Existence of Valuation Equilibria when Equilibrium Strategies Cannot Differentiate Between Equal Ties*

Philipp C. Wichardt†

Department of Economics, University of Bonn

This Version: February 24, 2011

Abstract This paper reconsiders the valuation equilibrium concept (Jehiel and Samet, 2007) and proposes an additional regularity condition concerning the players' equilibrium strategies. The condition, which requires equilibrium strategies to induce the same local behaviour at all nodes with "similar" optimal actions, increases both the predictive power and the internal consistency of the concept — especially when used as a tool to study boundedly rational behaviour in games with imperfect information and/or imperfect recall. It is shown not to conflict with existence.

Keywords: Bounded Rationality, Valuation Equilibrium, Existence, Imperfect Recall

JEL code: C72

*Acknowledgements: I am grateful to Anke Gerber, Philippe Jehiel, Christina Matzke, Christian Seel and Avner Shaked for helpful comments and discussions. The usual disclaimer applies.

†Postal address for correspondence: Economic Theory 3, University of Bonn, Adenauerallee 24-26, D-53113 Bonn, Germany; e-mail: philipp.wichardt@uni-bonn.de.

Introduction

The valuation equilibrium concept introduced by Jehiel and Samet (2005, 2007) proposes an intriguing way to capture boundedly rational behaviour in a formal game-theoretic equilibrium concept. Combining a simplified modelling of the players' perception of a strategic interaction, captured by a similarity grouping of actions, with an intuitive local optimisation procedure, it allows to analyse a variety of complex strategic interactions (and decision problems) under fairly weak conditions on the players' comprehension of the underlying formal structure. Moreover, in doing so, it provides an interesting tool to study the reliance of certain outcomes on the players' rationality (as reflected in the similarity grouping).

However, the original definition of equilibrium as proposed by Jehiel and Samet (2007) not only allows for an undesirable multiplicity of equilibria in many games. If the concept is applied to games with imperfect information or imperfect recall – two natural applications discussed also by Jehiel and Samet (2007, p. 173ff)[1] – it also gives rise to logical inconsistencies concerning the information possessed by the players and the information implicitly entailed in the technical description of their behaviour.

In the following, I briefly recap the definition of the valuation equilibrium concept, and illustrate the concept itself as well as the arising difficulties. Moreover, I propose an additional consistency condition which increases both the predictive power of the concept and its internal consistency (especially in view of interpretations of the grouping in terms of imperfect information and imperfect recall). Finally, I show that the additional condition does not to conflict with existence.

Valuation Equilibrium and the Problems of Inconsistent Tie-Breaking

Formally, (sequential) valuation equilibria, henceforth (S)VE, are defined for finite extensive form games with perfect information (see Jehiel and Samet, 2007, pp. 166-168, for a detailed definition). Thus, there is a finite set of players $\mathcal{I}$, a game tree (Z, H, r, A) with terminal nodes Z, non-terminal nodes H, an origin or root of the tree $r \in H$, and a set A of arcs, i.e. ordered pairs $(h, \tilde{h}) \in H \times H \cup Z$ such that $\tilde{h} \in H \cup Z$ is the immediate successor of $h \in H$. Furthermore, H is partitioned into subsets H_i, $i \in \mathcal{I}$, thereby assigning players to decision nodes; decisions of Nature, who is assumed to assign positive probability to all moves available to her, are gathered in $H_0 = H \setminus \cup_{i \in \mathcal{I}} H_i$. Moreover, player i's actions at some node $h \in H_i$ are identified with the nodes they lead to, i.e. $\tilde{h} \in M_i(h) := \{\tilde{h} \mid (h, \tilde{h}) \in A\}$ and the set of all player i's

[1] In discussing imperfect recall, the main point of Jehiel and Samet, however, is to clarify that, in general, the grouping of moves entailed in the valuation equilibrium approach is different from the grouping of nodes connected with imperfect recall.

actions is denoted by $\overline{M}_i$, i.e. $\overline{M}_i := \cup_{h \in H_i} M_i(h)$. Finally, payoffs are determined by a collection of functions $(f_i)_{i \in \mathcal{I}}$, $f_i : Z \to \mathbb{R}$.

