) to learn the features of the product produced by a competitor.

The two firms first simultaneously choose a message about their location. We assume that firms cannot lie. That is, the true location should be consistent with the message that is chosen. One way to think about this grain-of-truth assumption is that information is verifiable and that there is a large fine for providing information that turns out to be false. The assumption is in line with regulations concerning advertisement or other disclosure mechanisms requiring that firms provide truthful information. Firms can either send a rather vague message, indicating that their location is somewhere on the product line, as one extreme, or a much more precise message, indicating the exact location, as the other extreme, or anything in between. After firms have sent their messages, they both simultaneously choose prices. Consumers decide where to buy the product after observing the messages and the prices. Given the information they receive, consumers update their beliefs about the location of the two firms and buy from the firm they expect to have the best fit with their preferences.

In a series of papers, Anderson and Renault (2006, 2009) consider a similar framework and study the incentives of firms to disclose their product characteristics through advertising. They find that if products have both horizontal and vertical attributes and if qualities of firms' products are known and sufficiently different, only the firm with the lowest quality reveals its horizontal characteristic. The better quality firm remains silent as disclosure would induce it to set a lower price in order to retain the consumers who like its rival more. If firms' product qualities are identical, both firms

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<sup>2</sup>The competitive disclosure literature has also considered markets where firms do not know each others' type (see, e.g., Daughety and Reinganum (2007), Caldieraro, Shin and Stivers (2008) and Janssen and Roy (2011)).

reveal their horizontal characteristics. Despite the fact that the content message is similar, our paper differs in two important respects. First, in Anderson and Renault (2009) firms can only fully disclose their horizontal characteristic or stay silent. In contrast, we consider a model where firms can send any message concerning their product characteristics that satisfies the grain-of-truth assumption. Second, and more importantly, Anderson and Renault (2006, 2009) do not analyze their model as a game with private information where out-of-equilibrium beliefs are important, while we do. Our methodological innovation is that we develop an equilibrium notion that captures the incentives of firms to signal their type in a model like this. To see where out-of-equilibrium beliefs are important, consider a potential equilibrium where no type of firm discloses and all types set a high price. Anderson and Renault (2009) argue that this cannot be an equilibrium because of a standard Bertrand-like undercutting argument. However, undercutting the candidate equilibrium price is formally an out-of-equilibrium action and therefore consumers should form beliefs about the undercutting firm's product characteristics. If consumers believe this firm has disadvantageous product characteristics, the firm's demand would be lower than if it had not undercut, and therefore, the firm would not have an incentive to undercut in the first place. We argue that this nondisclosure equilibrium is not reasonable, but standard refinements like the Intuitive Criterion (Cho and Kreps, 1987) or D1 (Cho and Sobel, 1990) do not rule out this equilibrium as the profits of all types only depend on consumer beliefs about their location rather than on the actual location. We develop an equilibrium notion where consumers have so-called stable beliefs to capture the idea that if after observing an out-of-equilibrium action, consumers have no reason to discriminate between different types, their out-of-equilibrium beliefs should not discriminate between these types either.

The reason why in the environment we study all stable belief equilibria must be fully revealing is intimately related to the reason why in the standard Hotelling model with location choice, firms want to maximally differentiate from each other. Suppose that a firm would not choose a fully revealing strategy and would choose the same message for different locations. In this case, consumers will be uncertain about the true location of the firm and the updated beliefs of consumers will be such that they do not assign full probability mass to the extreme locations that send this particular message. At least one of these extreme locations has then an incentive to deviate for two reasons. First, by fully revealing its location, a firm can reduce the uncertainty concerning the location for consumers and any reduction in uncertainty increases demand *ceteris paribus*.<sup>3</sup> Second, by having a perceived

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<sup>3</sup>One way to interpret this is that convex transportation cost introduces an element of risk aversion in consumers' preferences: *ceteris paribus* a consumer rather buys at a known location than at an unknown location with the same expected value.

location that is further away from the competitor, firms charge higher price in the pricing game and this price effect outperforms the direct demand effect.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the results and Section 4 concludes.

## 2 Model

Consider a horizontally differentiated duopoly, where the variety produced by each firm is represented by a particular location on the unit interval. Let  $x_i$  denote the variety produced by firm  $i$  and  $x_i \in [0, 1]$ ,  $i = 1, 2$ . We focus on the disclosure policy of the firms and consider these varieties to be given for the firms. In the following,  $x_1$  and  $x_2$  will be referred to as locations or types of firms 1 and 2, respectively. We consider markets where firms know each others's location, but consumers are unaware of the specific location of firms. One way to think about this is that it requires resources to research the product characteristics of a firm and that rival firms are better equipped or have more incentives to do this than consumers. Production costs do not depend on firms' locations and without loss of generality are set to be equal to zero.

