Adversarial versus Inquisitorial Testimony*

Winand Emons†
Universität Bern
CEPR

Claude Fluet‡
Université du Québec
à Montréal, CIRPEE

Version: May 2011

Abstract

An arbiter can decide a case on the basis of his priors, or the two parties to the conflict may present further evidence. The parties may misrepresent evidence in their favor at a cost. At equilibrium the two parties never testify together. When the evidence is much in favor of one party, this party testifies. When the evidence is close to the prior mean, no party testifies. We compare this outcome under a purely adversarial procedure with the outcome under a purely inquisitorial procedure where it is for the arbiter to decide how much testimony he wants to hear.

Keywords: evidence production, procedure, costly state falsification, adversarial, inquisitorial.
JEL: D82, K41, K42

*We thank Tilman Börgers, Bob Cooter, Eddie Dekel, Sambuddha Ghosh, Navin Kartik, Simon Lörtscher, and Jennifer Reinganum for helpful comments. Emons acknowledges the hospitality of the Department of Economics at Boston University. The usual disclaimer applies.

†Departement Volkswirtschaftslehre, Universität Bern, Schanzeneckstrasse 1, Postfach 8573, CH-3001 Bern, Switzerland, Phone: +41-31-6313922, Fax: +41-31-6313383, Email: winand.emons@vwi.unibe.ch.

‡Université du Québec à Montréal and CIRPÉE, C.P. 8888, Suc. Centre-Ville, Montréal H3C 3P8, Canada, Phone: +1-514-9878386, Fax: +1-514-9878494, Email: fluet.claude-denys@uqam.ca.

1 Introduction

How much testimony will an arbiter hear in adversarial proceedings when the parties to the conflict may spend resources to misrepresent evidence in their favor? Will both parties come forward with boosted claims offsetting each other, or will only the party for whom the evidence is favorable testify? Are there circumstances where no party testifies? What are the efficiency properties of the outcome? Is the adversarial procedure where the parties to the conflict decide whether or not they testify better than the inquisitorial procedure where the arbiter decides how much testimony he wants to hear? In this paper we address these questions.

An arbiter has to decide on an issue which we take to be a real number, for example, the damages that one party owes to the other. The defendant wants the damages awarded to be small whereas the plaintiff wants them to be large. Both parties know the actual amount owed to the plaintiff and both would like to influence the arbiter's decision.

As a benchmark we first look at a pure disclosure framework. Parties can only submit hard information, thereby disclosing the true value. Presenting evidence involves a fixed cost. Alternatively, the parties may remain silent.

In a purely adversarial procedure the parties decide whether or not to present testimony. The arbiter is passive at the discovery stage. Once the parties have finished, he decides the case on the basis of his priors and of what can be inferred from the parties' actions. The arbiter seeks to minimize adjudication error, implying that his sequentially rational decision is to adjudicate the posterior mean. When he hears no testimony, given the symmetry of the parties' actions, the posterior mean equals the prior, which the arbiter therefore adjudicates. When he hears testimony, he knows and adjudicates the true value.

The pure disclosure game has a unique equilibrium. The defendant testifies for low values of damages, the plaintiff for high values, and for values in between both parties remain silent. We measure welfare by summing the social loss from inaccurate adjudication and the parties' submission costs.

The equilibrium has the following virtues. When actual damages are close to the prior mean so that the informational value is small, the parties remain silent. There are no submission costs, yet some inaccuracy. Only when the informational value is high, do the parties come forward and testify.

Under a purely inquisitorial procedure the arbiter decides how much testimony he wants to hear. He announces whether he wants to hear no, one, or both parties. When he hears no testimony, submission costs are zero and the arbiter adjudicates the mean leading to a loss from inaccurate decisions. In the pure disclosure setup the arbiter will never ask both parties to testify: one testimony reveals the truth. If the arbiter asks a party to testify, he adjudicates the true value, yet at the expense of the submission cost. Under the inquisitorial procedure there is no fine-tuning as to the realization of damages. But the arbiter has full control over which kind of costs he incurs.

Under the adversarial procedure the arbiter always expects both, silence and testimony. Under the inquisitorial procedure he has full control over the amount of testimony. When he cares little about accurate decisions, he refuses to hear the parties; when he cares a lot, he asks a party to testify. Therefore, the inquisitorial procedure does better than the adversarial one when the arbiter cares little or a lot about accurate decisions. The adversarial procedure, by contrast, does better when the arbiter cares about both, accurate decisions and submission costs.

Next we allow parties to inflate testimony. The parties can boost the evidence in either direction, but distorting the evidence involves additional costs: the greater the distortion, the higher the cost. For instance, expert witnesses charge more the more they distort the truth. We now have a two-sender signalling game.

The equilibrium under the adversarial procedure is similar to the pure disclosure equilibrium. No party testifies when the true value is close to the prior mean and thus influencing the arbiter has negligible private value. When, however, the evidence is sufficiently in favor of one party, this party comes forward and testifies.

But now parties inflate their testimony. If the plaintiff testifies, he overstates the true value; if the defendant testifies, he understates the true value. Boasting increases the more the true amount differs from the prior mean, yet at a decreasing rate. Accordingly, for sufficiently large deviations from the prior mean the equilibrium is revealing, but it involves falsification on the part of the party who testifies. The arbiter rationally corrects for the exaggerated amount and adjudicates the true value. Stated differently, because the marginal cost of slightly distorting the truth is negligible but the marginal return is not, the arbiter expects some falsification, leading parties to do so systematically. The equilibrium involves both falsification costs and error costs.

Under the inquisitorial procedure when the arbiter decides to hear one party, depending on who testifies, the party over- resp. understates the true value. The arbiter rationally corrects for the exaggerated amount and adjudicates the true value. Accordingly, the equilibrium is revealing but it involves falsification. When both parties submit, both testimonies involve falsification: one party over-reports while the other under-reports. The arbiter corrects for this by taking an average of the exaggerated testimonies. Interestingly, under joint testimony a party inflates less than if he is the only one to testify. Under joint testimony the arbiter attaches less weight to his claims than under single testimony. Total falsification costs are lower than under single testimony, but fixed submission costs are duplified. Yet, if fixed submission costs are low, joint testimony is cheaper than single testimony.

The welfare comparison yields the following results. When wasteful influence expenditures are not too large, the inquisitorial procedure performs better when the arbiter has strong views about error costs; otherwise the adversarial procedure does better. This result resembles our pure disclosure result. Nevertheless, when falsification expenditures are large, the inquisitorial procedure does better irrespective of the weight attached to accurate decisions.

It is standard in the literature to view accuracy in adjudication and procedural economy as the objectives at which legal procedures should aim. Adversarial systems of discovery clearly motivate parties to provide evidence. Nevertheless, they are often criticized (e.g., Tullock 1975, 1980) for yielding excessive expenditures through unnecessary duplication and costly overproduction of misleading information. We refer to legal procedures for concreteness. The same issues arise in regulatory or administrative hearings as well as in many other contexts.

One strand of literature has viewed the trial outcome as an exogenous function of the litigants' levels of effort or expenditure by using so-called success contest functions; see Cooter and Rubinfeld (1989), Farmer and Pecorino (1999), Katz (1988), and Parisi (2002). In these papers adjudication is a zero-one variable, i.e., a party either wins or loses. Parties engage in a rent-seeking game, leading to excessive expenditures. Our approach differs in that the arbiter's decisions are part of a perfect Bayesian equilibrium. In our set-up the arbiter is a sophisticated decision-maker who understands the parties' incentives to boost their claims.

In another well-known strand of literature, trials are modeled as persuasion games. Parties cannot falsify the evidence as such, but are able to misrepresent it by disclosing only what they see fit; see Sobel (1985), Milgrom and Roberts (1986), Lewis and Poitevin (1997), and Shin (1998). A variant of this literature includes models where the parties engage in strategic sequential search of favorable evidence; see Froeb and Kobayashi (1996) and Daughety and Reinganum (2000). In our main framework, by contrast, the parties do not have access to hard information; they dissipate resources in attempting to fabricate convincing stories. However, as benchmark, we will also discuss the case of hard information in order to compare with our signalling model.

Our paper is most closely related to the economics literature comparing adversarial with inquisitorial procedures of truth-finding. In this literature, "inquisitorial" usually refers to a system where a neutral investigator searches for evidence, "adversarial" to one where the parties to the conflict control the uncovering and presentation of evidence; see Shin (1998), Dewatripont and Tirole (1999), Froeb and Kobayashi (2001), and Palumbo (2001). However, in civil litigation and by contrast with criminal trials, the presentation of evidence essentially rests with the parties even in so-called inquisitorial systems. The main difference is the judge's involvement in controlling the litigants' presentation of evidence through bench requests, questions, and the like; see Langbein (1985) or Parisi (2002) for a comparative description, along these lines, of adversarial and inquisitorial systems. Demougin and Fluet (2008) present an analysis of active versus passive judging in a persuasion game set-up. They show that a more active or inquisitorial arbiter may eliminate inefficient equilibria. However, they doe not deal with influence costs as such nor with the trade-off between submission costs and accuracy.

The paper is organized as follows. In the next section we describe a simplified set-up where the parties cannot falsify. In section 3 we extend the set-up to inflated testimonies. Section 4 presents the welfare analysis for the signalling model. Section 5 concludes. Most proofs are relegated to the Appendix.

2 Pure Disclosure

A plaintiff P has sued a defendant D. The issue to be settled is the amount of damages $x \in \mathbb{R}$. The plaintiff wants damages to be large while the defendant wants them to be small. The evidence available so far about x is given by the normal distribution with mean μ and variance σ^2. We denote the density by $f(x)$ and the cumulative by $F(x)$.[1] At the beginning of the trial all parties involved, i.e., plaintiff , defendant, and arbiter, know the distribution of x. The mean is such that, given the expected outcome at trial, the plaintiff's claim has positive net value.[2]

[1] Support over the whole line is assumed in order to avoid boundary conditions. The probability of extreme values of x can be made, however, arbitrarily small.

[2] We make this precise at the end of this section. We take x to be damages. Yet other examples abound. In a divorce case x may be the amount of support P should get from

Once the trial has started, both plaintiff and defendant learn the realization of x. The fact that they have become perfectly informed is common knowledge. The trial cannot be stopped at this point; the adjudicator has to decide the case. In particular, we rule out any out-of-court settlement negotiations. The arbiter can adjudicate solely on the basis of his priors at that stage of the procedure as given by $f(x)$. Alternatively, he may receive further evidence submitted from the perfectly informed but self-interested plaintiff and defendant.