As an illustrating example, albeit without moves of Nature, consider the sequential perfect information matching pennies game depicted in Figure 1; player 1 moves first and player 2 moves second, i.e. $H_1 = \{r\}$, $H_2 = \{H, T\}$ and $Z = \{Hh, Ht, Th, Tt\}$.

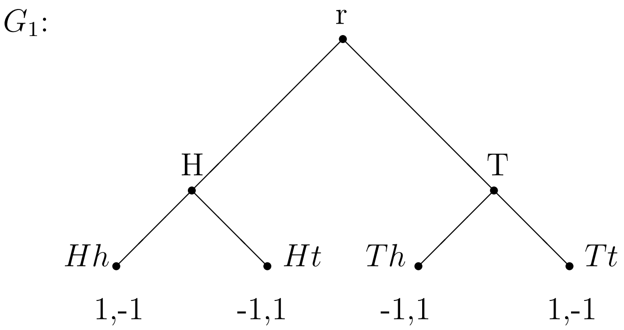

Figure 1: A sequential matching pennies game.

The constraints on the players information – be it for technical reasons or reasons of bounded rationality – then, are accounted for by a similarity grouping of actions. Formally, this is modelled by some equivalence relation defined on the set of player i's actions, $\overline{M}_i$, such that any two actions considered similar belong to the same equivalence class. The resulting partition of $\overline{M}_i$ is denoted by Λ_i with elements λ_i, or $\lambda_i(h)$ if reference to some specific $h \in \overline{M}_i$ is needed.

In the present example, for instance, the usual imperfect information structure of the matching pennies game can be obtained by assuming that player 2 groups both heads-actions, Hh and Th, as well as both tails-actions, Ht and Tt, into one similarity class each, say $\lambda^h = \{Hh, Th\}$ and $\lambda^t = \{Ht, Tt\}$. In general, the similarity grouping can, of course, also account for more complex informational constraints such as imperfect recall (see the example discussed further below) or combinations of imperfect information and imperfect recall (see also Jehiel and Samet, 2007).

Moreover, players are assumed to build average evaluations of (similarity classes of) actions. The resulting valuations are then paired with a local maximisation procedure requiring players to follow those actions with the locally highest valuations. In order to capture this, plans of actions are modelled using behaviour strategies (Kuhn, 1953), i.e players are assumed to randomise locally over available actions and not ex ante over available pure strategies.[2] Finally, in equilibrium, valutions have to be consistent with the average payoff earned when using such actions.

[2]See, e.g., Wichardt (2008) for an example illustrating the particularities of behaviour strategies.

More formally, a *sequential valuation equilibrium* (SVE) consists of a profile of behaviour strategies σ, and a collection of *valuations* $(\nu_i)_{i\in\mathcal{I}}$ attached to similarity classes of actions, i.e. $\nu_i : \Lambda_i \to \mathbb{R}$, so that

1. valuations are *sequentially consistent* with the strategy profile σ, i.e. there is a sequence of completely mixed strategy profiles $(\sigma^k)_{k=1}^{\infty}$ converging to σ such that for all i the sequence of valuations ν_i^k induced by σ^k converges to ν_i, i.e.

$$\nu_i^k(\lambda) = E^{\sigma^k}(f_i \mid Z(\lambda)) = \sum_{z\in Z(\lambda)} \frac{P^{\sigma^k}(z)f_i(z)}{P^{\sigma^k}(Z(\lambda))} \to \nu_i(\sigma),$$

   where $Z(\lambda)$ denotes the set of terminal nodes which are descendants of some $h \in \lambda$ and $P^{\sigma}(Z(\lambda))$ [$P^{\sigma}(z)$] is the probability that $Z(\lambda)$ [z] is reached under σ;[3]

2. for each player i, σ_i is *optimal given valuation* ν_i, i.e. for each $h \in H_i$ and each $\tilde{h} \in M_i(h)$ it holds that $\sigma_i(h)(\tilde{h}) = 0$ if $\tilde{h} \notin \mathit{argmax}_{h'\in M_i(h)}\nu_i(\lambda(h'))$ [at each node actions with inferior valuations are played with probability zero].

