LIMITATIONS ON LEARNING OF QUANTUM PROCESSES

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Michal Sedlak, Alessandro Bisio, Mario Ziman

OUTLINE

1. **Quantum paradigm in learning**
2. **Learning of quantum processes**
3. **Limitations**

WARNING:
Not exactly about machine learning.
Efficiency/complexity is not an issue.

QUANTISATION

MACHINE LEARNING
is a field of computer science that uses statistical techniques to give computer systems the ability **to "learn"** (e.g., progressively improve performance on a specific task) **with data**, **without being explicitly programmed**.

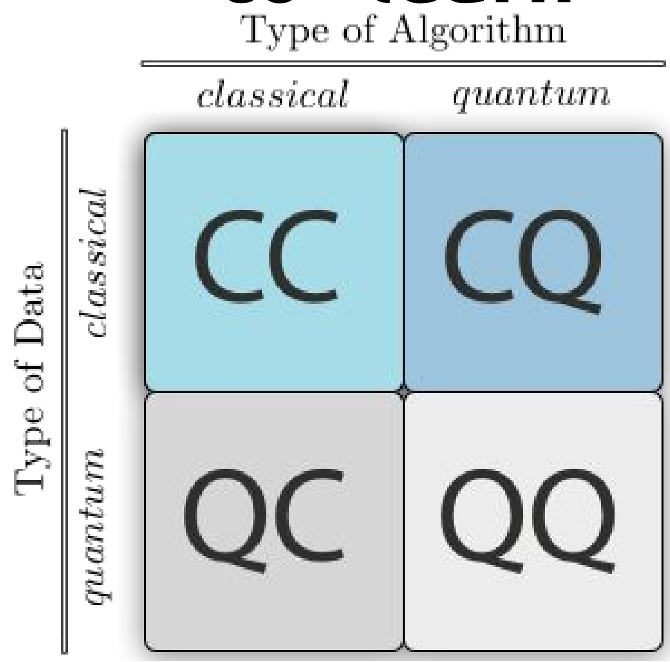
QUANTUM MACHINE LEARNING
(a) an attempt to **quantize** machine learning
(b) new **buzzword** in quantum computation

HOW TO QUANTIZE?

QUANTISATION

to "learn"

with data

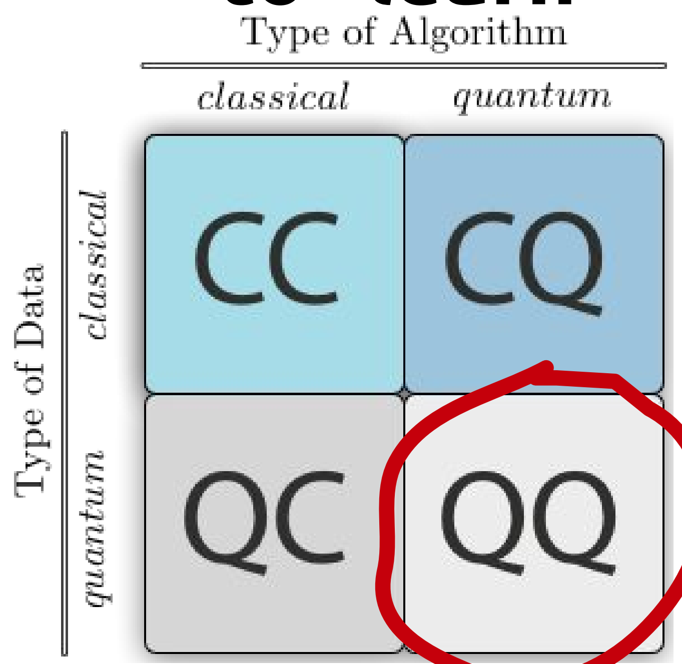


without being explicitly **programmed**

V. Dunjko, J.M. Taylor, H.J. Briegel "Quantum-Enhanced Machine Learning", Phys. Rev. Lett. 117, 130501 (2017)

QUANTISATION

to "learn"