After plaintiff and defendant have become informed, they may testify. Testimony is costly. A submission is of the form "the value of the quantity at issue is x_i", $i = P, D$. In the pure disclosure setup the parties can only submit hard information, thereby disclosing the true value, i.e., claims are restricted to $x_i = x$. Alternatively, the parties may refrain from testifying, which is denoted by $\emptyset_i$, $i = P, D$. A party's action is therefore $s_i \in \{\emptyset_i, x\}$. The cost is

$$c_i(s_i, x) = \begin{cases} \gamma, & \text{if } s_i = x; \\ 0, & \text{if } s_i = \emptyset_i, \end{cases}$$

$i = P, D$ where $\gamma > 0$. The total cost of testimony is $C = c_P + c_D$. The arbiter observes the defendant's and the plaintiff's actions and then adjudicates $\hat{x}(s_P, s_D) \in \mathbb{R}$.

Society is concerned about the loss from inaccuracy in adjudication and the cost of testimony. Accordingly, there is a potential trade-off between procedural costs and the social benefits of correct adjudication. The total social loss is

$$L = l + C$$

where l is the societal loss from inaccurate adjudication or "error costs". Let $\hat{x}$ be the arbiter's decision. The loss from inaccurate adjudication is

$$l(\hat{x}, x) = \theta(\hat{x} - x)^2$$

D; in regulatory hearings x may the rental charge for a local loop, the incumbent wants the charge to be high whereas the entrant wants it to be low. More generally, parties with conflicting interests face an arbiter who will deliver a decision.

where $\theta > 0$ is the rate at which society trades off accuracy against submission costs. If the true value is adjudicated, error costs are zero. The more the decision errs in either direction, the higher the losses from inaccurate adjudication and such losses increase at an increasing rate the further one moves away from the truth.

2.1 Adversarial and inquisitorial procedures

Under a purely adversarial procedure, it is for the parties to decide whether they testify or not. The procedure is as follows. The parties observe x and then simultaneously choose s_P and s_D. The arbiter observes the parties' actions and then adjudicates $\hat{x}$. The arbiter is a perfect agent and adjudicates so as to minimize expected error costs.

The parties choose s_P and s_D so as to maximize π_P and π_D where

$$\begin{aligned} \pi_P(\hat{x}, s_P, x) &= \hat{x} - c_P(s_P, x) \quad \text{and} \\ \pi_D(\hat{x}, s_D, x) &= -\hat{x} - c_D(s_D, x). \end{aligned}$$

After the arbiter has observed the agents' choices, he updates his beliefs; these are given by the probability distribution over x in the information set given by the parties' actions.

The pure disclosure game has a straightforward unique equilibrium. The defendant testifies for low values of x, the plaintiff for high values, and for values in between both parties remain silent.

Proposition 1: *In the unique equilibrium of the adversarial procedure the plaintiff discloses when $x \geq \mu + \gamma$ and is silent otherwise. The defendant discloses when $x \leq \mu - \gamma$ and is silent otherwise. The judge adjudicates the true x when he hears testimony; otherwise, he adjudicates $\hat{x} = \mu$.*

Proof: Given the judge's concern about correct decisions, he adjudicates $\hat{x} = x$ when he hears testimony. When he hears no testimony suppose he adjudicates some $\hat{x} = \nu$. Given the judge's behavior, the parties will never testify together. If, say, the defendant deviates to no testimony while the plaintiff testifies, the defendant doesn't change the arbiter's decision and saves the submission cost γ.

Suppose the defendant is silent. The plaintiff will disclose if $x - \gamma \geq \nu$; otherwise, he is better off remaining silent. Likewise, the defendant will disclose if $-x - \gamma \geq -\nu$ and is silent otherwise given the plaintiff is silent. Therefore, the arbiter knows that $x \in (\nu - \gamma, \nu + \gamma)$ when he hears no testimony. To minimize error he adjudicates the posterior mean. Given that $f(x)$ is normal, $\nu = E(x|\nu - \gamma < x < \nu + \gamma)$ is possible only if $\nu = \mu$. ■

The social loss is the sum of error cost over the interval where the parties remain silent and the submission cost over the range of x where the parties testify. At equilibrium

$$L^A(\theta, \gamma) = \theta \int_{\mu-\gamma}^{\mu+\gamma} (x - \mu)^2 f(x)\, dx + [F(\mu - \gamma) + 1 - F(\mu + \gamma)]\gamma, \qquad (1)$$

where the superscript A stands for the adversarial procedure.

We express the social loss as a function of the submission cost γ and the weight θ given to the error cost because we will perform our welfare analysis in this space. We immediately obtain $\lim_{\gamma \to 0} L^A(\theta, \gamma) := L^A(\theta, 0) = 0$. A reduction in γ not only reduces the cost of testimony but also the interval where parties are silent, thus also error costs. We also have $\lim_{\gamma \to \infty} L^A(\theta, \gamma) := L^A(\theta, \infty) = \theta\sigma^2$. For very high γ it becomes extremely unlikely to hear testimony at all.[3]

Let us now turn to a purely inquisitorial procedure. Under this procedure the arbiter decides how much testimony he wants to hear. He does so as to minimize the total social loss $L = l + C$. Specifically, the arbiter first announces whether he wants to hear no, one, or both parties. Afterwards the arbiter adjudicates.

When the arbiter refuses testimony, submission costs are zero. The arbiter then minimizes expected error costs solely on the basis of the priors implying $\hat{x} = \mu$. The expected total loss is then $L^I = \theta\sigma^2$ where the superscript I

[3] $F(\mu - \gamma)$ and $1 - F(\mu + \gamma)$ decrease exponentially fast when γ goes to infinity. Hence, the second term in (1) goes to zero.

indicates the inquisitorial procedure. In the pure disclosure set-up the arbiter will never ask both parties to testify. One testimony reveals the truth and a second testimony only adds to submission costs. Therefore, the inquisitorial judge will order at most one party to talk, leading to the loss $L^I = \gamma$. The arbiter chooses the action leading to the smallest social loss, thus

$$L^I(\theta, \gamma) = \min(\gamma, \theta\sigma^2). \tag{2}$$

2.2 Welfare comparison

From a welfare point of view the adversarial procedure has the following virtues. When the social value of information is small (i.e., x is close to the prior μ), the private benefit of testifying is also small; the parties then remain silent and do not spend resources on testifying, yet at the expense of some inaccuracy in adjudication. When the social value of information is large, the private benefit of disclosing is also large; the parties then testify thus enabling correct decisions, yet at the expense of the cost of disclosing. Nevertheless, the parties' incentives to testify need not be perfectly aligned with the social value of information. By contrast, the inquisitorial procedure is all-or-nothing: ex ante it either enforces or forbids testimony; there is no fine-tuning.

Comparing (1) and (2), it is obvious that the inquisitorial procedure does better when θ is either sufficiently small or sufficiently large. We have the following result.

Proposition 2: *For all $\gamma > 0$, there exists $0 < \underline{\theta}(\gamma) < \overline{\theta}(\gamma)$ such that $L^A < L^I$ if $\theta \in (\underline{\theta}(\gamma), \overline{\theta}(\gamma))$ and $L^I \leq L^A$ otherwise.*

Figure 1 depicts the social loss under each procedure as a function of θ. Under the inquisitorial procedure, the loss is the ex ante value of information, $\theta\sigma^2$, so long as this is smaller than the disclosure cost. Under the adversarial procedure, the loss is a straight line with slope less than σ^2 and vertical intercept less than the disclosure cost. In the Appendix we show that the L^A and L^I lines always intersect. Thus, for any positive disclosure cost, which procedure is better depends on the importance given to accuracy in adjudication. When the arbiter does not care too much about error costs or conversely when he cares a lot, he does better with the inquisitorial procedure where he fully controls which kind of costs he incurs. When the value of accuracy is in some intermediate range, the adversarial procedure does better.

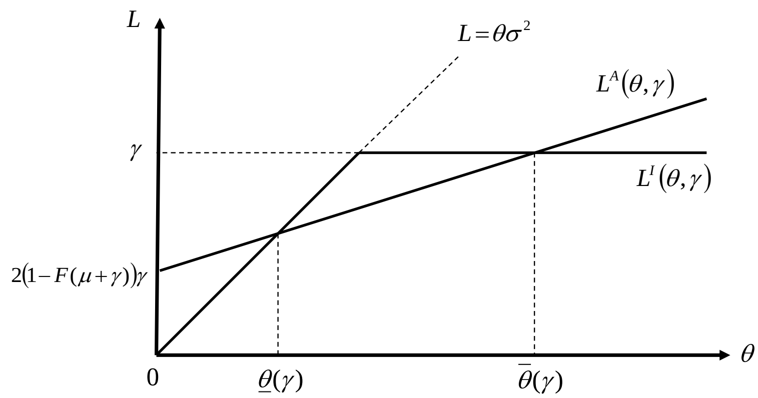

Figure 1: Inquisitorial versus adversarial

Our next result provides a characterization in the (θ, γ)-space. To provide intuition, we first compare the adversarial procedure with a first-best scenario. Suppose a social planner observes x together with the parties. The planner cannot adjudicate, which is the arbiter's responsibility, but he can force or forbid disclosure. When there is disclosure, the arbiter adjudicates $\hat{x} = x$; when there is no disclosure, he will rationally adjudicate $\hat{x} = \mu$. Obviously, the planner forces at most one party to disclose. He does so when the informational value is worth the cost, i.e., when $\theta(x-\mu)^2 \geq \gamma$. He will thus forbid disclosure when $x \in (\mu - \sqrt{\gamma/\theta}, \mu + \sqrt{\gamma/\theta})$ and he will enforce disclosure otherwise.

Therefore, when $\gamma = \sqrt{\gamma/\theta}$ or equivalently $\gamma = 1/\theta$, the parties' decisions under the adversarial procedure are socially efficient. When $\gamma > 1/\theta$, the plaintiff and defendant inefficiently remain silent for some values of x. When $\gamma < 1/\theta$, there is inefficient disclosure for some values.[4] Generically the amount of testimony is inefficient but is sometimes very close to the first best. Under the inquisitorial procedure, there is also either too much or too little testifying compared to the first best, but the outcome is then all or nothing.

Corollary 1: *If* $\theta \leq \underline{\theta}(\gamma)$*, then* $\gamma \in (\theta\sigma^2, 1/\theta)$*; if* $\theta \geq \overline{\theta}(\gamma)$*, then* $\gamma \in (1/\theta, \theta\sigma^2)$.

The result is illustrated in Figure 2. The lines $\gamma = 1/\theta$ and $\gamma = \theta\sigma^2$ partition the (θ, γ)-space into four regions. Together with Proposition 1, the Corollary tells us that the inquisitorial procedure does better than the adversarial one only in the interior of regions 2 and 4. Along the line $\gamma = 1/\theta$ the adversarial outcome yields the first-best. In region 1 there is too little disclosure under the adversarial procedure. However, there is no disclosure at all under the inquisitorial one, so that adversarial does better than inquisitorial. In region 3 we have too much disclosure under the adversarial procedure; yet there is even more disclosure under the inquisitorial one so that again adversarial does better than inquisitorial.