The demand side of the economy is represented by a continuum of consumers. Each consumer has a preference for the ideal variety of the good that she would like to buy, denoted by  $\lambda$ . The value of  $\lambda$ , or consumers' location on  $[0, 1]$ , follows a uniform distribution.<sup>4</sup> A consumer's net utility from buying variety  $x_i$  at price  $P_i$ ,  $i = 1, 2$ , is  $v - t(\lambda - x_i)^2 - P_i$ , where  $v$  is the gross utility of a consumer when the variety of the good,  $x_i$ , matches with her ideal variety,  $\lambda$ , perfectly (i.e., when  $x_i = \lambda$ ) and  $t$  measures the degree of disutility a consumer incurs when  $x_i$  and  $\lambda$  differ from each other. We assume that  $v$  is sufficiently large so that the market is fully covered.<sup>5</sup> Each consumer then chooses to buy the good from the firm where her expected utility is maximized. The consumer has unit demand and if she buys from firm  $i$ , then firm  $i$ 's payoff from the transaction is  $P_i$ ; otherwise, the payoff of firm  $i$  is zero.

The timing of the game is as follows. At stage 0, Nature *independently* selects location  $x_1$  for firm 1 and  $x_2$  for firm 2 from a strictly positive density function  $f(x)$ . The locations are known to both firms, but not to consumers. At stage 1, firms send a costless message  $M_i \subset [0, 1]$ ,  $i = 1, 2$ , about their location, where  $M_i = [0, 1]$  can be interpreted as "no message at all", or full non-disclosure of

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<sup>4</sup>This specification with a continuum of consumers whose preferences for variety are uniformly distributed over the unit interval, is identical to the specification with a single consumer who has a privately known taste for a variety drawn from the uniform density function defined over  $[0, 1]$ .

<sup>5</sup>In the concluding section 4 we argue that large  $v$  is essentially the most interesting case to consider as it highlights the difference between the competitive and non-competitive setting.

information by firm  $i$ . Messages have to contain a grain of truth in the sense that  $x_i \in M_i$  for  $i = 1, 2$ . That is, firms cannot lie about their location. In the following we will refer to this assumption as the grain of truth assumption. At stage 2, firms simultaneously set prices. Finally, at stage 3, consumers observe the messages and the prices of the two firms and decide where to buy. At the end of the game, the payoffs of all players – firms and consumers – are realized. All aspects of the game are common knowledge.

Two important observations are in order at this point. First, the quadratic term in the utility function of consumers (the transportation costs) implies risk aversion with respect to  $x_i$ . That is, a consumer dislikes uncertainty about the variety of the good and given two messages with the same conditional mean, favors the one with the smaller variance. Second, even though consumers are assumed to have unit demand for the good, the probability of a purchase from a given firm declines with its price so that the expected demand function faced by each firm is downward sloping.

To proceed with the more formal analysis, we define the strategy spaces as follows. The reporting strategy of firm  $i$  is denoted by  $m_i(x_i, x_j)$ . The image of  $m_i$  belongs to all subsets of  $[0, 1]$  such that  $x_i \in m_i$ . The pricing strategy of firm  $i$  is denoted by  $p_i(x_i, x_j | M_i, M_j)$  where the messages sent by the two firms are  $M_i$  and  $M_j$ , respectively. Similarly, let the vector  $b(\lambda, M_i, M_j, P_i, P_j)$  describe the buying strategy of a consumer with preferred variety  $\lambda$ , where  $b = (1, 0)$  if the consumer buys the good from firm 1 and  $b = (0, 1)$  if she buys from firm 2. Finally, let  $\mu_i(z | M_i, M_j, P_i, P_j)$  be the probability density that consumers assign to  $x_i = z$  when the firms send messages  $M_i, M_j$  and set prices  $P_i, P_j$ . Note that at the moment when consumers have to decide from which firm to buy, they form beliefs  $\mu_i$  not only on the basis of the observed messages and prices, but also on the basis of the equilibrium strategies, that is, equilibrium messages and prices,  $m_i^*(x_i, x_j)$  and  $p_i^*(x_i, x_j | M_i, M_j)$ . All consumers process the information received in the same way and therefore have symmetric beliefs.

Before providing the details of the equilibrium notion which we will use to analyze the game, we consider the decision making of a consumer. To do so, we first find the ideal variety,  $\hat{\lambda}$ , of the *indifferent* consumer, who obtains the same expected net utility of buying from either of the two firms, given the observed set of messages and prices. Then all consumers with ideal varieties below  $\hat{\lambda}$  buy from the firm with the most left perceived location and all others buy from the other firm. Therefore,  $\hat{\lambda}$  determines the expected demand faced by each firm and allows describing optimal prices and messages chosen by the firms at the previous two stages of the game.

Given the updated beliefs, the ideal variety  $\hat{\lambda}$  of the indifferent consumer is defined by the equality between the expected net utility of buying from firm 1 and the expected net utility of buying from

firm 2:

$$v - tE\left((\widehat{\lambda} - x_1)^2|\mu_1\right) - P_1 = v - tE\left((\widehat{\lambda} - x_2)^2|\mu_2\right) - P_2 \quad (2.1)$$

In this expression,  $tE\left((\widehat{\lambda} - x_i)^2|\mu_i\right)$ ,  $i = 1, 2$ , is the expectation of the transportation costs of the indifferent consumer associated with buying from firm  $i$ , conditional on consumers' beliefs.