Applied to the matching pennies example, this gives rise to a variety of SVE. For example, any strategy profile σ with $\sigma_1(H) \in (0,1)$ and either $\sigma_2 = (Hh, Tt)$ or $\sigma_2 = (Ht, Th)$ is an SVE of G_1: strategies induce (and, hence, confirm) equal valuations both for player 1's actions, namely 1 if $\sigma_2 = (Hh, Tt)$ and -1 otherwise, and for player 2's similarity classes λ^h and λ^t, namely -1 if $\sigma_2 = (Hh, Tt)$ and 1 otherwise; and, by definition, player 2 is free to vary his tie-breaking behaviour between different nodes with similar optimal actions, so that, given the induced indifference, consistency of strategies with valuations is immediate. Moreover, a similar argument shows that any strategy profile σ with $\sigma_1 = 1/2H + 1/2T$ and player 2 randomising at both nodes such that $\sigma_2(Hh) \equiv \sigma_2(Tt)$ is an SVE. And further equilibria are given by (a) $\sigma_1(H) \in \{0,1\}$ and again $\sigma_2 = (Hh, Tt)$ or $\sigma_2 = (Ht, Th)$, (b) $\sigma_1 = H$ and $\sigma_2 = (Hh, Th)$, and (c) $\sigma_1 = T$ and $\sigma_2 = (Ht, Tt)$.[4]

While already the resulting multiplicity of equilibria itself is arguably problematic in view of applications, it is noteworthy that almost all SVE described above are incompatible with the interpretation of player 2's similarity grouping in terms of imperfect information. In fact, but for the two equilibria with player 2 playing h /

[3] Jehiel and Samet (2007) also define a more general version of valuation equilibria requiring only consistency with equilibrium strategies but not sequential consistency. As this is of less relevance for the present discussion, I refer the interested reader to Jehiel and Samet (2007) for a definition as well as a motivation for both consistency requirements.

[4] The necessary off-path valuations can be supported by appropriate limiting strategies.

t for sure at both nodes and the equilibrium with both players randomising 50-50 at all instances, all equilibria effectively rely on player 2 conditioning his behaviour on player 1's action. Such conditioning, however, is obviously impossible if player 2 is not informed about player 1's choice when called upon to move.[5]

A possible way to remedy this deficiency is to introduce an additional regularity condition for the players' equilibrium strategies which requires these strategies to induce *the same* probability distribution over optimal actions whenever the optimal similarity classes a player locally chooses from are identical.[6] Formally, this can be defined as follows:

Definition 1 (Uniform Tie-Breaking Behaviour) *Let $Max_i(h, \nu_i)$ be the set of player i's similarity classes of those actions available at node h which have the (locally) highest valuation, i.e.*

$$Max_i(h, \nu_i) := \{\lambda_i \mid \lambda_i \cap M_i(h) \neq \emptyset \text{ and } \nu_i(\lambda_i) = \max_{\tilde{h} \in M_i(h)} \nu_i(\lambda(\tilde{h}))\}.$$

A strategy σ_i, then, is said to satisfy uniform tie-breaking behaviour*, UTB for short, if for all $h, h' \in H_i$ with $Max_i(h, \nu_i) = Max_i(h', \nu_i)$ it holds that $\sigma_i(h)(\tilde{h}) = \sigma_i(h')(\tilde{h}')$ for all $\tilde{h} \in M(h)$ and $\tilde{h}' \in M(h)$ with $\lambda(\tilde{h}) = \lambda(\tilde{h}')$ and $\lambda(\tilde{h}), \lambda(\tilde{h}') \in Max_i(h, \nu_i) = Max_i(h', \nu_i)$.*