In region 2 there is too little disclosure under the adversarial procedure while disclosure always occurs under the inquisitorial procedure. From Proposition 1 we know that there exists an area such as I_2 where the inquisitorial procedure does better. In region 4 there is too much disclosure under the adversarial procedure and no disclosure at all under the inquisitorial one. From Proposition 1 again, there is an area such as I_4 where the inquisitorial procedure yields a smaller social cost. In the Appendix we show that the areas are as represented in the figure. In particular, the boundary of region I_2 gets asymptotically close to $\gamma = 1/\theta$ or $\gamma = \theta\sigma^2$ when γ becomes arbitrarily small or large.

[4]Inefficient non-disclosure arises for x in $(\mu + \sqrt{\gamma/\theta}, \mu + \gamma)$ or $(\mu - \gamma, \mu - \sqrt{\gamma/\theta})$; inefficient disclosure for x in $(\mu + \gamma, \mu + \sqrt{\gamma/\theta})$ or $(\mu - \sqrt{\gamma/\theta}, \mu - \gamma)$.

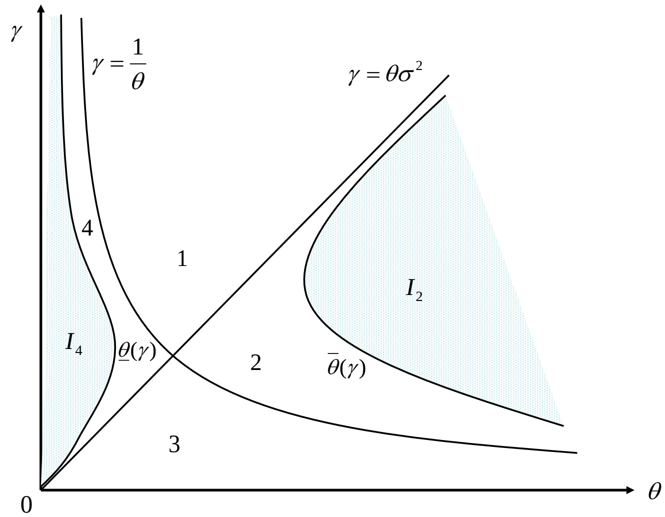

Figure 2: Optimal procedures under pure disclosure

To sum up: The inquisitorial procedure does better when either (i) it forbids testimony, there is too much disclosure under the adversarial regime, and the arbiter does not care too much about correct decisions, or (ii) when it enforces testimony, there is too little disclosure under the adversarial system, and the arbiter cares a lot about correct decisions. Otherwise the adversarial procedure does a better job than the inquisitorial one.

Let us finally look at the plaintiff's decision to sue. Under the adversarial procedure, the plaintiff's expected payoff at equilibrium is $\mu - \gamma(1 - F(\mu + \gamma))$. Under the inquisitorial procedure, his expected payoff is $\mu - \gamma$ if he thinks he will be required to testify; otherwise it is μ. The plaintiff sues if his expected payoff is positive. Under either procedure, $\mu > \gamma$ is clearly a sufficient condition.

3 Inflated Testimony

Let us now extend our pure disclosure model to a set-up where the parties can falsify the evidence at a cost. Again a submission is of the form "the value of the quantity at issue is x_i", $i = P, D$. Now it should be thought of as a story or argument rendering x_i plausible, together with the supporting documents, witnesses, etc. The cost of a presentation is $\gamma + \kappa (x_i - x)^2$ where $\kappa > 0$. The actual value is x, which is observed by the party, and x_i is the testimony or the statement submitted.

A distorting testimony is more costly than simply reporting the naked truth as it involves more fabrication. We take a quadratic function to capture the idea that the cost of misrepresenting the evidence increases at an increasing rate the further one moves away from the truth: it becomes more difficult to produce the corresponding documents or experts charge more the more they distort the truth. Falsification costs increase with the parameter κ; for κ going to zero falsification becomes costless and for κ arbitrarily large our pure disclosure set-up applies.[5]

A party's action is now $s_i \in \{\emptyset_i \cup \mathbb{R}\}$ with cost

$$c_i(s_i, x) = \begin{cases} \gamma + \kappa (x_i - x)^2, \text{ if } s_i = x_i \in \mathbb{R}; \\ 0, \text{ if } s_i = \emptyset_i, \end{cases} \tag{3}$$

$i = P, D$. We now have a signalling game. It differs from the usual signalling model in that two senders share the same information.

The message x_i in (3) is simply a costly action. Its meaning or rather the inference drawn from it will therefore be determined by the arbiter's beliefs at equilibrium. From a formal point of view, given the symmetry of the cost function (3), it would not matter if, say, we were to model the plaintiff as attempting to influence the arbiter's beliefs in his favor by playing $x_P < x$ (rather than $x_P > x$). In this perspective, "less" would simply be interpreted as meaning "more". However, we will stick to the interpretation that x_P has the literal meaning "the true state is x_P". The plaintiff then tells the truth if $x_P = x$; he boosts his case, that is, he "lies" or "falsifies" in his favor if $x_P > x$. At equilibrium, of course, the arbiter may not believe a testimony's

[5]Maggi and Rodríguez-Clare (1995) use the same lying cost function and interpret κ as capturing the publicness of information. If $\kappa = 0$, falsification is costless; therefore, information is purely private. As κ increases, it becomes more costly to falsify information and for an arbitrarily large κ the public-information model obtains.

literal meaning and may take into account a party's incentive to make inflated claims.[6]

3.1 Adversarial procedure

In principle we can have equilibria where testimony provides some information or we can have totally uninformative pooling equilibria. The latter possibility is easily discarded. First, standard refinements such as Grossman-Perry (1986) rule out equilibria where both parties would always remain silent. Suppose on the contrary that they do and the judge rationally adjudicates $\hat{x} = \mu$. Now take some out-of-equilibrium message x_P. In the Appendix we show the existence of an interval $(\underline{x}, \overline{x})$ such that, for x in this interval, the plaintiff has a payoff strictly larger than his equilibrium payoff from action x_P if the judge were to correctly infer that $x \in (\underline{x}, \overline{x})$ and accordingly update his beliefs using Bayes' rule. Secondly, it cannot be the case that both parties always testify with claims that are invariant with the true state.[7]

We focus on revealing equilibria; by revealing we mean that the arbiter infers the true state when the parties testify. We impose the following structure.

No-understatement: At equilibrium, if P testifies at x, he claims $x_P \geq x$; if D testifies at x, he claims $x_D \leq x$. The condition is in keeping with the convention that testimony has literal meaning.

Monotonicity: At equilibrium, if P testifies at x, he also testifies at $x' > x$ and $x_P(x') \geq x_P(x)$; if D testifies at x, he also testifies at $x'' < x$ and $x_D(x'') \leq x_D(x)$.

Minimality: At an out-of-equilibrium information set the arbiter believes

[6]For developments along these lines see Kartik (2009).

[7]Suppose that P always claims x_P and D always claims x_D. The judge adjudicates $\hat{x} = \mu$. If P deviates to $\emptyset_P$, he adjudicates some ν. But then P's payoff from the deviation is $\nu > \mu - \gamma - \kappa(x_P - x)^2$ when x differs sufficiently from x_P. The same argument holds if D were to always play $\emptyset_D$.

that it was reached with the minimum number of deviations from the equilibrium strategies.[8]

To clarify minimality, consider an out-of-equilibrium pair (s_P, s_D), meaning there does not exist x satisfying $(s_P(x), s_D(x)) = (s_P, s_D)$. If neither s_P nor s_D is ever observed at equilibrium, minimality imposes no restriction on beliefs. However, if s_P is never observed but s_D is, the arbiter assigns unit probability to the event $\{x : s_D(x) = s_D\}$, i.e., he infers that P has deviated and that D has not; we impose no restriction on how probability is distributed over the relevant set: if, say, $S_D = \emptyset$ and at equilibrium D is silent for $x \in [\underline{x}, \overline{x}]$, by minimality the arbiter puts unit mass on this interval and any distribution is admissible. The same holds if we reverse the role of s_P and s_D. Finally, if both s_P and s_D are observed at equilibrium, although never simultaneously, the arbiter assigns unit probability to the event $\{x : s_D(x) = s_D \text{ or } s_P(x) = s_P\}$, i.e., he infers that one party must have deviated but may not be sure which one. Again we impose no restriction on the distribution over the relevant set.

In the Appendix we prove the following result:

Lemma 1: *In a revealing equilibrium, (i) each party's strategy involves both silence and testimony; (ii) the plaintiff and the defendant never testify together.*

The lemma implies that there exists $x_D^0 < x_P^0$, both finite, such that the defendant testifies when $x \leq x_D^0$ and is otherwise silent, while the plaintiff testifies when $x \geq x_P^0$ and is otherwise silent. The consequence is that, for $x \geq x_P^0$, the plaintiff's equilibrium separating strategy $x_P(x)$ can be derived using the well-known methods of one-sender signalling games.[9] The only minor difference is that the equilibrium profile $(x_P(x), \emptyset_D)$ will need to be

[8] A similar restriction on beliefs has been used by Bagwell and Ramey (1991), Schultz (1999) or Emons and Fluet (2009); see also Hetzendorf and Overgaard (2001) and Fluet and Garella (2001). These papers also involve two-sender signalling games with perfectly correlated information.

[9] See Mailath (1987) for signalling games with a continuum of types.

supported by out-of-equilibrium beliefs preventing a deviation by D. The same observations apply to the defendant's separating strategy over the half-line $x \leq x_D^0$.