We solve this equality for  $\widehat{\lambda}$ . Notice that

$$\begin{aligned} E\left((\widehat{\lambda} - x_i)^2|\mu_i\right) &= \widehat{\lambda}^2 + E(x_i^2|\mu_i) - 2\widehat{\lambda}E(x_i|\mu_i) \\ &= \widehat{\lambda}^2 + \text{var}(x_i|\mu_i) + E^2(x_i|\mu_i) - 2\widehat{\lambda}E(x_i|\mu_i) \end{aligned}$$

so that (2.1) becomes:

$$\begin{aligned} \widehat{\lambda}^2 + \text{var}(x_1|\mu_1) + E^2(x_1|\mu_1) - 2\widehat{\lambda}E(x_1|\mu_1) + \frac{P_1}{t} &= \\ &= \widehat{\lambda}^2 + \text{var}(x_2|\mu_2) + E^2(x_2|\mu_2) - 2\widehat{\lambda}E(x_2|\mu_2) + \frac{P_2}{t}. \end{aligned}$$

Thus, the ideal variety of the indifferent consumer is equal to:

$$\widehat{\lambda} = \frac{1}{2} \frac{\frac{P_2 - P_1}{t} + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)}{E(x_2|\mu_2) - E(x_1|\mu_1)} + \frac{1}{2} (E(x_1|\mu_1) + E(x_2|\mu_2)) \quad (2.2)$$

This result has an immediate implication for the form of the expected demand functions of firms 1 and 2. In fact, since consumers with preferred variety  $\lambda < \widehat{\lambda}$  ( $\lambda > \widehat{\lambda}$ ) buy from the firm with the most left (right) perceived location and since the value of consumer's best-preferred variety is distributed uniformly over  $[0, 1]$ ,  $\widehat{\lambda}$  is also the value of the expected demand faced by the firm with the most left perceived location, given the prices  $P_1$  and  $P_2$  and the messages  $M_1$  and  $M_2$ . Accordingly,  $1 - \widehat{\lambda}$ , the remaining share of the market, is the expected demand of the other firm. Without loss of generality, throughout the paper we consider that  $E(x_1|\mu_1) \leq E(x_2|\mu_2)$ . Then in case of strict inequality,  $\widehat{\lambda}$  is the expected demand of firm 1 and  $1 - \widehat{\lambda}$  is the expected demand of firm 2. The case when  $E(x_1|\mu_1) = E(x_2|\mu_2)$ , in which  $\widehat{\lambda}$  is not well defined, will be addressed separately later.

The derivation of expected demand helps to define the equilibrium notion we use. From (2.2) it follows that apart from the price difference, a firm's expected demand and, hence, its profit only depend on *expected* locations and on the precision of the messages about these locations, but (and this is important) *not* on actual locations. This is true both on the equilibrium and off the equilibrium path. That is, any type of firm that sends the same equilibrium message concerning location has equal incentives to set any out-of-equilibrium price. Given this fact, price of a firm cannot reasonably act as a signal of its location.

A similar argument applies to the inability of a firm to signal the location of its competitor. Note



that as a firm knows its own location and the location of the competitor, a firm's type is, in principle, a two-dimensional object  $(x_1, x_2)$ . Therefore, consumers could, in principle, make some inference on the location of the competitor upon observing a firm's out-of-equilibrium message and/or price. Given the above, this, however, would not be a reasonable inference. Consider that all types  $(x_1, x_2)$  of firm 1 (the case of firm 2 is analogous) with the same  $x_1$  component can— *given the grain of truth assumption* — send the same equilibrium message. Therefore, in equilibrium their payoff should be the same as otherwise one of the types would have an incentive to send another message. Moreover, all types of a firm that have sent the same equilibrium message concerning location and that therefore are believed to have an identical location along the equilibrium path have equal incentives to make consumers believe that their competitor has a certain location. Therefore, if consumers would infer a certain location of the competitor after observing a particular out-of-equilibrium message and/or price, either all types  $(x_1, x_2)$  of the firm with the same  $x_1$  component would want to deviate to that out-of-equilibrium message and/or price or no type would. But then it would be unreasonable for consumers to discriminate between firm types  $(x_1, x_2)$  and  $(x_1, x'_2)$  that differ only in the location of the competitor. Hence, the competitor's location cannot be reasonably deduced from firm's out-of-equilibrium message and/or price.

In principle, the same could apply to a firm trying to signal its own location. But here is where the grain of truth assumption becomes relevant. If a firm would deviate to a very precise message, then because of the grain of truth assumption only few (or in the limit, no) other types with different own location can imitate that signal. Thus, the grain of truth assumption makes it possible for out-of-equilibrium messages to signal some information about own location.

