Manipulability in Matching Markets: Conflict and Coincidence of Interests*

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Abstract

We study comparative statics of manipulations by women in the men-proposing deferred acceptance mechanism in the two-sided one-to-one marriage market. We prove that if a group of women weakly successfully manipulates or employs truncation strategies, then all other women weakly benefit and all men are weakly harmed. We show that our results do not appropriately generalize to the many-to-one college admissions model.

Keywords: matching, deferred acceptance, manipulability, welfare.

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1 Introduction

We study the effect of strategic agents on non-strategic agents in two-sided matching markets. Consider the marriage market introduced by Gale and Shapley (1962) where the two (finite) sides of the market are "men" and "women," each agent having preferences over the other side of the market and the prospect of being alone. An outcome for a marriage market is a matching in which each agent either marries an agent from the other side of the market or remains single. A key property for a matching is stability. A

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matching is stable if each agent has an acceptable match and there is no pair of a man and a woman who like each other better than their current matches. Using their deferred acceptance algorithm, Gale and Shapley (1962) constructively proved that there exists a stable matching for each profile of preferences. Moreover, Knuth (1976) showed that the set of stable matchings is a distributive lattice with respect to the preferences of the agents. An important consequence is that on the set of stable matchings each side of the market has common interests that are in conflict with those of the other side.[1]

In this note, we show that the conflict and coincidence of interests extends to the effects of manipulations in the direct-revelation games based on the deferred acceptance algorithm.[2] Consider the direct-revelation mechanism induced by the men-proposing deferred acceptance algorithm. It is in the best interest of each man to report his true preferences (Dubins and Freedman, 1981, and Roth, 1982), but women typically have incentives to misreport their true preferences. Concerning her strategic options, a woman needs to consider only truncation strategies, which are the strategies obtained by removing a tail of men (i.e., some least preferred men) from her (true) ordered list of acceptable men. More precisely, for any (general) manipulation by a woman, there is a truncation strategy which is at least as good. We show that under the men-proposing deferred acceptance mechanism,

- any weakly successful group manipulation[3] by women is weakly beneficial to all other women and weakly harmful to all men (Proposition 3.2), and
- truncating preferences by some women is weakly beneficial to all other women and weakly harmful to all men (Proposition 3.3).

Finally, we consider extending our results to the many-to-one college admissions model where students have to be assigned to colleges (with possibly multiple seats). A minor adaptation of the proof of Proposition 3.3 shows that under the student-proposing deferred acceptance mechanism, any truncation of preferences by some colleges is weakly beneficial to the other colleges and weakly harmful to all students. However, Kojima and Pathak (2009) showed that under the student-proposing deferred acceptance mechanism, truncation strategies typically do not exhaust the strategic options of the colleges. They proved that so-called dropping strategies constitute a class of exhaustive strategies. A dropping strategy of a college is obtained by removing some students from its (true) ordered lists of acceptable students (i.e., not necessarily a tail of least preferred students).

[1] See also Roth (1984) and Roth (1985b) for further results on polarization of interests in two-sided markets.

[2] For the important role of the deferred acceptance algorithm in both matching theory and many real-life applications we refer to Roth (2008).

[3] That is, none of the manipulating agents is strictly worse off.

We show that neither of our results extends to the college admissions model in an appropriate way: there are dropping strategies and successful manipulations that strictly harm some other college and strictly benefit some student.

Our results complement work by Crawford (1991) and Ma (2010). Crawford (1991) studied general many-to-one matching markets and investigated the effect of the entrance of an agent on the welfare of the other agents. When restricted to the marriage market, his result is the particular case of our second result in which a woman submits an empty truncation strategy. Ma (2010) analyzed the equilibria of the games induced by the men-proposing and student-proposing deferred acceptance mechanisms. He showed that in equilibrium either the matching is not stable or the receiving side is favored.

2 Marriage Model

In Gale and Shapley's (1962) marriage market there are two non-empty, finite, and disjoint sets of agents M (men) and W (women). A generic man, woman, and agent are denoted by m, w, and i, respectively. Each agent i has a complete, transitive, and strict preference relation P_i over the agents on the other side of the market and the prospect of being alone. Let $P = (P_i)_{i \in M \cup W}$ denote the profile of all agents' preferences. For notational convenience we denote a singleton set $\{x\}$ by x.