For $x \geq x_P^0$, the arbiter's strategy is $\hat{x}(x_P, \emptyset_D)$ where x_P is the plaintiff's testimony. The plaintiff chooses x_P to maximize $\hat{x}(s_P, \emptyset_D) - c_P(x_P, x)$. If $x_P(x)$ is separating, the function is one-to-one and the arbiter therefore adjudicates $\hat{x}(s_P, \emptyset_D) = x_P^{-1}(x_P)$. Because the strategy is optimal for the plaintiff, he chooses x_P to maximize $x_P^{-1}(x_P) - c_P(x_P, x)$. Substituting from (3), the first-order condition to the plaintiff's problem then yields the differential equation

$$(x_P(x) - x))\, x_P'(x) = \frac{1}{2\kappa}, \quad x \geq x_P^0, \tag{4}$$

We solve this equation using the non-decreasing solution and given the initial condition $x_P(x_P^0) = x_P^0$; the latter characterizes the least-cost signalling strategy, the so-called Riley equilibrium. The defendant's strategy solves

$$(x - x_D(x)))\, x_D'(x) = \frac{1}{2\kappa}, \quad x \leq x_D^0, \tag{5}$$

with $x_D(x_D^0) = x_D^0$.[10]

The thresholds at which the parties decide to testify are a property of the equilibrium. In the proof of the next Proposition (see the Appendix), we show that $x_D^0 = \mu - \gamma$ and $x_P^0 = \mu + \gamma$. Thus, the parties' decision whether or not to testify is the same as in the pure disclosure framework. Solving the differential equations then yields:

(i) The plaintiff's testimony $x_P(x)$ is $x_P \geq x$ satisfying

$$x = x_P - \frac{1 - e^{-2\kappa(x_P - \mu - \gamma)}}{2\kappa}, \quad x \geq \mu + \gamma. \tag{6}$$

(ii) The defendant's testimony $x_D(x)$ is $x_D \leq x$ satisfying

$$x = x_D + \frac{1 - e^{-2\kappa(\mu - \gamma - x_D)}}{2\kappa}, \quad x \leq \mu - \gamma. \tag{7}$$

[10] The solutions satisfy the conditions for a global maximum to the parties' optimization problem. See Mailath (1987).

To sum up:

Proposition 3: *Under the adversarial procedure with least-cost separating strategies, the defendant claims x_D solving (7) when $x \leq \mu - \gamma$, while the plaintiff is silent. When $x \in (\mu - \gamma, \mu + \gamma)$, neither party testifies. When $x \geq \mu + \gamma$, the plaintiff claims x_P solving (6) and the defendant is silent. If one party testifies, the arbiter infers and adjudicates the true x; if neither party testifies, the arbiter rationally expects and adjudicates μ.*

The strategies are represented in Figure 3. At the threshold where he decides to testify, the plaintiff claims the truth. For $x > \mu + \gamma$, he inflates his claim, i.e., $x_P(x) > x$ with $\lim_{x \to \infty} x_P(x) = x + 1/(2\kappa)$. Boasting increases with the true state, yet at a decreasing rate. The defendant's strategy is similar. For $x < \mu - \gamma$, $x_D(x) < x$ with $\lim_{x \to -\infty} x_P(x) = x - 1/(2\kappa)$.

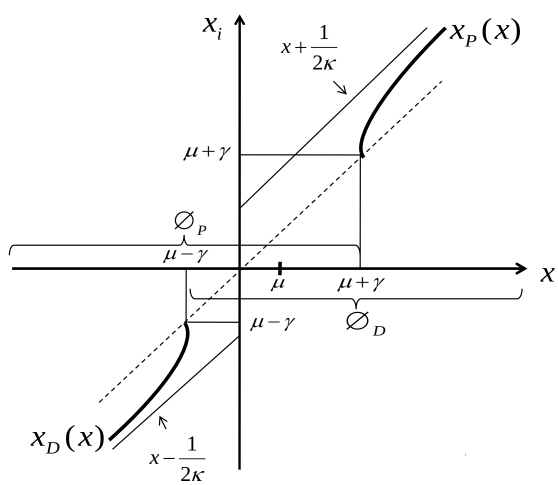

Figure 3: The least cost signalling strategies $x_P(x)$ and $x_D(x)$

3.2 Inquisitorial procedure

When the arbiter requires the plaintiff to testify, the separating strategy $x_P(x)$ will also solve the differential equation (4) but without the boundary condition; that is, the equation must hold for all $x \in \mathbb{R}$.[11] Similarly, when the defendant is required to testify, $x_D(x)$ will solve the differential equation (5) over the whole real line.

The least-cost separating strategies, i.e., the ones with the least inflated claims are given by $x_P(x) = x + 1/(2\kappa)$ and $x_D(x) = x - 1/(2\kappa)$. The amount of boasting is the same as asymptotically under the adversarial procedure. Under the inquisitorial procedure there is no finite starting point upon which the party can build in order to make his claims. For instance, under the adversarial procedure the plaintiff knows that, if he testifies, the arbiter will expect the true x to be at least $\mu + \gamma$; in particular, the plaintiff will be believed to speak the truth if he claims $x_P = \mu + \gamma$. This opportunity is not available under the inquisitorial procedure: if the plaintiff wants to convince the arbiter that $x = x + \mu$, he has to boost.

However, the inquisitorial arbiter has an additional option: he may require joint testimony. The idea is that, by forcing confrontation, both the plaintiff and the defendant will be induced to falsify less because boosting their claim is less productive. Under simultaneous testimony, the parties' equilibrium strategies $x_D(x)$ and $x_P(x)$ are again one-to-one functions spanning the whole space. Hence, they can be inverted. When the arbiter observes the pair (x_D, x_P), he knows that the true x is equal to $x_D^{-1}(x_D) = x_P^{-1}(x_P)$ when the equality holds. When it does not hold, it must be that at least one party deviated from his equilibrium strategy. By the minimality condition, he then believes that at most one did. In the least-cost separating profile, given the symmetry of lying costs, he assigns an equal chance to a deviation by the plaintiff or the defendant. He therefore adjudicates

$$\hat{x}(x_D, x_P) = \frac{1}{2}x_D^{-1}(x_D) + \frac{1}{2}x_P^{-1}(x_P).$$

The plaintiff then chooses x_P to maximize

$$\frac{1}{2}x_D^{-1}(x_D) + \frac{1}{2}x_P^{-1}(x_P) - c_P(x_P, x).$$

[11]This section borrows heavily from Emons and Fluet (2009). There we provide a more detailed justification of the statements that follow.

The first-order conditions to this problem yields the differential equation

$$(x_P(x) - x))\, x_P'(x) = \frac{1}{4\kappa}.$$

Similarly, the defendant's optimization problem yields

$$(x - x_D(x)))\, x_D'(x) = \frac{1}{4\kappa}.$$

These differential equations should be compared with the corresponding ones under single testimony. The solutions, choosing the ones with the smallest falsification, are $x_P(x) = x + 1/(4\kappa)$ and $x_D(x) = x - 1/(4\kappa)$.

Under joint testimony, a party inflates his claim only half as much as he would if he were the only one to testify. The reason is that the arbiter now attaches to his testimony only half as much importance as he would under single testimony. A party falsifies less because lying is costly and it now has less influence on the arbiter's decision.

3.3 The cost of testimony

We now collect some results that will be useful in our welfare comparison. Consider first the inquisitorial procedure. Under single testimony, the lying cost of the testifying party is $k := \kappa\,(1/2\kappa)^2 = 1/4\kappa$. The total cost of testimony is then $\gamma + k$. The easier it is to falsify, the larger the value of k. Under joint testimony, each party bears the cost $\gamma + \kappa\,(1/4\kappa)^2 = \gamma + k/4$. Summing over both parties, the total cost of testimony is then $2\gamma + k/2$. Joint testimony is cheaper than single testimony when $2\gamma + k/2 < \gamma + k$ or equivalently $\gamma < k/2$. Thus, for any fixed cost, joint testimony will be cheaper than single testimony if falsification is sufficiently easy.

Consider next the adversarial procedure. Because we will be looking at the family of equilibria generated by different values of γ, we write the parties' strategies explicitly as a function of that parameter. For the plaintiff, the falsification cost at equilibrium is $v^P(x,\gamma) := \kappa\,(x_P(x,\gamma) - x)^2, \quad x \geq \mu + \gamma$. From the previous discussion, we know that the function is increasing and concave in x with $v^P(\mu+\gamma,\gamma)=0$ and $v^P(\infty,\gamma)=k$. For the defendant $v^D(x,\gamma) := \kappa\,(x-x_D(x,\gamma))^2, \quad x \leq \mu-\gamma$, where $v^D(\mu-\gamma,\gamma)=0$ and $v^D(-\infty,\gamma)=k$. In the Appendix Lemma 2 summarizes some properties of the falsification expenditure functions.

Testimonies depend on γ only through the initial condition, i.e., the curves $x_D(x,\cdot)$ and $x_P(x,\cdot)$ simply shift horizontally when the fixed cost changes. When the parties are more reluctant to testify they also falsify less, everything else equal.

Over the range where a party testifies, the average falsification expenditure is

$$\overline{v}(\gamma) := \int_{-\infty}^{\mu-\gamma} v^D(x,\gamma)\frac{f(x)}{F(\mu-\gamma)}\,dx = \int_{\mu+\gamma}^{\infty} v^P(x,\gamma)\frac{f(x)}{1-F(\mu+\gamma)}\,dx.$$

The equality follows from symmetry. The expected falsification expenditure under the adversarial procedure is then $[F(\mu-\gamma)+1-F(\mu+\gamma)]\overline{v}(\gamma)$. It follows immediately from Lemma 2 that the expected falsification expenditure is decreasing in γ. Adding γ gives us the expected cost of testimony.

Finally, let us reconsider the plaintiff's incentive to file suit. Under the adversarial procedure, his expected payoff is $\mu-(1-F(\mu+\gamma))(\gamma+\overline{v}(\gamma))$. Under the inquisitorial procedure, it is at worst $\mu-\gamma-k$. Under either procedure, a sufficient condition to sue is, therefore, $\mu>\gamma+k$.

3.4 Comparing the adversarial with the inquisitorial procedure

It is useful to take pure disclosure as a benchmark. Let the fixed cost of testimony under claim falsification be equal to the submission cost under pure disclosure. Several results then follow immediately.

Consider the adversarial procedure. Even though the parties now attempt to boost their claims, their decisions whether or not to testify are the same as under pure disclosure: parties testify when $x \notin (\mu-\gamma,\mu+\gamma)$. Because the arbiter infers the truth from the parties' testimony, the social loss from inaccurate adjudication is equal to the loss under pure disclosure. However, testimony is more costly.

When parties may falsify, testimony under the inquisitorial procedure is also more costly than in the pure disclosure framework. Because the arbiter trades-off the cost of testimony against error costs, he will be more reluctant to allow testimony than under pure disclosure. Denote by $\overline{\gamma}(\theta)$ the frontier between testimony and no-testimony in the (θ, γ)-space, i.e., testimony is allowed only when $\gamma \leq \overline{\gamma}(\theta)$. With pure disclosure, $\overline{\gamma}(\theta) = \theta\sigma^2$. When claims are inflated, the cost of testimony is either $\gamma + k$ (single testimony) or $2\gamma + k/2$ (joint testimony), whichever is cheaper. Then $\overline{\gamma}(\theta) = \max\left[\theta\sigma^2 - k, \theta\sigma^2/2 - k/4, 0\right]$; see Figures 4 and 5. Under the inquisitorial procedure, adjudication will therefore more often be inaccurate when parties may falsify than when they may not.

Finally, the cost of testimony will differ between procedures when parties may boost. When single testimony is optimal under the inquisitorial procedure, the cost of testimony is larger than with the adversarial procedure. When joint testimony is best, it is not clear at first sight which procedure is cheaper.