These considerations imply that in the context of our model where profits are only governed by prices and expected locations (and not by real locations), it is reasonable to confine attention to equilibria where consumer out-of-equilibrium beliefs concerning a firm's location only depend on firm's own message and not on its pricing decision. Also, due to the grain of truth assumption, upon observing the one dimensional message  $M_i$  and price  $p_i$  of firm  $i$  off the equilibrium path, consumers interpret a firm's message and price as being uninformative about the location of the competitor. We call such a perfect Bayesian equilibrium a stable belief equilibrium. Essentially, a stable belief equilibrium is an equilibrium that satisfies certain restrictions on the out-of-equilibrium beliefs of consumers as described by part (ii) of condition 4 in the definition below.

**Definition** A *stable belief equilibrium* is a set of reporting and pricing strategies  $m_1^*, m_2^*, p_1^*, p_2^*$  of the two firms, strategy  $b^*$  of a consumer, and the probability density functions  $\mu_1^*, \mu_2^*$  which satisfy

the following conditions:

- (1) For all  $M_1, M_2, P_1$  and  $P_2$ ,  $b^*$  is a consumer's best buying decision as defined below:

$$b(\lambda, M_1, M_2, P_1, P_2) = \begin{cases} (1, 0) & \text{if } \int_0^1 (v - t(\lambda - x_1)^2 - P_1)\mu_1(x_1|M_1, M_2, P_1, P_2)dx_1 \geq \\ & \int_0^1 (v - t(\lambda - x_2)^2 - P_2)\mu_2(x_2|M_1, M_2, P_1, P_2)dx_2 \\ (0, 1) & \text{if } \int_0^1 (v - t(\lambda - x_2)^2 - P_2)\mu_2(x_2|M_1, M_2, P_1, P_2)dx_2 \geq \\ & \int_0^1 (v - t(\lambda - x_1)^2 - P_1)\mu_1(x_1|M_1, M_2, P_1, P_2)dx_1 \end{cases} \quad (2.3)$$

- (2) Given (1) and given the messages sent by the two firms and the price set by the competitor,  $p_i^*$  is the price that maximizes the expected profit of firm  $i$ ,  $i = 1, 2$ .
- (3) Given (1), (2) and given the message sent by the competitor,  $m_i^*$  is the message that maximizes the expected profit of firm  $i$ ,  $i = 1, 2$ , subject to the constraint that  $x_i \in m_i$ .
- (4) For all  $M_1, M_2, P_1$  and  $P_2$ , a consumer updates his or her beliefs,  $\mu_i$ , regarding the location of firm  $i$  in the following way:<sup>6</sup>
- (i) according to Bayes' rule on the equilibrium path,
  - (ii) arbitrarily but subject to the condition  $\mu_i(z|M_i, M_j, P_1, P_2) = \mu_i(z|M_i, M'_j, P'_1, P'_2)$  off the equilibrium path.

Part (1) of the definition states that for any observed messages and prices, a consumer buys a unit of the product from the firm, where her expected net utility, given the updated beliefs, is maximized. Each firm rationally anticipates the best response of consumers to any given messages and prices, and chooses the price and message that maximize its expected profit. This is stated in parts (2) and (3). Finally, part (4) claims that consumers update beliefs about the locations using Bayes' rule for any  $M_1, M_2, P_1$  and  $P_2$  that occur with positive density along the equilibrium path and that, as discussed above, off-the-equilibrium path beliefs about firm  $i$ 's location cannot depend on prices and on the message sent by the other firm.

<sup>6</sup>Note that Bayes' rule cannot be applied when  $M_i$  or a subset of  $M_i$  is discrete. For example, if  $M_i = \{y, z\}$ , then both events,  $x_i = y$  and  $x_i = z$  have ex-ante zero probability. In this case, updating proceeds as follows:

$$\mu_i(z|M_1, M_2, P_1, P_2) = \lim_{\varepsilon \rightarrow 0} \frac{F(z + \varepsilon) - F(z)}{F(z + \varepsilon) - F(z) + F(y + \varepsilon) - F(y)}$$

Using l'Hôpital's rule,

$$\mu_i(z|M_1, M_2, P_1, P_2) = \lim_{\varepsilon \rightarrow 0} \frac{f(z + \varepsilon)}{f(z + \varepsilon) + f(y + \varepsilon)} = \frac{f(z)}{f(z) + f(y)}$$

### 3 Results

Given that we have already derived consumer demand in the previous section, we start our analysis by studying the pricing decision of firms.

Each firm anticipates the optimal behavior of consumers and chooses price so as to maximize its expected profit, for any given messages of the firms sent at the previous stage. Expression (2.2) for  $\hat{\lambda}$  implies that the profits of firms 1 and 2 are given by

$$\begin{aligned}\pi_1 &= P_1 \left( \frac{1}{2} \frac{\frac{P_2 - P_1}{t} + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)}{E(x_2|\mu_2) - E(x_1|\mu_1)} + \frac{1}{2} (E(x_1|\mu_1) + E(x_2|\mu_2)) \right) \\ \pi_2 &= P_2 \left( 1 - \frac{1}{2} \frac{\frac{P_2 - P_1}{t} + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)}{E(x_2|\mu_2) - E(x_1|\mu_1)} - \frac{1}{2} (E(x_1|\mu_1) + E(x_2|\mu_2)) \right).\end{aligned}$$