For $w, w' \in W \cup m$, we write $w \, P_m \, w'$ if man m strictly prefers w to w' ($w \neq w'$), and $w \, R_m \, w'$ if m likes w at least as well as w' ($w \, P_m \, w'$ or $w = w'$). Similarly, we write $m \, P_w \, m'$ and $m \, R_w \, m'$. A woman w is acceptable to a man m if $w \, P_m \, m$. Analogously, m is acceptable to w if $m \, P_w \, w$.

With some abuse of notation we also represent a man m's preferences P_m as an ordered list of the elements in $W \cup m$. For instance, $P_m = w_3 \, w_2 \, m \, w_1 \, \ldots \, w_4$ indicates that m prefers w_3 to w_2 and he prefers remaining single to any other woman. Similarly, woman w's preferences can be represented as an ordered list P_w of the elements in $M \cup w$. We often omit the unacceptable agents from agent i's ordered list P_i.

A marriage market is a triple (M, W, P), or P for short. A matching is a one-to-one function μ from $M \cup W$ to itself, such that for each $m \in M$ and for each $w \in W$ we have $\mu(m) = w$ if and only if $\mu(w) = m$, $\mu(m) \notin W$ implies $\mu(m) = m$, and similarly $\mu(w) \notin M$ implies $\mu(w) = w$. If $\mu(m) = w$, then man m and woman w are matched to one another. If $\mu(i) = i$, then agent i is unmatched or single. Agent $\mu(i)$ is called i's match at μ. We sometimes use a vector of men (or women) to denote a matching, e.g., $\mu = (m_3, \, m_1, \, m_2)$ denotes the matching where w_1 is matched to m_3, w_2 to m_1, and w_3 to m_2.

A matching μ is individually rational if $\mu(i) \, R_i \, i$ for all $i \in M \cup W$. A pair (m, w) is a blocking pair for a matching μ if $w \, P_m \, \mu(m)$ and $m \, P_w \, \mu(w)$. A matching is stable if it is individually rational and if there are no blocking pairs. Gale and Shapley (1962) proved constructively that each marriage market has at least one stable matching. For this they introduced the deferred acceptance (DA) algorithm. Let Q be a profile of ordered lists of acceptable agents. The men-proposing DA algorithm applied to Q, denoted by DA(Q) for short, finds a matching through the following steps.

Step 1: Each man m proposes to the woman that is ranked first in Q_m (if there is no such woman then m remains single). Each woman w tentatively accepts the best man among her proposers (using the list Q_w). All other proposers are rejected.

Step k, $k \geq 2$: Each man m that is rejected in Step $k-1$ proposes to the next woman in his list Q_m (if there is no such woman then m remains single). She tentatively accepts the best man among the new proposers and the tentatively matched man from the previous step, if any (using the list Q_w). All other proposers are rejected.

The algorithm stops when no man is rejected. Then, all tentative matches become final. With some abuse of notation, let $\mu(Q)$ denote the matching. For $i \in M \cup W$, let $\mu(Q, i)$ denote the match of agent i at $\mu(Q)$. Gale and Shapley (1962) proved that for preference profile Q matching $\mu(Q)$ is the best (worst) stable matching for the men (women). Dubins and Freedman (1981) and Roth (1982) proved that under the direct-revelation mechanism induced by μ it is a weakly dominant strategy for the men to reveal their true preferences. Therefore, we will assume that men are truthful and that women are the only strategic agents. Whenever there are at least two stable matchings some woman have incentives to misreport their true preferences (see for instance Roth and Sotomayor, 1990, Corollary 4.12).

3 Results

Before we present our results on the direct-revelation mechanism induced by the men-proposing deferred acceptance algorithm, we first provide the formal definitions of two classes of manipulations.

Let P be a marriage market. A **(group) manipulation** by a group of women W' is a strategy-profile $P'_{W'} = (P'_w)_{w \in W'}$. If $|W'| = 1$, then $P'_{W'}$ is an **individual manipulation**. A manipulation is **weakly successful** if for all $w \in W'$, $\mu(P', w)\, R_w\, \mu(P, w)$ where $P' = (P'_{W'}, P_{-W'})$. A manipulation is **successful** if for all $w \in W'$, $\mu(P', w)\, R_w\, \mu(P, w)$ and for some $w' \in W'$, $\mu(P', w')\, P_{w'}\, \mu(P, w')$.