with data

without being explicitly **programmed**

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FRAMEWORK

input/output data → **bits of information**
to learn → **in/out function**

result of learning
→ a program simulating action f
→ stored in bits

QUANTUM LEARNING

input/output data → **quantum states**
to learn → **quantum transformation**

result of learning
→ a program simulating the process
→ stored in bits

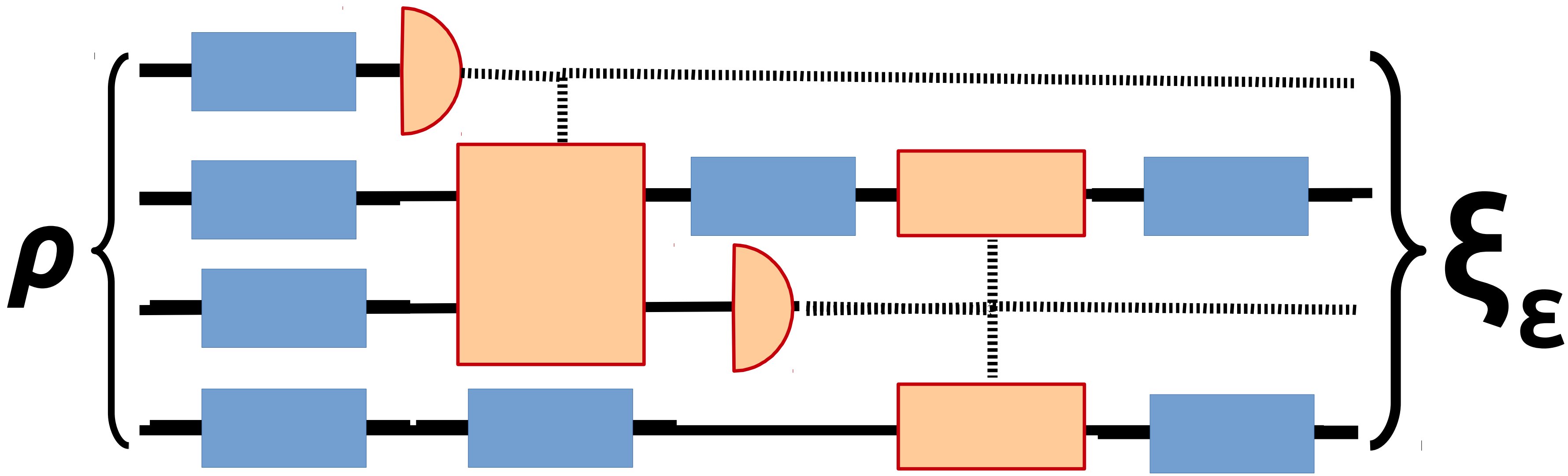
QUANTUM LEARNING

input/output data → **quantum states**
to learn → **quantum transformation**

result of learning
→ a program simulating the process
→ stored in ~~bits~~ qubits

QUANTUM TRAINING

Sampling the transformation performance
→ use of entangled input states
→ known processes and measurements



Storing process in quantum machine (memory)