By construction, the UTB-condition rules out all cases where players condition their tie-breaking behaviour for equal ties on the node they choose at. Thus, once the condition is applied to the matching pennies example, the only equilibria that remain for G_1 are: (a) $\sigma_1(H) = 0.5$ and $\sigma_2(Hh) = \sigma_2(Th) = 0.5$ with valuation 0 for all moves, and (b) $\sigma_1(H) = 1$ $(= 0)$ and $\sigma_2(Hh) = \sigma_2(Th) = 1$ $(= 0)$ with player 2 assigning valuation -1 to both λ^t and λ^h; and both equilibria are consistent with the presumed underlying information structure.

As a second example, this time focusing on the analysis of games with imperfect recall, consider the simple one-player game G_2 depicted in Figure 2, which can be interpreted as a modification of the absent minded driver example discussed by Piccione and Rubinstein (1997).[7]

[5] It is worth noting that also more generally, if a player can vary his equilibrium behaviour between nodes with similar optimal actions, then "something" must allow the respective player to recognise differences between these nodes (he must know when to behave how). Yet, if there is such a something, then it seems hard to argue why this something does not also allow the player to recognise differences between the actions which originate from the respective nodes as well.

[6] The focuses on similarity of optimal actions (and not of available actions per se) is chosen as

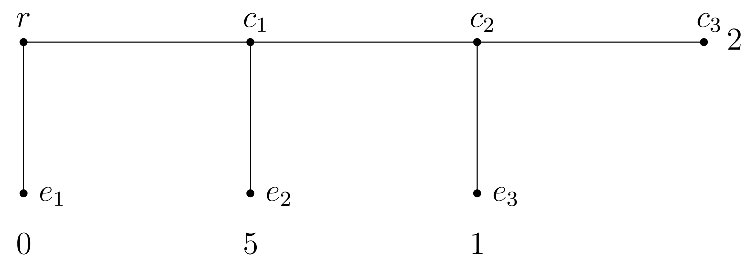

Figure 2: G_2: A modified absent minded driver game; actions labelled c refer to decisions to continue on the highway while actions labelled e refer to exit decisions.

In order to make the situation one of imperfect recall, assume that player 1 groups all decisions to continue (c_1, c_2 and c_3) and all his exit decision (e_1, e_2 and e_3) in one similarity class each, denoted by λ^c and λ^e, respectively. Then, already accounting for the UTB-condition, the sequential valuation equilibria of G_2 are given by: (a) $\sigma_1 = (c_1, c_2, c_3)$ with a valuation of 2 for λ^c and some valuation for λ^e which is smaller than 2, and (b) equal randomisation at all nodes such that the induced valuations for λ^c and λ^e coincide. Moreover, both equilibria have a plausible interpretation. In case (a) player 1 can be seen as somehow having "learned" that playing e is unprofitable and, therefore, chooses c all the way through. In case (b), one can think of player 1's experience as having been such that it has made him indifferent while his actual behaviour essentially confirms this experience thereby justifying the respective randomisation.

By contrast, without the additional UTB-condition, also any strategy $(c_1, \sigma^2(c_2), e_3)$ with $\sigma^2(c_2) \in [0,1]$, where $\sigma^2(c_2)$ denotes the probability of c_2 at the second node, constitutes a sequential valuation equilibrium of G_2. The induced valuations for λ^c and λ^e are identical at all nodes by construction.[8] And, by definition, player 1 is free to vary his local tie-breaking behaviour between different nodes with similar optimal actions. Thus, strategies are consistent with and confirm valuations. Such equilibria, again, are not only undesirable in view of the multiplicity of predictions they allow, though. They also conflict with the earlier interpretation of the grouping in terms of imperfect recall. In particular, if player 1 is unable to differentiate between different exits, it is difficult argue how he shall condition his local tie-breaking behaviour

these are effectively the only actions a player locally considers.