A truncation strategy (Roth and Rothblum, 1999) of a woman w is a strategy (or equivalently, an ordered list) P'_w obtained from P_w by making a tail of acceptable men unacceptable. Formally, P'_w is a **truncation strategy** if for all $m, m' \in M$, (a) [if $m\, R'_w\, m' R'_w\, w$ then $m\, R_w\, m' R_w\, w$], and (b) [if $m\, P'_w\, w$ and $m'\, P_w\, m$ then $m'\, P'_w\, w$].

Note that not every truncation strategy is a weakly successful manipulation. For instance, an empty truncation strategy leaves the woman unmatched. Likewise, not every weakly successful, individual manipulation is a truncation strategy (see, for instance, Example 1). However, truncation strategies are exhaustive in the sense that any weakly successful, individual manipulation can be replicated or improved upon by some truncation strategy.[4]

The following well-known result states that men and women have opposite interests whenever a manipulation leads to a stable matching.

Lemma 3.1. *Under the men-proposing DA mechanism, a group manipulation by some women W' is weakly beneficial to all women and weakly harmful to all men if the induced matching is stable. If the matching is not stable then each blocking pair contains a woman from W'.*

Proof. Let $P'_{W'}$ be a group manipulation and let $P' = (P'_{W'}, P_{-W'})$. By assumption, $\mu(P')$ is stable for the market P. Hence, by men-optimality of $\mu(P)$, all women weakly prefer $\mu(P')$ to $\mu(P)$ and all men weakly prefer $\mu(P)$ to $\mu(P')$. The second statement follows from the observation that $\mu(P')$ is stable for P' and that for each pair (m, w) with $w \notin W'$, $P_m = P'_m$ and $P_w = P'_w$. □

The following example illustrates that a manipulation may lead to an unstable matching, even if the manipulating women are strictly better off at the new matching.

Example 1. (**A successful manipulation that yields an unstable matching.**) Consider the matching market with 3 men, 3 women, and preferences P given by the columns in the table below. For instance, $w_3 \, P_{m_1} \, w_1 \, P_{m_1} \, w_2 \, P_{m_1} \, m_1$. One easily verifies

Men			Women		
m_1	m_2	m_3	w_1	w_2	w_3
$\boxed{w_3}$	$\boxed{w_2}$	$\boxed{w_1}$	m_1	$\boldsymbol{m_1}$	$\boldsymbol{m_3}$
w_1	$\boldsymbol{w_1}$	$\boldsymbol{w_3}$	$\boldsymbol{m_2}$	$\boxed{m_2}$	$\boxed{m_1}$
$\boldsymbol{w_2}$	w_3	w_2	$\boxed{m_3}$	m_3	m_2

that $\mu(P) = (m_3, m_2, m_1)$— the boxed matching in the table. Suppose that woman w_1 submits the list $P'_{w_1} = m_2$. Then, $\mu(P') = (m_2, m_1, m_3)$— the boldfaced matching in the table. Note P'_{w_1} is a successful manipulation since $\mu(P', w_1) = m_2 \, P_{w_1} \, m_3 = \mu(P, w_1)$.

[4]To see this, let P'_w be an individual manipulation. Let $m = \mu\left((P'_w, P_{-w}), w\right) \in M \cup w$. Consider the truncation strategy P''_w obtained from P_w by making all men that are strictly less preferred than m unacceptable. One easily verifies that $\mu\left((P''_w, P_{-w}), w\right) R_w \, \mu\left((P'_w, P_{-w}), w\right)$.

But $\mu(P')$ is not stable with respect to the true preferences P (the unique blocking pair is (m_1, w_1)). $\diamond$

In Example 1, all women that do not manipulate weakly benefit and all men are weakly harmed. Since the resulting matching is not stable this observation does not follow from Lemma 3.1. Nevertheless, we will prove that the observed opposed interests are a feature of two interesting classes of group manipulations: weakly successful group manipulations and group truncation strategies.

To prove our results we introduce the following additional notation. For every integer $k \geq 1$, let $X(Q, w, k)$ be the set of men that will have proposed to woman w by step k under DA(Q), i.e., in some step $l \in \{1, \ldots, k\}$ of DA(Q). Let $X(Q, w)$ be the set of men that will have proposed to w by the last step of DA(Q), i.e., $X(Q, w) = \cup_k X(Q, w, k)$.

The proofs of our results are similar to Crawford's (1991). We include them since for marriage markets the arguments are shorter and more transparent.