Function  $\pi_i$ ,  $i = 1, 2$ , is a strictly concave, quadratic function of  $P_i$ . Hence, the profit-maximization problem of each firm is well-defined and the first-order conditions yield the price at which  $\pi_i$  is maximized:<sup>7</sup>

$$\begin{aligned}-\frac{P_1}{t(E(x_2|\mu_2) - E(x_1|\mu_1))} + \frac{1}{2} \left( \frac{\frac{P_2}{t} + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)}{E(x_2|\mu_2) - E(x_1|\mu_1)} + E(x_1|\mu_1) + E(x_2|\mu_2) \right) &= 0 \\ 1 - \frac{P_2}{t(E(x_2|\mu_2) - E(x_1|\mu_1))} - \frac{1}{2} \left( \frac{-\frac{P_1}{t} + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)}{E(x_2|\mu_2) - E(x_1|\mu_1)} + E(x_1|\mu_1) + E(x_2|\mu_2) \right) &= 0\end{aligned}$$

The first (second) equation above represents the first-order condition for firm 1 (2). Solving these equations results in the solution of the price setting stage of the game:

$$\begin{aligned}P_1 &= t \left( \frac{2}{3} (E(x_2|\mu_2) - E(x_1|\mu_1)) + \frac{1}{3} (E^2(x_2|\mu_2) - E^2(x_1|\mu_1)) + \right. \\ &\quad \left. + \frac{1}{3} (\text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)) \right)\end{aligned}\tag{3.1}$$

$$\begin{aligned}P_2 &= t \left( \frac{4}{3} (E(x_2|\mu_2) - E(x_1|\mu_1)) - \frac{1}{3} (E^2(x_2|\mu_2) - E^2(x_1|\mu_1)) - \right. \\ &\quad \left. - \frac{1}{3} (\text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)) \right)\end{aligned}\tag{3.2}$$

Plugging expressions (3.1)–(3.2) for prices into the profit functions of the two firms, yields reduced-form profit functions that are expressed in terms of the conditional expectations and variances of  $x_1$

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<sup>7</sup>This derivation makes use of the fact that we are restricting our attention to stable belief equilibria.

and  $x_2$ :

$$\begin{aligned} \pi_1 &= \frac{t}{18(E(x_2|\mu_2) - E(x_1|\mu_1))} \cdot (2(E(x_2|\mu_2) - E(x_1|\mu_1)) + \\ &+ (E^2(x_2|\mu_2) - E^2(x_1|\mu_1)) + (\text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)))^2 \end{aligned} \quad (3.3)$$

$$\begin{aligned} \pi_2 &= \frac{t}{18(E(x_2|\mu_2) - E(x_1|\mu_1))} \cdot (4(E(x_2|\mu_2) - E(x_1|\mu_1)) - \\ &- (E^2(x_2|\mu_2) - E^2(x_1|\mu_1)) - (\text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)))^2 \end{aligned} \quad (3.4)$$

We can now consider the stage at which firms decide on the messages they will send. As a special case, consider first the situation where locations of both firms are fully revealed. This means that in all expressions above  $E(x_i|\mu_i) = x_i$ ,  $\text{var}(x_i|\mu_i) = 0$  and profits of firm 1 and 2 are functions of exact locations  $x_1, x_2$ . In particular, following the assumption that  $x_1 \leq x_2$  (the analogue of  $E(x_1|\mu_1) \leq E(x_2|\mu_2)$ ), the profits of firms 1 and 2, when inequality is strict, are given by:

$$\pi_1(x_1, x_2) = \frac{t}{18}(x_2 - x_1)(2 + x_1 + x_2)^2 \quad (3.5)$$

$$\pi_2(x_1, x_2) = \frac{t}{18}(x_2 - x_1)(4 - x_1 - x_2)^2 \quad (3.6)$$

Both these expressions are strictly positive as long as  $x_1 < x_2$ . If  $x_1 = x_2$ , then consumers buy from the firm with the lowest price. The usual Bertrand-type argument then establishes (cf., 3.1 and 3.2) that  $P_1 = P_2 = 0$  and so,  $\pi_1 = \pi_2 = 0$ .

Note that the profit of firm 1 in (3.5) is *decreasing* in  $x_1$ , while the profit of firm 2 in (3.6) is *increasing* in  $x_2$ . Indeed,

$$\begin{aligned} \frac{\partial \pi_1}{\partial x_1} &= (2 + x_1 + x_2) \left( \frac{t}{18}x_2 - \frac{3t}{18}x_1 - \frac{t}{9} \right) < 0 \\ \frac{\partial \pi_2}{\partial x_2} &= (4 - x_1 - x_2) \left( \frac{t}{18}x_1 - \frac{3t}{18}x_2 + \frac{2t}{9} \right) > 0, \end{aligned}$$

where the signs of the derivatives are implied by the fact that  $0 \leq x_1, x_2 \leq 1$ . This finding is consistent with the argument in the standard Hotelling model of location choice. Firms want to be located maximally far from each other as differentiation allows them to charge higher prices, which turns out to outweigh the adverse effect of a decline in demand. This functional dependence of profits on locations plays a key role in the proof of the first theorem:

**Theorem 3.1.** *There exists a stable belief equilibrium where firms fully disclose their location.*

Theorem 3.1 claims that full disclosure is always an equilibrium of the game. Clearly, the fully revealing equilibrium is not unique since there are many sets of messages with which firms are able to fully disclose their location. In the proof we use strategies where firms disclose their location

precisely. This facilitates the proof in the sense that it is impossible for types to imitate each others' message due to the restriction that messages must be truthful.