The social loss under the inquisitorial procedure is

$$L^I(\theta, \gamma) = \min\left[\theta\sigma^2, \gamma + k, 2\gamma + k/2\right]. \tag{8}$$

Under the adversarial procedure, it is

$$L^A(\theta, \gamma) = \theta \int_{\mu-\gamma}^{\mu+\gamma} (x-\mu)^2 f(x)\, dx + [F(\mu-\gamma) + 1 - F(\mu+\gamma)](\gamma + \overline{v}(\gamma)). \tag{9}$$

Taking γ as given, (8) and (9) describe functions of θ similar to the ones depicted in Figure 1. Indeed, when single testimony is optimal under the inquisitorial procedure, $L^I(\theta, \gamma) = \min\left[\theta\sigma^2, \gamma + k\right]$. Then the L^I and L^A lines will be as drawn in figure 1. Specifically, because $\overline{v}(\gamma) < k$, an argument similar to the one used in Proposition 1 shows that the lines necessarily intersect. Thus, the inquisitorial procedure will do better for either small or large values of θ, while the adversarial procedure will do better for intermediate values. The same argument cannot be used, however, when joint testimony is best, i.e., when $L^I(\theta,\gamma) = \min\left[\theta\sigma^2, 2\gamma + k/2\right]$. As we will show below, it is then possible that the L^I and L^A lines do not intersect. When this occurs, the inquisitorial procedure does better for all values of θ.

In Figures 4 and 5, the lines $\gamma = 1/\theta$ and $\gamma = \overline{\gamma}(\theta)$ are used to partition the (θ,γ)-space into four regions, as was done in section 2. In figure 4, the lines intersect on the no testimony-joint testimony frontier of the inquisitorial procedure. In figure 5, they intersect on the single testimony frontier. In the latter case, region 3 defined by $\gamma \leq \min[1/\theta, \overline{\gamma}(\theta)]$ has been further partitioned into two parts: the subregion 3a is for the case where $\gamma \geq k/2$ so that single testimony is required under the inquisitorial procedure; the subregion 3b is for the case where $\gamma < k/2$ and joint testimony is required. In the next result, regions 2 and 3 are taken to be closed sets, i.e., they include their frontier.

Lemma 3: *$L^A < L^I$ in regions 1 and 3a. If $\overline{v}(0) \leq k/2$, $L^A < L^I$ in the whole of region 3.*

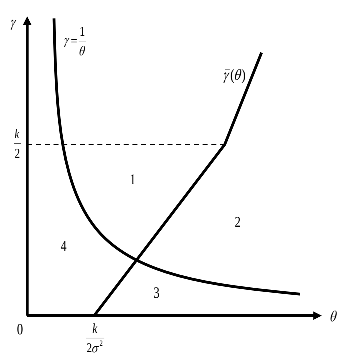

Figure 4: Large values of *k*

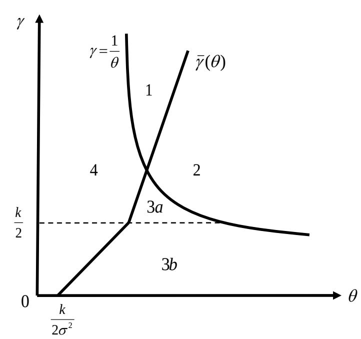

Figure 5: Smaller values of *k*

The argument for region 1 is similar to the one used under pure disclosure, except that falsification costs must now be taken into account; in the Appendix we show that when parties testify under the adversarial procedure, the social value of information always exceeds the cost of testimony. The argument for region 3a (when it is non-empty) is also similar to the one used for region 3 under pure disclosure: the adversarial procedure does better because unwarranted testimony arises less often than under the inquisitorial procedure, to which must now be added the fact that the cost of testimony in region 3a is smaller under the adversarial than under the inquisitorial procedure. In the rest of region 3, that condition cannot be guaranteed. It does hold, however, when $\overline{v}(0) \leq k/2$.

The foregoing condition plays an important role in what follows. Under the adversarial procedure the expected falsification expenditure is

$$F(\mu - \gamma) + 1 - F(\mu + \gamma)]\overline{v}(\gamma)$$

which is decreasing in γ. In the limiting case where γ tends to zero, the parties always testify under the adversarial procedure and the expression reduces to $\overline{v}(0)$. $\overline{v}(0)$ is thus the upper bound of the expected falsification expenditure under adversarial testimony. When $\overline{v}(0) \leq k/2$, the expected falsification expenditure under the adversarial procedure is therefore always smaller than under the inquisitorial one, for any value of γ. The same holds for the expected total cost of testimony including the fixed cost. When the inequality is reversed, however, there will be a range of γ-values where the inquisitorial procedure is cheaper because it yields smaller falsification costs.

From the preceding section we know that over the range where a party testifies, a party's falsification cost under the adversarial procedure varies between zero and k. When prior beliefs are very precise (i.e., when σ^2 is very small), most of the probability weight under will be concentrated close to the mean of the distribution. Because $v^D(\mu, 0) = v^P(\mu, 0) = 0$, we will then have $\overline{v}(0) < k/2$. Conversely, when prior beliefs are very diffuse, most of the probability weight will be on values of x far from the mean and we will then have $\overline{v}(0) > k/2$.

The next result is our main Proposition. It relies on the fact that the condition introduced in Lemma 3 is necessary and sufficient to ensure that the expected cost of testimony under the adversarial procedure is never greater than the cost of testimony under the inquisitorial procedure.

Proposition 4:

i) If $\overline{v}(0) \leq k/2$*, then for all* $\gamma > 0$ *there exists* $\overline{\theta}(\gamma) > \underline{\theta}(\gamma) > 0$ *such that* $L^A < L^I$ *if* $\theta \in (\underline{\theta}(\gamma), \overline{\theta}(\gamma))$ *and* $L^I \leq L^A$ *otherwise.*

ii) If $\overline{v}(0) > k/2$*, then there exists* $\hat{\gamma} > 0$ *such that* $L^I \leq L^A$ *for all* θ *if* $\gamma \leq \hat{\gamma}$*; if* $\gamma > \hat{\gamma}$ *there exists* $\underline{\theta}(\gamma)$ *and* $\overline{\theta}(\gamma)$ *as in i).*

The first part of the Proposition is illustrated in Figure 6. The result is similar to the one obtained under pure disclosure. However, the area I_2 is smaller whereas I_4 is larger than the corresponding areas under pure disclosure.

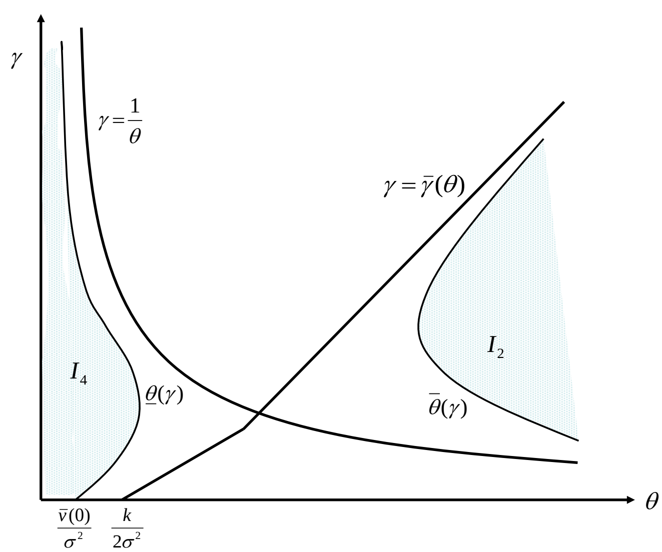

Figure 6: Small prior uncertainty

In region 2 the inquisitorial arbiter requires testimony. This region is smaller than under pure disclosure; the inquisitorial arbiter is more reluctant to require testimony because testimony is now more costly because of falsification costs. This is captured by the lower upper bound $\overline{\gamma}(\theta)$ for region 2 compared to pure disclosure. In addition, when $\overline{v}(0) \leq k/2$, the cost of testimony under the adversarial procedure is smaller than under the inquisitorial procedure. Hence, the area within region 2 where the inquisitorial does better is smaller: for a given θ, the inquisitorial procedure will do better only within a smaller range of γ-values.[12] In region 4 the inquisitorial arbiter refuses to hear any testimony at all, yielding the social loss $L^I = \theta\sigma^2$. The area I_4 is larger than under pure disclosure because the cost of adversarial testimony is now larger. For a given θ, the inquisitorial procedure will now do better within a larger range of γ-values.

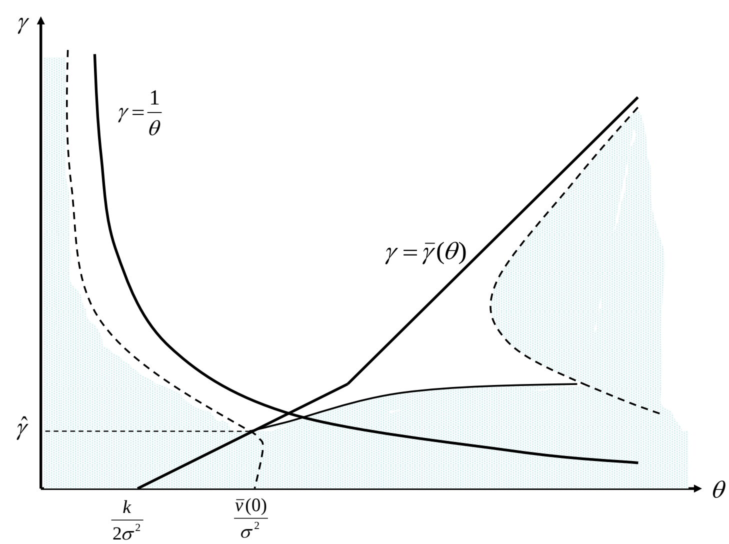

Figure 7: Large prior uncertainty

The second part of Proposition 4 is illustrated in Figure 7. This differs markedly from the pure disclosure set-up. The inquisitorial procedure does better in the shaded area. To see how this area is obtained, suppose that the

[12]One can show that I_2 is bounded below by the curve $\gamma = g(\theta) := (1 + \sqrt{1 + \theta/k})/2\theta$. This curve is above the $\gamma = 1/\theta$ curve but tends to it asymptotically when θ becomes arbitrarily large.

inquisitorial arbiter does not have the option of requiring joint testimony, so that $L^I(\theta, \gamma) = \min[\theta\sigma^2, \gamma + k]$ irrespective of the values of γ and k. Then the inquisitorial procedure does better in the shaded area below the two dashed curves, i.e., either for small for or large values of θ as in part i) of Proposition 4.