[7]Following the original story, player 1 is on his way home over the highway and faces a repeated choice between continuing on the highway, c, and exiting e. However, being absent minded, he does not recall earlier decisions and therefore is unable to distinguish the different possible exits (2 in the original game).

[8]Whatever the realised outcome, both similarity classes must have been reached. Thus, any outcome equally affects the valuation of both similarity classes.

between λ^c and λ^e on the corresponding exit. This is not to say, of course, that the actual tie-breaking for a recurrent indifference may not differ for a single play of the game. Yet, it seems difficult to support such inconsistencies as a constituent part of the players' equilibrium strategies; i.e. while realised actions may differ for single plays the underlying selection procedure should not.

Thus, also when applied to games with imperfect recall, the additional UTB-condition increases both the predictive power of the valuation equilibrium concept and its internal consistency. In fact, once the UTB-condition is added, I believe that the modified valuation equilibrium concept offers an interesting alternative to the common Nash-approach with behaviour strategies (cf. Kuhn, 1953) for the analysis of such games. In particular, the grouping of actions together with the entirely *local* maximisation entailed in the concept is not only intuitively appealing in such instances. The modified SVE concept also avoids potential inconsistencies between ex ante optimality and (local) on-path incentives entailed in the Nash-approach (see Piccione and Rubinstein, 1997a, or Rubinstein, 1998, for an exposition of the problem, and Piccione and Rubinstein, 1997b, for a synthesis of the respective discussion and further references). Moreover, as I demonstrate below, valuation equilibria with uniform tie-breaking behaviour always exist while common Nash-equilibria in behaviour strategies may not (cf. Wichardt, 2008).

Redefining Valuation Equilibria and Proving Existence

In the remainder of this paper, I provide a formal definition of the modified valuation equilibrium concept and show that the new consistency requirement does not conflict with existence. For comparison purposes, I refer to those (S)VE for which equilibrium strategies also satisfy UTB as (S)VE*.[9]

Definition 2 *A strategy profile* $\sigma = (\sigma_i)_{i\in\mathcal{I}}$ *is a* (sequential) valuation equilibrium with uniform tie-breaking behaviour*, (S)VE*, if:*

1. *σ is a (S)VE, i.e. if there is a profile of valuations $\nu = (\nu_i)_{i\in\mathcal{I}}$ such that for all i, σ_i is optimal given ν_i, and ν_i is (sequentially) consistent with σ,*

2. *for all i, strategy σ_i satisfies UTB given ν_i.*

As for (S)VE, it suffices to establish the existence of SVE* as each SVE* also is an VE*.

[9]VE here refers to the more general definition suggested by Jehiel and Samet (2007); cf. footnote 3.

Proposition 1 *For any finite extensive form game with perfect information and any similarity grouping of actions, there exists at least one sequential valuation equilibrium with uniform tie-breaking behaviour.*

Proof. The proof of Proposition 1 follows the same lines as the existence proof for SVE given by Jehiel and Samet (2007) except that now the additional UTB requirement has to be accounted for.

To begin with, let $\varepsilon > 0$ and consider the set Σ^ε of all strategy profiles σ^ε such that $\sigma_i(h)(\tilde{h}) \geq \varepsilon$ for all i, $h \in H_i$ and $\tilde{h} \in M_i(h)$. As all $\sigma^\varepsilon \in \Sigma^\varepsilon$ are fully mixed, each $\sigma^\varepsilon \in \Sigma^\varepsilon$ induces a unique valuation $\nu(\sigma^\varepsilon)$ which is consistent with σ^ε. Thus, for a given ε, the mapping $\nu : \Sigma^\varepsilon \to \mathbb{R}$, $\sigma^\varepsilon \mapsto \nu(\sigma^\varepsilon)$, is a continuous function (continuity follows from the definition of the valuations).