$$\varepsilon \rightarrow \xi_{\varepsilon}$$

QUANTUM TRAINING

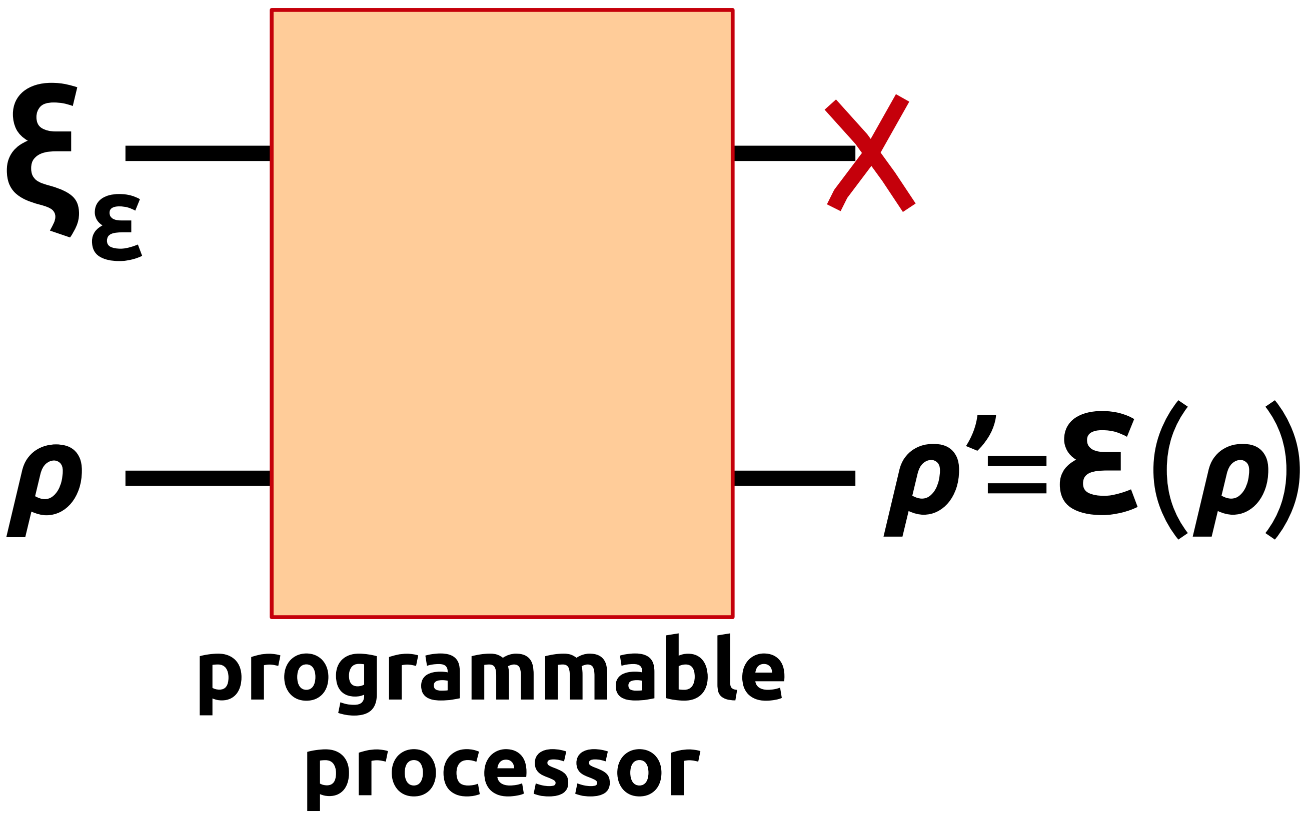
Storing process in quantum machine (memory)

$$\mathcal{E} \rightarrow \xi_{\mathcal{E}}$$

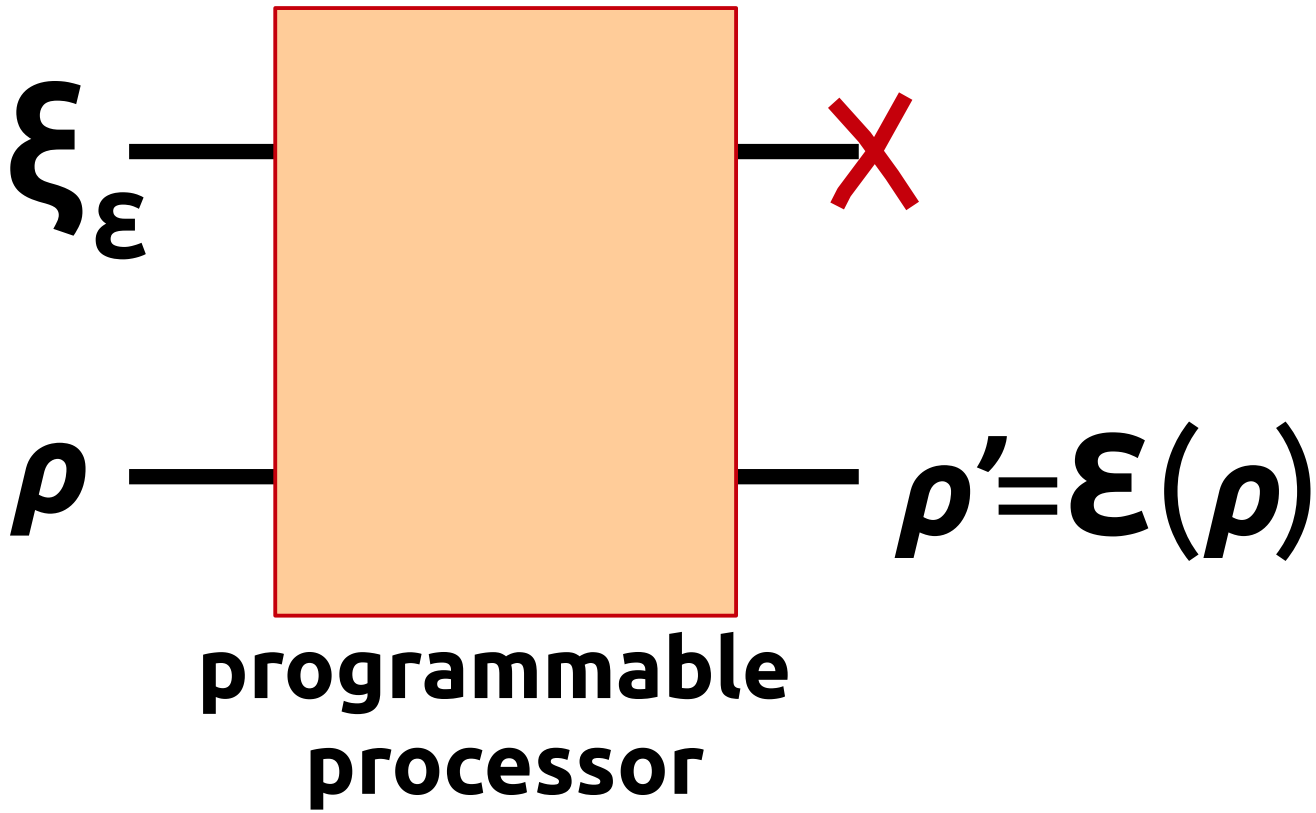
No-cloning theorem
→ impossible to reuse the stored resource
→ number of uses must be considered during the training phase
→ **limitations on training**