When the option of joint testimony becomes available, the rest of the shaded area must be added: confronting the parties is cheaper than single testimony for $\gamma < k/2$. Therefore, for all $\gamma \leq \hat{\gamma}$, the inquisitorial procedure does better than the adversarial one for all values of θ; here $\hat{\gamma}$ is the value of γ at which the shaded area crosses the joint testimony-no testimony frontier. In the Appendix we show that the relevant areas are as depicted.

To sum up: If lying costs are not too important compared to the fixed cost of testifying (i.e., $\gamma \geq k/2$) or if there is little prior uncertainty (i.e., σ^2 is small so that $\overline{v}(0) \leq k/2$), the inquisitorial procedure is better than the adversarial one when accuracy in adjudication is not too important or, conversely, when it is very important; otherwise, the adversarial procedure does better. When lying costs are important and there is significant prior uncertainty ($k/2$ is sufficiently large and $\overline{v}(0) > k/2$), the inquisitorial procedure does better irrespective of the importance of accuracy in adjudication.

4 Concluding Remarks

We have derived the equilibrium testifying behavior under adversarial arbitration. When the true value of the amount at issue differs only slightly from the prior mean, the parties remain silent and do not spend resources on falsification. This comes at the expense of incorrect decisions, but the social loss from inaccurate adjudication will then also be small. Only when the true value differs sufficiently from the prior mean do parties testify. This enables correct decisions, yet at the expense of falsification costs.

Moreover, we have compared the adversarial with the inquisitorial procedure, taking into account submission costs and accuracy in adjudication.

When wasteful influence expenditures are not too large, the inquisitorial procedure performs better when the arbiter has strong views about error costs; otherwise, the adversarial procedure does better. However, when falsification expenditures are an important component of the cost of testimony, the inquisitorial procedure does better irrespective of the weight attached to accuracy in adjudication.

We have assumed extreme forms both for the adversarial and inquisitorial procedures. Under the former, our arbiter is passive and can just wait for testimony by the parties. Under the latter, the arbiter does not have the option to let the parties freely decide whether they want to testify: he can only either summon them to testify or refuse to hear them. Obviously, an active arbiter who also has the option to let the parties freely testify would yield the best of both worlds. On matters where accuracy has negligible social value, he would refuse to hear the parties. When accuracy has very large social value, he could summon one or both parties to testify. In intermediate cases, he could let the parties decide whether or not they want to testify. He then relies on the parties' superior private information about the true state to reach the best compromise between submission costs and accuracy. This is not unlike the justification often given for "managerial judges" who participate in activities such as pretrial discovery and settlement bargaining (see Schrag, 1999).

Appendix

Proof of Proposition 2. We complete the argument in the text by showing that $L^A(\gamma/\sigma^2,\gamma) < L^I(\gamma/\sigma^2,\gamma)$, i.e.,

$$(\gamma/\sigma^2)\int_{\mu-\gamma}^{\mu+\gamma}(x-\mu)^2 f(x)\,dx + \gamma[F(\mu-\gamma)+1-F(\mu+\gamma)] < \gamma$$

or equivalently

$$\int_{\mu-\gamma}^{\mu+\gamma}(x-\mu)^2\left(\frac{f(x)}{F(\mu+\gamma)-F(\mu-\gamma)}\right)dx < \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx.$$

Given symmetry, the distribution $f(x)$ is a mean-preserving spread of the conditional distribution $f(x)/(F(\mu+\gamma)-F(\mu-\gamma))$. The inequality then follows from the strict convexity of $(x-\mu)^2$ with respect to x. ■

Proof of Corollary 1. If $\theta \leq \underline{\theta}(\gamma)$, it follows directly from Lemma 1 and Figure 1 that $\gamma > L^A(\theta,\gamma) \geq L^I(\theta,\gamma) = \theta\sigma^2$. Similarly, if $\theta \geq \overline{\theta}(\gamma)$, it follows that $\theta\sigma^2 > L^A(\theta,\gamma) \geq L^I(\theta,\gamma) = \gamma$. Using symmetry, the inequality $L^A(\theta,\gamma) \geq \theta\sigma^2$ is easily seen to be equivalent to

$$\int_{\mu+\gamma}^{\infty} [\gamma - \theta(x-\mu)^2] f(x)\, dx \geq 0. \tag{10}$$

The inequality $L^A(\theta,\gamma) \geq \gamma$ is equivalent to

$$\int_{\mu}^{\mu+\gamma} [\gamma - \theta(x-\mu)^2] f(x)\, dx \leq 0. \tag{11}$$

Now, observe that

$$\int_{\mu+\gamma}^{\infty} [\gamma - \theta(x-\mu)^2] f(x)\, dx < \int_{\mu+\gamma}^{\infty} \gamma(1-\gamma\theta) f(x)\, dx \tag{12}$$

and

$$\int_{\mu}^{\mu+\gamma} [\gamma - \theta(x-\mu)^2] f(x)\, dx > \int_{\mu}^{\mu+\gamma} \gamma(1-\gamma\theta) f(x)\, dx. \tag{13}$$

If $\gamma\theta \geq 1$, (12) implies that (10) cannot hold. Thus, $\theta \leq \underline{\theta}(\gamma)$ implies $\gamma < 1/\theta$ as well as $\gamma > \theta\sigma^2$. If $\gamma\theta \leq 1$, (13) implies that (11) cannot hold. Thus, $\theta \geq \overline{\theta}(\gamma)$ implies $\gamma > 1/\theta$ as well as $\gamma < \theta\sigma^2$. ■

Properties of the functions $\underline{\theta}(\gamma)$ and $\overline{\theta}(\gamma)$. $\underline{\theta}(\gamma)$ is the unique solution to $L^A(\theta,\gamma) = \theta\sigma^2$. Because $L^A(\theta,0) = 0$, $\underline{\theta}(0) = 0$. Moreover, for any $\gamma > 0$, there exists θ sufficiently small such that (γ,θ) is in I_4. Hence, the curve $\underline{\theta}(\gamma)$ is tangent to the curve $\gamma = \theta\sigma^2$ at $\gamma = \theta = 0$. Indeed,

$$\underline{\theta}'(\gamma) = \frac{L^A_\gamma(\gamma,\theta)}{\sigma^2 - 2\int_\mu^{\mu+\gamma}(x-\mu)^2 f(x)\, dx}$$

where

$$L^A_\gamma(\gamma,\theta) = 2\,[1-F(\mu+\gamma)] - 2\gamma(1-\theta\gamma) f(\mu+\gamma).$$

Hence, $\underline{\theta}'(0) = 1/\sigma^2$. It is easily verified that $\underline{\theta}''(0) < 0$. Similarly, for γ large, the curve cannot intersect the vertical axis. Hence, it must extend upwards as shown.

$\overline{\theta}(\gamma)$ is the unique solution to $L^A(\gamma,\theta) = \gamma$. For any $\gamma > 0$, there exists θ sufficiently large such that (γ,θ) is in I_2. Therefore, the curve must extend indefinitely downwards bounded only by $\gamma = 1/\theta$, and indefinitely upwards bounded only by $\gamma = \theta\sigma^2$. Indeed

$$\overline{\theta}'(\gamma) = \frac{1 - L^A_\gamma(\gamma,\theta)}{2\int_\mu^{\mu+\gamma}(x-\mu)^2 f(x)\, dx},$$

When γ and θ tend to infinity, $\overline{\theta}'(\gamma)$ tends to $1/\sigma^2$.

Proof that pooling with no testimony is no equilibrium under the Grossman-Perry refinement.

Take some $x_P > \mu + \gamma$. Suppose P expects a judgement $\hat{x} \geq \gamma$. Then P makes no loss when the true x satisfies $\hat{x} - \gamma - \kappa(x_P - x)^2 \geq \mu$ and benefits if the inequality is strict. Let $\underline{x}$ and $\overline{x}$ be the solutions for the equality in this condition and write $\underline{x} = x_P - h(\hat{x})$, $\overline{x} = x_P + h(\hat{x})$, where $h(\hat{x})$ is strictly decreasing with $h(\mu + \gamma) = 0$. Now define $\phi(\hat{x}) = E(x | x_P - h(\hat{x}) \leq x \leq x_P + h(\hat{x}))$. Note that $\phi(\mu + \gamma) = x_P > \mu + \gamma$. For x_P sufficiently large, $x_P - h(\hat{x}) > \mu$ so that $f(x)$ is decreasing over the interval. Because x_P is the midpoint, it then follows that $\phi(\hat{x}) < x_P$. In particular $\phi(x_P) < x_P$. By continuity there exist $\hat{x}^* \in (\mu + \gamma, x_P)$ such that $\phi(\hat{x}^*) = \hat{x}^*$. Thus, when $x \in (x_P - h(\hat{x}^*), x_P + h(\hat{x}^*))$, P would do strictly better by reporting x_P expecting $\hat{x}^*$ and the judge would be right to infer the posterior mean $\hat{x}^*$. ■

Proof of Lemma 1.

(i) Suppose first that both parties always testify. Let (x'_P, x'_D) be the equilibrium pair at x'. Because x' is revealed, it follows from monotonicity that either $x_P(x)$ or $x_D(x)$ is strictly increasing at x'. Let this be true of $x_P(x)$. If D deviates to $\emptyset_D$, by minimality the arbiter then infers x' from the observation of $(x'_P, \emptyset_D)$. Therefore, D is better off because he saves on the cost of testimony without affecting the arbiter's decision.

Next, suppose P always testifies but D does so only at $x \leq x^0_D$. If $x_D(x)$ is strictly increasing at some $x' < x^0_D$, the preceding argument shows that P would be better off deviating to $\emptyset_P$. So it must be that $x_D(x)$ is constant for all $x \leq x^0_D$, the true state being revealed only through $x_P(x)$. If P deviates to $\emptyset_P$ at some $x \leq x^0_D$, by minimality the arbiter will adjudicate some $\nu \leq x^0_D$. But then P saves on the cost of testimony and is better off deviating whenever $x < \nu$. Finally, suppose P always testifies but D never does. The argument is then similar: if P deviates to $\emptyset_P$ at some x, the arbiter will adjudicate some ν and P will be better off if x is sufficiently small.

We conclude that D testifies for $x \leq x^0_D$ and is otherwise silent, P testifies for $x \geq x^0_P$ and is otherwise silent, where x^0_D and x^0_P are finite.