The proof also uses specific out-of-equilibrium beliefs that discourage firms to deviate from their equilibrium strategies. These out-of-equilibrium beliefs are somewhat extreme in the sense that consumers believe that an out-of-equilibrium message is sent by one of the types with the lowest equilibrium profit across the set of all types that are consistent with the message, even though all other types in this set could also have truthfully sent such a message. However, one can show that these extreme out-of-equilibrium beliefs are reasonable in the sense that they satisfy the logic of the D1 criterion.<sup>8</sup>

The next result is probably even more important for the general message of the paper than Theorem 3.1 stating the existence of a fully disclosing equilibrium. Theorem 3.2 shows that when there is competition between firms, there cannot be a stable belief equilibrium where firms do not perfectly disclose the variety they produce. Thus, even though the fully revealing equilibrium strategies are themselves not unique, the equilibrium outcome of full disclosure is.

**Theorem 3.2.** *There does not exist a stable belief equilibrium where firms do not fully disclose their location.*

The intuition behind the result of Theorem 3.2 is related to the argument in the standard Hotelling model with location choice, where firms have an incentive to maximally differentiate from each other. If a firm does not follow a fully revealing strategy, there are locations within its non-fully revealing message that are further away from the perceived location of the rival firm than the own perceived location. These locations have an incentive to deviate by fully disclosing themselves. The reason for this is twofold. First, by fully revealing its location, a firm reduces the uncertainty associated with the location for consumers and given the quadratic transportation costs, any reduction in uncertainty increases the demand *ceteris paribus*. Second, by having a perceived location that is further away from the competitor, a firm can charge higher price and this price effect outperforms the direct effect of a decline in demand.

The proof relies on the fact that a firm can always make consumers know its exact location, without at the same time also changing the beliefs about the location of the competitor. Here is

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<sup>8</sup>Intuitively, the D1 criterion requires that for a given observed deviation from the equilibrium strategy, consumers believe that such deviation was chosen by the type of a firm that has "most incentives" to deviate. To define which type has "most incentives" to deviate, observe that the profits are completely determined by consumer beliefs about location and not by location itself. Thus, after sending an out-of-equilibrium message, the profits of a deviating firm are independent of its type. Therefore, the incentive to deviate is largest for the type (or types) in  $\widehat{M}_i$  with the smallest equilibrium payoff. This way to modify the D1 criterion to this game is suggested by a similar adaptation in Janssen and Roy (2010).

where the restriction to stable belief equilibrium is used. Consider a potential equilibrium where a firm does not fully disclose. If by revealing its exact location, consumers would believe that a firm would only fully reveal if its rival's location is the same (or very close), then severe price competition would reduce the profits of such a full disclosure strategy to (almost) zero and then the deviation to full disclosure would not be profitable. We have argued that such beliefs by consumers are not reasonable.

## 4 Discussion and conclusions

In this paper we developed a duopoly model of horizontal product differentiation. We studied the incentives of a firm to disclose its horizontal product characteristic when this characteristic is known to both firms, but not to consumers. Firms first simultaneously choose a message about their location, such that this message is *truthful*, that is, the true location of a firm is consistent with the message. The messages can range from being very precise (indicating the exact location) to very vague. After firms have sent their messages, they simultaneously choose prices. Given the messages and the prices, consumers update their beliefs about firms' locations and decide where to buy.

As profits in this environment only depend on expected locations and prices, but not on real locations, or types, and as we insist that messages have to contain a grain of truth, we define a stable belief equilibrium where consumers' beliefs concerning a firm's location after observing some out-of-equilibrium message or prices only depend on a firm's own message concerning its location. We argue that this is the natural equilibrium notion in this environment. We find that all stable belief equilibria of the game are such that both firms fully reveal their locations. In other words, there always exists an equilibrium where firms fully disclose their location and there does not exist a stable belief equilibrium where firms do not disclose. This full-disclosure result contrasts with the finding of possible non-disclosure in the literature on horizontal product differentiation in a monopolistic set-up, suggesting that competition plays a key role in determining incentives for firms to disclose.