Next, consider the correspondence ψ^ε that maps each strategy profile $\sigma^\varepsilon \in \Sigma^\varepsilon$ to the set of profiles $\hat{\sigma}^\varepsilon$ such that: (1.) for each player i, strategy $\hat{\sigma_i}^\varepsilon$ is *ε-optimal given valuation* $\nu_i(\sigma^\varepsilon)$, i.e. for all $h \in H_i$ and $\tilde{h} \in M_i(h)$ if $\tilde{h} \notin argmax_{h' \in M_i(h)} \nu_i(\lambda_i(h'))$ then $\hat{\sigma}(h)(\tilde{h}) = \varepsilon$; (2.) each $\hat{\sigma_i}^\varepsilon$ satisfies UTB$^\varepsilon$, i.e. $Max_i(h, \nu(\sigma^\varepsilon)) = Max_i(h', \nu(\sigma^\varepsilon))$ implies

$$\frac{\hat{\sigma}_i^\varepsilon(h)(\tilde{h})}{1 - (|M_i(h)| - |Max_i(h, \nu_i(\sigma^\varepsilon))|) \cdot \varepsilon} = \frac{\hat{\sigma}_i^\varepsilon(h')(\tilde{h}')}{1 - (|M_i(h')| - |Max_i(h', \nu_i(\sigma^\varepsilon))|) \cdot \varepsilon}$$

for all $\tilde{h} \in M(h)$ and $\tilde{h}' \in M(h)$ with $\lambda(\tilde{h}) = \lambda(\tilde{h}')$ and $\lambda(\tilde{h}), \lambda(\tilde{h}') \in Max_i(h, \nu_i(\sigma^\varepsilon)) = Max_i(h', \nu_i(\sigma^\varepsilon))$.[10] Then, if ψ^ε is upper hemi-continuous with non-empty, closed and convex values, Kakutani's fixed point theorem guarantees the existence of a strategy profile σ^ε such that for each player i, σ_i^ε is ε-optimal given $\nu_i(\sigma^\varepsilon)$ and satisfies UTB$^\varepsilon$. If these fixed points exist, it follows by the compactness of the strategy space that there are a strategy profile σ, a valuation ν, and a sequence σ^{ε^k}, $\varepsilon^k \to 0$, such that $\sigma^{\varepsilon^k} \to \sigma$ and $\nu(\sigma^{\varepsilon^k}) \to \nu$. Moreover, as valuations depend continuously on the strategy profile, σ is optimal given ν. Finally, as all $\sigma_i^{\varepsilon^k}$ satisfy UTB$^\varepsilon$, the limits σ_i satisfy UTB; this can be seen as follows: For all i and $h, h' \in H_i$, if $Max_i(h, \nu_i(\sigma)) = Max_i(h', \nu_i(\sigma)) =: Max_i(h, h', \sigma)$, then the continuity of $\nu(.)$ implies that $Max_i(x, \nu_i(\sigma^{\varepsilon^k})) \subseteq Max(h, h', \sigma)$, for $x = h, h'$ and k sufficiently large, say $k > \overline{k}$. Since $Max_i(h, h', \sigma) \subseteq M_i(x)$, $x = h, h'$, it follows that $Max_i(h, \nu_i(\sigma^{\varepsilon^k})) \subseteq M_i(h')$, $k > \overline{k}$, and vice versa. Accordingly, $Max_i(h, \nu_i(\sigma)) = Max_i(h', \nu_i(\sigma))$ implies $Max_i(h, \nu_i(\sigma^{\varepsilon^k})) = Max_i(h', \nu_i(\sigma^{\varepsilon^k}))$, for $k > \overline{k}$. And, by UTB$^\varepsilon$, σ^{ε^k} puts equal (adjusted) weight on all $\tilde{h} \in M_i(h)$ and $\tilde{h}' \in M_i(h')$ with

[10] The adjustment to relative weights is necessary as, in general, $Max_i(h, \nu_i) = Max_i(h', \nu_i)$ need not imply $|M_i(h)| = |M_i(h')|$, $h, h' \in H_i$; as usual, $|M|$ refers to the number of elements of M.