(ii) It remains to show that $x^0_D < x^0_P$. Suppose on the contrary that there is some range $[x^0_P, x^0_D]$ where both parties testify. We show that this yields a contradiction.

a) We first show that $x_D(x)$ is strictly increasing in a left-neighborhood of x^0_D and that $x_D(x^0_D) = x^0_D$; hence $x_D(x) = x_D(x^0_D)$ has the unique solution $x = x^0_D$. Suppose that $x_D(x^0_D) < x^0_D$. Then, the action x^0_D is cheaper for D than the equilibrium play of $x_D(x^0_D)$. By monotonicity, x^0_D is then never played at equilibrium. By minimality, when he observes the out-of-equilibrium pair $(x_P(x^0_D), x^0_D)$, the arbiter therefore discards x^0_D and infers x from the observation of $x_P(x^0_D)$ alone. Because strategies are monotone, he must infer some $\hat{x} \leq x^0_D$. Consequently, D

would deviate from the postulated strategy. Therefore, $x_D(x_D^0) = x_D^0$. Suppose next that $x_D(x)$ is not strictly increasing in the left-neighborhood. By monotonicity, it must then be constant, i.e., $x_D(x) = x_D^0$. But for $x < x_D^0$ this yields $x_D(x) > x$, contradicting the no-understatement condition.

b) We next show that $c_P(x_P(x_D^0+\varepsilon), x_D^0) \geq c_P(x_P(x_D^0), x_D^0)$ for $\varepsilon > 0$. Because the equilibrium is revealing, and given monotonicity, $x_P(x_D^0+\varepsilon) \geq x_P(x_D^0)$ and $x_P(x_D^0+\varepsilon)$ is strictly increasing in ε. By minimality, when he observes the out-of-equilibrium pair $(x_P(x_D^0+\varepsilon), x_D(x_D^0))$, the arbiter infers some $\hat{x} \in [x_D^0, x_D^0+\varepsilon]$. That is, if he thinks that P has deviated, he must conclude that $x = x_D^0$; if he thinks that D has deviated, he must conclude that $x = x_D^0+\varepsilon$; and he may assign positive probabilities to both possibilities. When he observes the equilibrium pair $(x_P(x_D^0), x_D(x_D^0))$, the arbiter correctly infers $x = x_D^0 \leq \hat{x}$. For P not to deviate from $x_P(x_D^0)$ to $x_P(x_D^0+\varepsilon)$, it must therefore be that

$$c_P(x_P(x_D^0+\varepsilon), x_D^0) - c_P(x_P(x_D^0), x_D^0) \geq 0, \text{ all } \varepsilon > 0. \tag{14}$$

c) Finally, we show that (14) cannot in fact hold for ε sufficiently small. Denote by $\widetilde{x}$ the arbiter's belief upon observing the out-of-equilibrium pair $(x_P(x_D^0), \emptyset_D)$. At $x = x_D^0$, and recalling that $x_D(x_D^0) = x_D^0$, D's equilibrium payoff is $-x_D^0-\gamma$. For D not to deviate to $\emptyset_D$, it must be that

$$-x_D^0-\gamma \geq -\widetilde{x}. \tag{15}$$

At $x_D^0+\varepsilon$, the equilibrium pair is $(x_P(x_D^0+\varepsilon), \emptyset_D)$. For P not to deviate to $x_P(x_D^0)$, it must be that

$$x_D^0+\varepsilon - c_P(x_P(x_D^0+\varepsilon), x_D^0+\varepsilon) \geq \widetilde{x} - c_P(x_P(x_D^0), x_D^0+\varepsilon). \tag{16}$$

Combining (15) and (16) yields

$$c_P(x_P(x_D^0+\varepsilon), x_D^0+\varepsilon) - c_P(x_P(x_D^0), x_D^0+\varepsilon) \leq \varepsilon - \gamma. \tag{17}$$

For ε positive but arbitrarily small, (14) and (17) cannot simultaneously hold given $\gamma > 0$ and the continuity of the cost functions. ■

Proof of Proposition 3. We briefly complete the argument in the text. Rather than attempting to solve (4) and (5) directly, it is easier to work with the equations expressed in terms of the inverse of $x_P(x)$ and $x_D(x)$, which we write $x(x_P)$ and $x(x_D)$ respectively. The differential equations then become:

$$2\kappa(x_P - x(x_P)) = x'(x_P), \tag{18}$$

$$2\kappa(x(x_D) - x_D) = x'(x_D). \tag{19}$$

The general solution to (18), given the condition $x(x_P) \leq x_P$, is

$$x = x_P - \frac{1 - Ke^{-2\kappa x_P}}{2\kappa}$$

for some constant K. Choosing the constant to satisfy the initial condition $x(x_P^0) = x_P^0$ yields

$$x = x_P - \frac{1 - Ke^{-2\kappa(x_P - x_P^0)}}{2\kappa}.$$

Similarly,

$$x = x_D + \frac{1 - Ke^{-2\kappa(x_D^0 - x_P)}}{2\kappa}.$$

It remains to determine the constants x_P^0 and x_D^0. The argument is the same as in Proposition 1. When neither party submits, the arbiter's posterior mean is

$$\varphi_\emptyset := E\left(x \mid \emptyset_P, \emptyset_D\right) = E\left(x \mid x_D^0 < x < x_P^0\right). \tag{20}$$

At $x = x_P^0$ party P is just indifferent between submitting and not submitting. If the party submits, the true state is revealed at the cost of γ. If the party does not submit, the arbiter adjudicates $\varphi_\emptyset$. Thus party P is indifferent if $x_P^0 - \gamma = \varphi_\emptyset$. Using the same argument, when $x = x_D^0$, party D is indifferent between submitting and not if $-x_D^0 - \gamma = -\varphi_\emptyset$. Combining with (20) yields

$$\varphi_\emptyset = E\left(x \mid \varphi_\emptyset - \gamma < x < \varphi_\emptyset + \gamma\right).$$

Thus, the updated expectation given that x is in the interval $[\varphi_\emptyset - \gamma, \varphi_\emptyset + \gamma]$ must be the mid point $\varphi_\emptyset$. Given the normal density, this is possible only if $\varphi_\emptyset$ equals the prior mean μ. Consequently, $x_P^0 = \mu + \gamma$, $x_D^0 = \mu - \gamma$.

Finally, we specify the arbiter's beliefs for out-of-equilibrium moves. We discuss only the beliefs that prevent a unilateral deviation by P; deviations by D would be dealt with in the same way. There are two relevant situations:

(i) The true state is $x > \mu - \gamma$; the equilibrium pair is $(\emptyset_P, \emptyset_D)$ if $x < \mu + \gamma$, it is $(x_P(x), \emptyset_D)$ with $x_P(x) \geq \mu + \gamma$ if $x \geq \mu + \gamma$. Suppose the arbiter observes $(x_P, \emptyset_D)$ with $x_P < \mu + \gamma$. By minimality, he believes that P has deviated while D played his equilibrium strategies. Hence he puts unit probability on the event $x > \mu - \gamma$. One possibility is that he infers $x = \mu$, in which case the deviation by P is clearly unprofitable.

(ii) The true state is $x \leq \mu - \gamma$ so that the equilibrium pair is $(\emptyset_P, x_D(x))$, where $x_D(x) \leq \mu - \gamma$. Suppose the arbiter observes $(x_P, x_D(x))$ with $x_P < \mu + \gamma$. By minimality, he believes that P has deviated while D played at equilibrium. Hence he disregards x_P and infers the state from the play of x_D alone, which clearly makes the deviation unprofitable. Next, suppose the arbiter observes $(x_P, x_D(x))$ with $x_P \geq \mu + \gamma$. Then the arbiter does not know who has deviated. By minimality, he believes that at most one did. One possibility is that he assigns an equal chance to either possibility and therefore adjudicates $x_D^{-1}(x_D)/2 + x_P^{-1}(x_P)/2$. P then earns the payoff

$$\pi := \tfrac{1}{2}x + \tfrac{1}{2}x_P^{-1}(x_P) - \gamma - \kappa(x_P - x)^2,$$

which must be compared with his equilibrium payoff of x. Because $x_P(\cdot)$ satisfies (4),

$$\frac{\partial \pi}{\partial x_P} = \frac{1}{2}\frac{\partial x_P^{-1}(x_P)}{\partial x_P} - 2\kappa(x_P - x) = -\kappa(x_P - x) < 0.$$

Hence, we need only check whether a deviation to $x_P = \mu + \gamma$ might be profitable. The payoff is then

$$\pi = \tfrac{1}{2}x + \tfrac{1}{2}(\mu + \gamma) - \gamma - \kappa(\mu + \gamma - x)^2.$$

This is increasing in x. At $x = \mu - \gamma$, $\mu - \gamma - \kappa(2\gamma)^2 < \mu - \gamma = x$. ■

Lemma 2: $v_x^P(x,\gamma) \in (0,1]$ with $v_x^P(\mu+\gamma,\gamma) = 1$; $v_x^D(x,\gamma) \in [-1,0)$ with $v_x^D(\mu-\gamma,\gamma) = -1$; $v_{xx}^i(x,\gamma) < 0$, $v_\gamma^i \in [-1,0)$, $i = D, P$, with $v_\gamma^D(\mu-\gamma,\gamma) = v_\gamma^P(\mu+\gamma,\gamma) = -1$.

Proof. Define

$$\varphi(x - \mu - \gamma) := x_P(x,\gamma) - x, \quad x \geq \mu + \gamma,\ \gamma \geq 0.$$

where $\varphi(\cdot)$ is the "claim inflation function" defined over R_+ and which is invariant across equilibria. From (4) and given the initial condition, $\varphi(0) = 0$ and

$$\varphi(y)\left(1 + \varphi'(y)\right) = \frac{1}{2\kappa},\ y \geq 0. \tag{21}$$

Also $\varphi(y) < 1/(2\kappa)$ and $\varphi''(y) < 0$ with $\varphi'(0) = \infty$ and $\varphi'(\infty) = 0$. The plaintiff's falsification expenditure is

$$v^P(x,\gamma) \equiv \kappa\varphi(x - \mu - \gamma)^2, \quad x \geq \mu + \gamma.$$

For $x > \mu + \gamma$ and using (21),

$$v_x^P(x,\gamma) = 2\kappa\varphi(x-\mu-\gamma)\varphi'(x-\mu-\gamma) = 1 - 2\kappa\varphi(x-\mu-\gamma)$$

$$v_\gamma^P(x,\gamma) = -\left(1 - 2\kappa\varphi(x-\mu-\gamma)\right).$$

Therefore $v_{xx}^P < 0$, $v_x^P(x,\gamma) \in (0,1]$ and $v_\gamma^P(x,\gamma) \in (-1,0)$. At $x = \mu + \gamma$, the preceding partial derivative is not defined. We use

$$v_\gamma^P(\mu+\gamma,\gamma) := \lim_{x \downarrow \mu+\gamma} v_\gamma^P(x,\gamma) = -1.$$

For D, $x - x_D(x,\gamma) = \varphi(\mu - \gamma - x)$, $x \leq \mu - \gamma$, and the argument is the same. ■

Proof of Lemma 3.