Intuitively, the reason why in the competitive environment all equilibria must be fully revealing is related to the reason why in the standard Hotelling model with location choice, firms want to maximally differentiate from each other. Suppose that a firm would not fully reveal its location, choosing the same message for different locations. Consumers will then be uncertain about the true location of the firm and hence, will form beliefs such that the extreme locations, sending that particular message, will not obtain full probability mass. These extreme locations have then incentive to deviate to full disclosure for two reasons. On one hand, by fully revealing its location, a firm reduces

the uncertainty concerning the location for consumers and with quadratic transportation costs, this increases their demand. On the other hand, by indicating a location that is perceived by consumers as being further away from the competitor, price competition is softened and this price effect outweighs the direct effect of a decline in demand.

The finding of full disclosure does not seem to critically depend on the assumption that the market is fully covered ( $v$  is sufficiently large). We have considered the case of a fully covered market to distinguish our results from the monopoly results that have recently been established. If the market is not covered, then for any locations of the two firms, the consumers who do not buy may be located in three potential areas: at one or two extremes of the  $[0,1]$  interval or "in the middle", between the locations of the firms. If the non-buyers are only located at the extreme(s), then this effectively just shortens the relevant interval and the same reasoning as before should lead to full revelation. If instead the non-buyers are also located in the middle (or only in the middle), then the market turns out to be split into two monopolistic markets, with firm 1 effectively being a monopolist in one of the submarkets and firm 2 effectively being a monopolist in the other. In this case, the main result of Celik (2011) for low value of  $v$  implies that both firms fully disclose their location. Thus, in either case full revelation should follow. The formal analysis would, however, be somewhat tedious as different cases have to be considered. Therefore, we do not discuss the situation with non-fully covered market in this paper in any detail.

In the present model we have considered a simple framework where consumers are uniformly distributed over the unit interval and have quadratic transportation costs. Moreover, disclosure is completely costless and firms know not only their own location, but also the location of their competitor. Disclosure decisions are considered to be long-term strategies and are therefore modelled as taken place before pricing decisions. We have considered this simple framework to focus on the role of competition in providing incentives for firms to fully disclose. Future work should focus on whether similar conclusions hold when some of these assumptions are replaced by others. For example, the case where firms have purely private information about their product characteristics could be of considerable interest. It is not difficult to see that under purely private information of product characteristics, full disclosure is also an equilibrium outcome and in this respect the result of the current paper easily generalizes. The main challenge is to argue that no other equilibria exist. This is not an easy task as without knowing the location of the competitor, disclosing own location has advantages for certain, but not all locations of the rival firm.

## Appendix

*Proof of Theorem 3.1.* Suppose that firms fully reveal their location by truthfully announcing it, i.e., every firm with location  $x_i$  sends message  $M_i = \{x_i\}$ . Since firms cannot lie, the firm of any given type  $x_i$  cannot imitate the message of another type. Any deviating message is therefore an out-of-equilibrium message and the proof of an equilibrium then requires to construct a set of out-of-equilibrium beliefs such that given these beliefs, no firm has an incentive to deviate.

Let us consider the following out-of-equilibrium beliefs. For any out-of-equilibrium message  $\widehat{M}_i$  sent by firm  $i$ , consumers assign probability one to firm  $i$  being of type  $\widehat{x}(M_i)$  where

$$\widehat{x}(M_i) \in \arg \min_{x_i \in \widehat{M}_i} \pi_i(x_1, x_2)$$

i.e.,  $\widehat{x}(M_i)$  is any selection from the set of minimizers of the function  $\pi_i(x_1, x_2)$  on the set  $\widehat{M}_i$ .

Observe that given these out-of-equilibrium beliefs, no type of any firm wishes to deviate from the candidate equilibrium strategies. If firm  $i$  of type  $x_i$  deviates and sends some admissible message  $\widehat{M}_i \neq \{x_i\}$ , then the subsequent choice of consumers will be as if the true type of firm  $i$  is  $\widehat{x}(M_i)$  for sure, and since all that matters for the payoff of firm  $i$  is her perceived type (and not her true type), the expected continuation payoff after this deviation is exactly equal to  $\pi_i(\widehat{x}(M_i), x_j)$ . As  $x_i \in M_i$  it follows that  $\pi_i(\widehat{x}(M_i), x_j) \leq \pi_i(x_i, x_j)$ . Therefore, the deviation is not gainful.  $\square$

*Proof of Theorem 3.2.* Suppose that at least one of the two firms does not follow a fully revealing strategy and chooses the same message for different locations. Let types  $x_1 \in S_1$  send identical message  $M_1$  and types  $x_2 \in S_2$  send identical message  $M_2$ , where at least one of the sets  $S_1, S_2$  contains two or more types.<sup>9</sup> Without loss of generality, assume that firm 1 does not fully disclose its location (while firm 2 may disclose or not disclose). In this case, consumers are uncertain about the true location of the non-disclosing firm/firms and form expectations about this location and resulting transportation costs. Again, without loss of generality, we restrict the analysis to the case when  $E(x_1|\mu_1) \leq E(x_2|\mu_2)$ . If the inequality is strict, the profits of firms 1 and 2 are given by (3.3)–(3.4). If instead  $E(x_1|\mu_1) = E(x_2|\mu_2)$ , then consider the firm (referred to as firm  $i$ ) whose equilibrium message has the largest variance (referred to as firm  $j$ ), that is,  $\text{var}(x_i|\mu_i) \geq \text{var}(x_j|\mu_j)$ . The profit of firm  $i$  is equal to zero because firm  $j$  is at least as attractive to consumers and hence, can either push firm  $i$  out of the market by setting  $P_j = t(\text{var}(x_i|\mu_i) - \text{var}(x_j|\mu_j))$  (if  $\text{var}(x_i|\mu_i) > \text{var}(x_j|\mu_j)$ ) or share the market with firm  $i$  but at zero prices (if  $\text{var}(x_i|\mu_i) = \text{var}(x_j|\mu_j)$ ).