$\lambda(\tilde{h}) = \lambda(\tilde{h}') \in Max_i(h, \nu_i(\sigma^{\varepsilon^k})) = Max_i(h', \nu_i(\sigma^{\varepsilon^k}))$, $k > \overline{k}$. Taking limits, this implies that σ_i satisfies UTB as

$$\lim \frac{\sigma_i^{\varepsilon^k}(x)(\tilde{x})}{[1 - (|M_i(x)| - |Max_i(x, \nu_i(\sigma^{\varepsilon^k}))|) \cdot \varepsilon]} = \lim \sigma_i^{\varepsilon^k}(x)(\tilde{x}) = \sigma_i(x)(\tilde{x})$$

for all $x \in H_i$ and $\tilde{x} \in M_i(x)$ with $\lambda(\tilde{x}) \in Max_i(x, \nu_i(\sigma^{\varepsilon^k}))$. Thus, σ is a SVE*.

That ψ^ε is indeed upper hemi-continuous with non-empty, closed and convex values can be seen as follows: Upper hemi-continuity of ψ^ε is a consequence of the continuity of ν in σ and the fact that ψ^ε has closed values (see further below). As regards the values of ψ^ε, non-emptiness is immediate. Values are closed because their complements are open; for any σ^ε and $\tilde{\sigma}^\varepsilon \notin \psi^\varepsilon(\sigma^\varepsilon)$, i.e. (at least) some $\tilde{\sigma}_i^\varepsilon$ contained in $\tilde{\sigma}^\varepsilon$ is not both ε-optimal given $\nu(\sigma^\varepsilon)$ and satisfies UTB$^\varepsilon$, there is an open ball $B_\delta(\tilde{\sigma}^\varepsilon)$ around $\tilde{\sigma}^\varepsilon$, $\delta > 0$, such that $B_\delta(\tilde{\sigma}^\varepsilon) \cap \psi^\varepsilon(\sigma^\varepsilon) = \emptyset$ (note that if a strategy $\tilde{\sigma}_i^\varepsilon$ does not satisfy UTB$^\varepsilon$, then also small variations in $\tilde{\sigma}_i^\varepsilon$ cannot change this). Finally, the values of ψ^ε are convex because, apart from the optimality requirement, also the UTB$^\varepsilon$ condition is compatible with taking convex combinations. ■

References

Jehiel, P., Samet, D., 2005. Learning to Play Games in Extensive Form by Valuation. Journal of Economic Theory 124, pp. 129-148.

Jehiel, P., Samet, D., 2007. Valuation Equilibrium. Theoretical Economics 2, pp. 163-185.

Kuhn, H.W., 1953. Extensive Games and the Problem of Information. Pp. 193-216 in *Contributions to the Theory of Games II*, Princeton University Press, Princeton, New Jersey.

Piccione, M., Rubinstein, A., 1997a. On the Interpretation of Decision Problems with Imperfect Recall. Games and Economic Behavior 20, pp. 3-24.

Piccione, M., Rubinstein, A., 1997b. The Absent-Minded Driver's Paradox: Synthesis and Responses. Games and Economic Behavior 20, pp. 121-130.

Rubinstein, A., 1998. *Modeling Bounded Rationality.* MIT Press, Cambridge, Massachusetts.

Wichardt, P., 2008. Existence of Nash Equilibria in Finite Extensive Form Games with Imperfect Recall: A Counterexample. Games and Economic Behavior 63, pp. 366-369.