(i) In Region 1, $\theta\gamma \geq 1$ and $L^I(\theta,\gamma) = \theta\sigma^2$. Using symmetry,

$$\Delta(\theta,\gamma) := L^I(\theta,\gamma) - L^A(\theta,\gamma) = 2\int_{\mu+\gamma}^{\infty} \psi(x) f(x)\, dx,$$

where

$$\psi(x) := \theta\,(x-\mu)^2 - \left(\gamma + v^P(x,\gamma)\right),\ x \geq \mu+\gamma.$$

It suffices to show that $\psi(x)$ is always positive. Applying Corollary 2, $\psi''(x) > 0$. Moreover, $v^P(\mu+\gamma,\gamma) = 0$ and $v_x^P(\mu+\gamma,\gamma) = 1$. Hence

$$\psi(\mu+\gamma) = \gamma(\theta\gamma - 1) \geq 0,$$

$$\psi'(\mu+\gamma) = 2\theta\gamma - v_x^P(\mu+\gamma,\gamma) \geq 1.$$

Therefore $\psi(x) > 0$ for all $x > \mu+\gamma$.

(ii) In Region 3a, if it exists, $\theta\gamma \leq 1$ and $L^I(\theta,\gamma) = \gamma + k$. Then

$$\begin{aligned}\frac{\Delta(\theta,\gamma)}{2} &= \int_{\mu}^{\mu+\gamma} \left[\gamma + k - \theta\,(x-\mu)^2\right] f(x)\, dx + \int_{\mu+\gamma}^{\infty} \left(k - v^P(x,\gamma)\right) f(x)\, dx \\ &> \int_{\mu}^{\mu+\gamma} \left[k + \gamma(1-\theta\gamma)\right] f(x)\, dx + \int_{\mu+\gamma}^{\infty} \left(k - v^P(x,\gamma)\right) f(x)\, dx > 0.\end{aligned}$$

(iii) Consider now the area defined by $\gamma \leq k/2$ and $\gamma \leq \min\left[\overline{\gamma}(\theta), 1/\theta\right]$. When $\overline{\gamma}(\theta)$ and $\gamma = 1/\theta$ intersect at some $\gamma \leq k/2$, this area is the whole of region 3; when the intersection occurs at some $\gamma > k/2$, the area is region 3b. In either case, $L^I(\theta,\gamma) = 2\gamma + k/2$. Hence

$$\frac{\Delta(\theta,\gamma)}{2} = \gamma + \frac{k}{4} - \int_{\mu}^{\mu+\gamma} \theta\,(x-\mu)^2 f(x)\, dx - \int_{\mu+\gamma}^{\infty} \left(\gamma + v^P(x,\gamma)\right) f(x)\, dx.$$

When $\overline{v}(0) \leq k/2$,

$$\Delta(\theta,0) = \frac{k}{2} - \overline{v}(0) \geq 0.$$

Differentiating with respect to γ,

$$\frac{\Delta_\gamma(\theta,\gamma)}{2} = 1 + \gamma(1-\theta\gamma) f(\mu+\gamma) + \int_{\mu+\gamma}^{\infty} \left(1 + v_\gamma^P(x,\gamma)\right) f(x)\, dx > 0.$$

By Corollary 2, the integrand of the second term is positive. Therefore $\Delta(\theta,\gamma) > 0$ in this area as well. ■

Proof of Proposition 4. Let $\Delta(\theta,\gamma) = L^I(\theta,\gamma) - L^A(\theta,\gamma)$. From the argument in the text, we know that: for any $\gamma > 0$, if $\Delta(\theta,\gamma) > 0$ for some θ', there exists $\overline{\theta}(\gamma) > \underline{\theta}(\gamma) > 0$ such that $\Delta(\theta,\gamma) > 0$ if $\theta \in (\underline{\theta}(\gamma), \overline{\theta}(\gamma))$ and $L^I \leq L^A$ otherwise.

i) By Lemma 2, when $\overline{v}(0) \leq k/2$, $\Delta(\theta, \gamma) > 0$ in the regions 1 and 3. For any γ, one can choose θ so that (θ, γ) is in region 1 or in region 3. Hence the condition $\Delta > 0$ can always be satisfied for some θ, which proves the first part of the Proposition.

ii) Suppose now $\overline{v}(0) > k/2$. For any $\gamma \geq k/2$, one can always choose θ such that (θ, γ) is in region 1 (as in Figure 4) or is in either region 1 or region 3a (as in Figure 5). The same argument as in i) can then be applied. We therefore restrict attention to $\gamma < k/2$. We then have $L^I(\theta, \gamma) = \min(\theta\sigma^2, 2\gamma + k/2)$. Moreover

$$\Delta(\theta, 0) = \min(\theta\sigma^2, k/2) - \overline{v}(0) < 0.$$

By continuity, therefore, $\Delta(\theta, \gamma) \leq 0$ for all θ if γ is not too large. We now characterize the area where the preceding inequality holds.

When $2\gamma + k/2 \leq \theta\sigma^2$ (i.e., when joint testimony is preferred to no testimony), $\Delta(\theta, \gamma) \leq 0$ implies $\Delta(\theta', \gamma) < 0$ for all $\theta' > \theta$. When $2\gamma + k/2 \geq \theta\sigma^2$ (i.e., when no testimony is preferred), $\Delta(\theta, \gamma) \leq 0$ implies $\Delta(\theta', \gamma) < 0$ for all $\theta' < \theta$. In particular, for points on the boundary between joint testimony and no testimony, $\Delta(\theta, \gamma) \leq 0$ implies $\Delta(\theta', \gamma) < 0$ for all $\theta' \neq \theta$.

On the boundary, the difference between the social losses is $\Delta(\theta, \overline{\gamma}(\theta))$, where $\overline{\gamma}(\theta) = \theta\sigma^2/2 - k/4$. At the horizontal intercept, $\Delta = k/2 - \overline{v}(0) < 0$. Let θ_0 be the smallest value at which $\Delta(\theta, \overline{\gamma}(\theta))$ changes sign, from negative to positive. Such a θ_0 necessarily exists because $\Delta > 0$ when $\overline{\gamma}(\theta)$ crosse the $\gamma = 1/\theta$ curve as in Figure 4 or the $\gamma = k/2$ line as in Figure 5. The critical $\hat{\gamma}$ referred to in the Proposition equals $\overline{\gamma}(\theta_0)$, i.e., $\Delta(\theta, \gamma) \leq 0$ for all θ can be true only for $\gamma \leq \overline{\gamma}(\theta_0)$. ■

References

Bagwell, Kyle and Garey Ramey. 1991. “Oligopoly Limit Pricing,” 22 *Rand Journal of Economics* 155-72.

Cooter, Robert and Daniel L. Rubinfeld. 1989. “Economic Analysis of Legal Disputes and their Resolution,” 27 *Journal of Economic Literature* 1067-1097.

Daughety, Andrew F. and Jennifer F. Reinganum. 2000. “On the Economics of Trials: Adversarial Process, Evidence, and Equilibrium Bias,” 16 *Journal of Law, Economics, and Organization* 365-94.

Demougin, Dominique and Claude Fluet. 2008. “Rules of Proof, Courts, and Incentives,” 39 *Rand Journal of Economics* 20-40.

Dewatripont, Mathias and Jean Tirole. 1999. “Advocates,” 107 *Journal of Political Economy* 1-39.

Emons, Winand and Claude Fluet. 2009. "Accuracy versus Falsification Costs: The Optimal Amount of Evidence under different Procedures," 25 *Journal of Law, Economics, and Organization* 134-156.

Farmer, Amy and Paul Pecorino. 1999. "Legal Expenditure as a Rent-Seeking Game," 100 *Public Choice* 271-288.

Fluet, Claude and Paolo Garella. 2002. "Advertising and Prices as Signals of Quality in a Regime of Price Rivalry," 20 *International Journal of Industrial Organization* 907-30.

Froeb, Luke and Bruce Kobayashi. 1996. "Naive, Biased, yet Bayesian: Can Juries Interpret Selectively Produced Evidence?" 12 *Journal of Law, Economics, and Organization* 257-76.

Froeb, Luke and Bruce Kobayashi. 2001. "Evidence Production in Adversarial vs. Inquisitorial Regimes," 72 *Economics Letters* 267-272.

Grossman, Sanford and Motto Perry. 1986. "Perfect Sequential Equilibrium," 39 *Journal of Economic Theory* 97-119.

Hetzendorf, Mark N. and Per B. Overgaard. 2001. "Price Competition and Advertising Signals: Signaling by Competing Senders," 10 *Journal of Economics and Management Strategy* 621-62.

Kartik, Navin. 2009. "Strategic Communication with Lying Costs", 76 *Review of Economic Studies* 1359-95.

Katz, Avery. 1988. "Judicial Decisionmaking and Litigation Expenditure," 8 *International Review of Law and Economics* 127-143.

Langbein, John. 1985. "The German Advantage in Civil Procedure," 52 *University of Chicago Law Review* 823-866.

Lewis, Tracy and Michael Poitevin. 1997. "Disclosure of Information in Regulatory Proceedings," 13 *Journal of Law, Economics, and Organization* 50-73.

Maggi, Giovanni and Andrés Rodríguez-Clare. 1995 "Costly Distortion of Information in Agency Problems," 26 *Rand Journal of Economics* 675-689.

Mailath, George. 1987 "Incentive Compatibility in Signaling Games with a Continuum of Types," 55 *Econometrica* 1349-1365.

Milgrom, Paul and John Roberts. 1986. "Relying on the Information of Interested Parties," 17 *Rand Journal of Economics* 18-32.

Palumbo, Giuliana. 2001. "Trial Procedures and Optimal Limits on Proof-Taking," 21 *International Review of Law and Economics* 309-327.

Parisi, Francesco. 2002. "Rent-seeking through Litigation: Adversarial and Inquisitorial Systems Compared," 22 *International Review of Law and Economics* 193-216.

Schrag, Joel L. 1999. "Managerial Judges: An Economic Analysis of Judicial Management of Legal Discovery," 30 *Rand Journal of Economics* 305-323.

Schultz, Christian. 1999. "Limit Pricing when Incumbents have Conflicting Interests," 17 *International Journal of Industrial Organization* 801-825.

Shin, Hyun S. 1998. "Adversarial and Inquisitorial Procedures in Arbitration," 29 *Rand Journal of Economics* 378-405.

Sobel, Joel. 1985. "Disclosure of Evidence and Resolution of Disputes: Who Should Bear the Burden of Proof?," in A. E. Roth, ed., *Game Theoretic Models of Bargaining*, Cambridge: Cambridge University Press, 341-361.

Tullock, Gordon. 1975. "On the Efficient Organization of Trials," 28 *Kyklos* 745-762.

Tullock, Gordon. 1980, *Trials on Trials: The Pure Theory of Legal Procedure.* New York: Columbia University Press.