<sup>9</sup>If both sets,  $S_1$  and  $S_2$ , contain only one type, then reporting strategies of both firms are fully revealing.



Suppose first that  $E(x_1|\mu_1) < E(x_2|\mu_2)$ . We prove that if  $\text{var}(x_1|\mu_1) \geq \text{var}(x_2|\mu_2)$ , the deviation to full disclosure is profitable for any type  $y$  of firm 1 such that  $y < E(x_1|\mu_1)$ . If the opposite inequality for variances holds, the deviation to full disclosure is profitable for any type  $z$  of firm 2 such that  $z > E(x_2|\mu_2)$ .<sup>10</sup> The proof of this claim relies on the following two observations.<sup>11</sup> First, profit functions  $\pi_1$  and  $\pi_2$  are monotonically decreasing in  $\text{var}(x_1|\mu_1)$  and  $\text{var}(x_2|\mu_2)$ , respectively. This is an immediate implication of (3.3) and (3.4). Second, profit function  $\pi_1$  in (3.3) is monotonically *decreasing* in  $E(x_1|\mu_1)$  when  $\text{var}(x_1|\mu_1) \geq \text{var}(x_2|\mu_2)$ , and profit function  $\pi_2$  in (3.4) is monotonically *increasing* in  $E(x_2|\mu_2)$  when the opposite inequality is true, i.e.,  $\text{var}(x_2|\mu_2) > \text{var}(x_1|\mu_1)$ . To demonstrate this second observation, consider the derivative of  $\pi_1$  with respect to  $E(x_1|\mu_1)$  and the derivative of  $\pi_2$  with respect to  $E(x_2|\mu_2)$  and evaluate their signs. Straightforward calculations lead to

$$\begin{aligned} \frac{\partial \pi_1}{\partial E(x_1|\mu_1)} &= \frac{p_1}{6(E(x_2|\mu_2) - E(x_1|\mu_1))^2} \left( (E(x_2|\mu_2) - E(x_1|\mu_1))(E(x_2|\mu_2) - 3E(x_1|\mu_1) - 2) + \right. \\ &\quad \left. + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1) \right) \\ \frac{\partial \pi_2}{\partial E(x_2|\mu_2)} &= \frac{p_2}{6(E(x_2|\mu_2) - E(x_1|\mu_1))^2} \left( (E(x_2|\mu_2) - E(x_1|\mu_1))(4 - 3E(x_2|\mu_2) + E(x_1|\mu_1)) + \right. \\ &\quad \left. + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1) \right) \end{aligned}$$

Given that  $0 \leq E(x_1|\mu_1), E(x_2|\mu_2) \leq 1$ , the first expression is strictly *negative* when  $\text{var}(x_1|\mu_1) \geq \text{var}(x_2|\mu_2)$  and the second expression is strictly *positive* when  $\text{var}(x_2|\mu_2) > \text{var}(x_1|\mu_1)$ .

Now, suppose that  $E(x_1|\mu_1) = E(x_2|\mu_2)$ . Then if  $\text{var}(x_1|\mu_1) \geq \text{var}(x_2|\mu_2)$ , the deviation by type  $y$  of firm 1 to the fully revealing message is beneficial simply because before the deviation its profit is zero and after the deviation it is positive:

$$\pi_1^D = \frac{t}{18(E(x_2|\mu_2) - y)} \left( 2(E(x_2|\mu_2) - y) + (E^2(x_2|\mu_2) - y^2) + \text{var}(x_2|\mu_2) \right)^2$$

Similarly, if  $\text{var}(x_2|\mu_2) > \text{var}(x_1|\mu_1)$ , then the deviation by type  $z$  of firm 2 to the fully revealing message is beneficial. Thus, at least one firm can always benefit by deviating. Therefore, an equilibrium where firms do not fully disclose their location does not exist.  $\square$

<sup>10</sup>Type  $y$  of firm 1 such that  $y < E(x_1|\mu_1)$  exists because a) by assumption, firm 1 does not fully disclose its location, so that  $S_1$  is not a singleton, and b) the probability density function  $f(x)$  is strictly positive. For the same reason, when  $\text{var}(x_2|\mu_2) > \text{var}(x_1|\mu_1)$ , type  $z$  of firm 2 such that  $z > E(x_2|\mu_2)$  exists.

<sup>11</sup>As the deviation is such that its effect on the variance and the effect on the expected location of consumers both increase profits, we can act as if these two effects can be achieved independently of each other.

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