Proposition 3.2. *Under the men-proposing DA mechanism, any weakly successful group manipulation by women is weakly beneficial to the other women and weakly harmful to all men.*

Proof. Let $P'_{W'}$ be a weakly successful manipulation of a group of women W' and let $P' = (P'_{W'}, P_{-W'})$. It is sufficient to show that for each woman w and each step k, $X(P, w, k) \subseteq X(P', w)$. For $k = 1$ the inclusion is obvious since at step 1 of DA(P) and DA(P') each man proposes to exactly the same woman.

Assume that the inclusion holds for k. We will show that the inclusion also holds for $k + 1$. Let $m \in X(P, w, k + 1)$. If $m \in X(P, w, k)$, then by induction, $m \in X(P', w)$. So, assume $m \in X(P, w, k + 1) \setminus X(P, w, k)$. Then, in DA($P$), man m proposed to w at step $k + 1$ but not at step k. So, m was rejected by some woman $\bar{w} \neq w$ at step k of DA(P). By the induction hypothesis, $m \in X(P, \bar{w}, k) \subseteq X(P', \bar{w})$. If $\bar{w} \notin W'$ then $\bar{w}$ will also reject m in DA(P') since $P'_{\bar{w}} = P_{\bar{w}}$. If $\bar{w} \in W'$ then $\mu(P', \bar{w}) \, R_{\bar{w}} \, \mu(P, \bar{w}) \, P_{\bar{w}} \, m$, which implies that in the last step of DA(P') woman $\bar{w}$ strictly prefers her match to m (according to her true preferences). Therefore, in either case $\bar{w}$ will also eventually reject m in DA(P'). Since m makes his proposals in the same order in DA(P) and DA(P'), he will have proposed to w by the last step of DA(P'). Hence, $m \in X(P', w)$. $\square$

For marriage markets, the next proposition generalizes the results of Crawford (1991) from an *individual empty* truncation strategy to *arbitrary group* truncation strategies. It also shows that we can replace the weakly successful group manipulations in Proposition 3.2 by (possibly unsuccessful) truncation strategies.

Proposition 3.3. *Under the men-proposing DA mechanism, any group manipulation by women that consists of truncation strategies is weakly beneficial to the other women and weakly harmful to all men.*

Proof. The proof is (almost) identical to that of Proposition 3.2: One only needs to replace the line "If $\bar{w} \in W'$ then ... (according to her true preferences)." by "If $\bar{w} \in W'$ then $\bar{w}$ will also have rejected m by the last step of DA(P') since $P'_{\bar{w}}$ is a truncation strategy obtained from $P_{\bar{w}}$." □

4 Extensions to College Admissions

In this section we consider extending our results to the many-to-one college admissions model where students have to be assigned to colleges with possibly multiple seats, strict preferences over individual students, and responsive preferences over groups of students.

There are two finite and disjoint sets of agents: a set S of students and a set C of colleges. We denote a generic student, college, and agent by s, c, and i, respectively. For each college c, there is a fixed quota q_c that represents the number of positions it offers.[5]

Each student s has a complete, transitive, and strict preference relation over the colleges and the prospect of being unmatched. Hence, student s's preferences can be represented by a strict ordering P_s of the elements in $C \cup s$. For $c, c' \in C \cup s$, we write $c\, P_s\, c'$ if student s strictly prefers c to c' ($c \neq c'$), and $c\, R_s\, c'$ if s likes c at least as well as c' ($c\, P_s\, c'$ or $c = c'$). If $c \in C$ such that $cP_s s$, then we call c an acceptable college for student s. Let $P_S = (P_s)_{s \in S}$.

Each college c has a complete, transitive, and strict preference relation over the *individual* students and the prospect of being unmatched. Hence, college c's preferences over individual students can be represented by a strict ordering P_c of the elements in $S \cup \{\emptyset\}$. For $s, s' \in S \cup c$, we write $s\, P_c\, s'$ if college c strictly prefers s to s' ($s \neq s'$), and $s\, R_c\, s'$ if c likes s at least as well as s' ($s\, P_c\, s'$ or $s = s'$). If $s \in S$ such that $sP_c\emptyset$, then we call s an acceptable student for college c. Let $P_C = (P_c)_{c \in C}$.

A set of students $S' \subseteq S$ is feasible for college c if $|S'| \leq q_c$. Each college c has a complete and transitive preference relation over feasible sets of students, which can be represented by a weak ordering P^*_c of the elements in $\mathcal{P}(S, q_c) \equiv \{S' \subseteq S : |S'| \leq q_c\}$. For $S', S'' \in \mathcal{P}(S, q_c)$, we write $S'\, P^*_c\, S''$ if college c strictly prefers S' to S'' ($S' \neq S''$), and $S'\, R^*_c\, S''$ if c likes S' at least as well as S''.

[5] The marriage model is the special case of one-to-one (two-sided) matching where for all $c \in C$, $q_c = 1$.

We will assume that for each college c, P_c^* is a responsive extension of P_c,[6] i.e., for all $S' \in \mathcal{P}(S, q_c)$,

(r1) if $s \notin S'$ and $|S'| < q_c$, then $(S' \cup s) P_c^* S'$ if and only if $s P_c \emptyset$ and

(r2) if $s \notin S'$ and $t \in S'$, then $((S' \backslash t) \cup s)\, P_c^* S'$ if and only if $s P_c t$.By responsiveness, P_c^* coincides with P_c on the set of individual students. Note also that P_c allows for multiple responsive extensions.

A college admissions market is a triple (S, C, P), where $P = (P_S, P_C^*)$. A matching for college admissions market (S, C, P) is a function μ on the set $S \cup C$ such that

(m1) each student is either matched to exactly one college or unmatched, i.e.,

for all $s \in S$, either $\mu(s) \in C$ or $\mu(s) = s$,

(m2) each college is matched to a feasible set of students, i.e.,

for all $c \in C$, $\mu(c) \in \mathcal{P}(S, q_c)$, and

(m3) a student is matched to a college if and only if the college is matched to the student, i.e.,

for all $s \in S$ and $c \in C$, $\mu(s) = c$ if and only if $s \in \mu(c)$.

Given matching μ, we call $\mu(s)$ student s's match and $\mu(c)$ college c's match.

A key property of matchings is stability. First, we impose a voluntary participation condition. A matching μ is individually rational if neither a student nor a college would be better off by breaking a current match, i.e., if $\mu(s) = c$, then $c P_s s$ and $\mu(c) P_c^* (\mu(c) \backslash s)$. By responsiveness of P_c, the latter requirement can be replaced by $s P_c \emptyset$. Thus alternatively, a matching μ is individually rational if any student and any college that are matched to one another are mutually acceptable. Second, if a student s and a college c are not matched to one another at a matching μ but the student would prefer to be matched to the college and the college would prefer to either add the student or replace another student by student s, then we would expect this mutually beneficial adjustment to be carried out. Formally, a pair (s, c), $s \notin \mu(c)$, is a blocking pair if $c P_s \mu(s)$ and (a) $[\,|\mu(c)| < q_c$ and $s P_c \emptyset\,]$ or (b) [there exists $t \in \mu(c)$ such that $s P_c t$].[7] A matching is stable if it is individually rational and there are no blocking pairs. Since stability does not depend on the particular responsive extensions of the colleges' preferences over individual students, a college admissions market is henceforth a triple $(S, C, P = (P_S, P_C))$, or simply P.

The deferred acceptance (DA) algorithm (Gale and Shapley, 1962) yields again a stable matching. Let $Q = (Q_S, Q_C)$ be a preference profile (of preferences over individual agents).[8] The student-proposing DA algorithm applied to Q, denoted by DA(Q) for short, finds a matching through the following steps.

[6]See Roth (1985a) and Roth and Sotomayor (1989) for a discussion of this assumption.

[7]Recall that by responsiveness (a) implies $(\mu(c) \cup s) P_c^* \mu(c)$ and (b) implies $((\mu(c) \backslash t) \cup s) P_c^* \mu(c)$.

[8]Note that the DA algorithm does not depend on the particular responsive extensions.

Step 1: Each student s proposes to the college that is ranked first in Q_s (if there is no such college then s remains single). Each college c considers its proposers and tentatively assigns its q_c positions to these students one at a time following the preferences Q_c. All other proposers are rejected.

Step k, $k \geq 2$: Each student s that is rejected in Step $k-1$ proposes to the next college in his list Q_s (if there is no such college then s remains single). Each college c considers the students it has been holding together with its new proposers and tentatively assigns its q_c positions to these students one at a time following the preferences Q_c. All other proposers are rejected.

The algorithm stops when no student is rejected. Then, all tentative matches become final. With some abuse of notation, let $\mu(Q)$ denote the matching. For $i \in S \cup C$, let $\mu(Q, i)$ denote the match of agent i at $\mu(Q)$. Gale and Shapley (1962) proved that for preference profile Q matching $\mu(Q)$ is stable. In fact, From Roth and Sotomayor (1990, Theorem 5.31) it follows that $\mu(Q)$ is the best (worst) stable matching for the students (colleges). Roth (1985a, Theorem 5*) proved that under the direct-revelation mechanism induced by μ it is a weakly dominant strategy for the students to reveal their true preferences. Therefore, we will assume that students are truthful and that colleges are the only strategic agents. Note that a college can manipulate not only its ordered list of students but also the number of available seats, i.e., the strategy space is richer than in one-to-one markets.

A minor adaptation of the proof of Proposition 3.3 shows that under the student-proposing DA mechanism, any group manipulation by colleges that consists of truncation strategies is weakly beneficial to the other colleges and weakly harmful to all students. However, Kojima and Pathak (2009) showed that under the student-proposing deferred acceptance mechanism, truncation strategies typically do *not* exhaust the strategic options of the colleges. More precisely, they presented a many-to-one market in which for some college there is a strategy such that any truncation strategy yields a strictly worse match. They also proved that so-called dropping strategies constitute a class of exhaustive strategies. A dropping strategy of a college is obtained by removing some students from its (true) ordered lists of acceptable students (i.e., not necessarily a tail of least preferred students).[9] Formally, for a college c with preferences P_c over individual students, P'_c is a **dropping strategy** if for all students s, s', $[s\, R'_c\, s' R'_c\, \emptyset$ implies $s\, R_c\, s' R_c\, \emptyset]$.

Therefore, a possible appropriate extension of Proposition 3.3 to college admissions would involve dropping strategies rather than truncation strategies. The next example, however, shows that neither of our results extends to the college admissions model in an

[9] The fact that dropping strategies are exhaustive implies that it suffices to focus on each college's submittable ordered lists of students.

appropriate way: there are dropping strategies and successful manipulations that strictly harm some other college and strictly benefit some student.

Example 2. (Propositions 3.2 and 3.3 cannot be appropriately generalized to college admissions.)
Consider the following matching market with students s_1, s_2, s_3, and s_4, and colleges c_1 and c_2. Each college has two seats. The preferences P over *individual* agents are given by the columns in the table below. We assume that the colleges' preferences over sets of students are responsive to the preferences over individual students and that both colleges prefer $\{s_1, s_4\}$ to $\{s_2, s_3\}$.[10] One easily verifies that $\mu(P) = (c_2, c_1, c_1, c_2)$— the boxed

Students				Colleges	
s_1	s_2	s_3	s_4	c_1	c_2
$\boxed{c_2}$	$\boxed{c_1}$	$\boxed{c_1}$	$\boldsymbol{c_1}$	$\boldsymbol{s_1}$	$\boxed{s_4}$
$\boldsymbol{c_1}$	$\boldsymbol{c_2}$	$\boldsymbol{c_2}$	$\boxed{c_2}$	$\boxed{s_2}$	$\boldsymbol{s_2}$
				$\boxed{s_3}$	$\boldsymbol{s_3}$
				$\boldsymbol{s_4}$	$\boxed{s_1}$

matching in the table. Suppose that college c_1 submits the dropping strategy $P'_{c_1} = s_1, s_4$. Then, $\mu(P') = (c_1, c_2, c_2, c_1)$— the boldfaced matching in the table. Note that P'_{c_1} is a successful dropping strategy since college c_1 prefers $\{s_1, s_4\}$ to $\{s_2, s_3\}$. Since college c_2 is strictly worse off and student s_4 is strictly better off under $\mu(P')$ it follows that Propositions 3.2 and 3.3 cannot be appropriately extended to college admissions. $\diamond$

Remark 1. In fact, using the many-to-one market in Example 2 one can construct a marriage market in which an individual (unsuccessful) dropping strategy of a woman makes another woman strictly worse off and some man strictly better off (cf. Proposition 3.2).[11] For two reasons we do not provide further details and present Example 2 instead. First, the class of dropping strategies contains the strictly smaller class of truncation strategies, which is already exhaustive for one-to-one markets. Second, the market in Example 2 shows not only the impossibility of appropriately generalizing Proposition 3.3 but also the impossibility of generalizing Proposition 3.2. $\diamond$

Finally, we note that Example 2 uncovers another difference between marriage markets and college admissions and adds to those already identified in Roth (1985a).

[10] Note that preferring $\{1, 4\}$ to $\{2, 3\}$ is compatible with responsiveness.
[11] We thank Bettina Klaus for pointing this out.

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