PROCESS RETRIEVAL

"Inverting" the training phase.



PROCESS RETRIEVAL



NO-PROGRAMMING THEOREM

There is no perfect universal programming machine → retrieval cannot be universally perfect

PROGRAMMABILITY

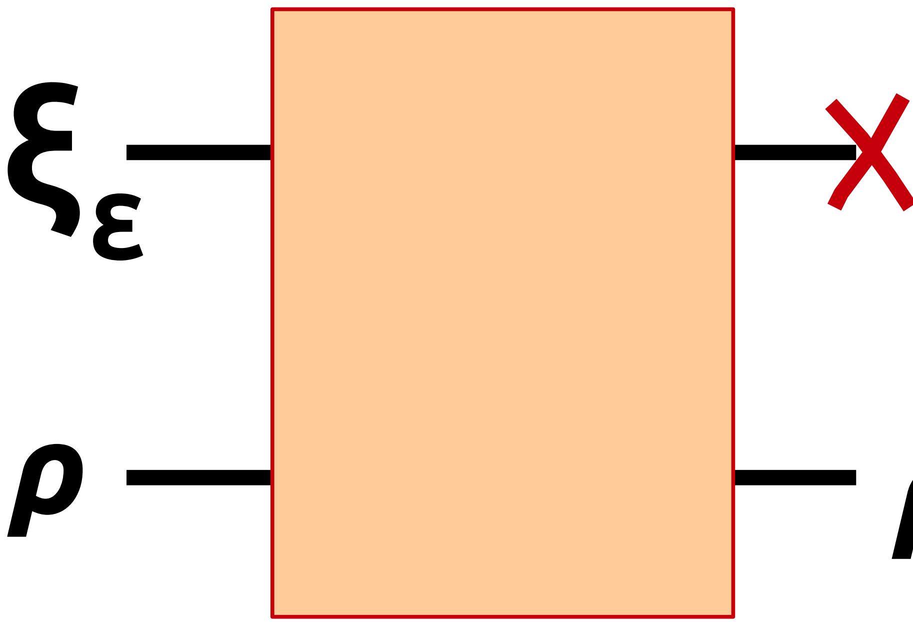
BEYOND NO-GO THEOREM

→ APPROXIMATE PERFORMANCE

→ PROBABILISTIC PERFORMANCE

PROGRAMMABILITY
BEYOND NO-GO THEOREM

→ APPROXIMATE PERFORMANCE



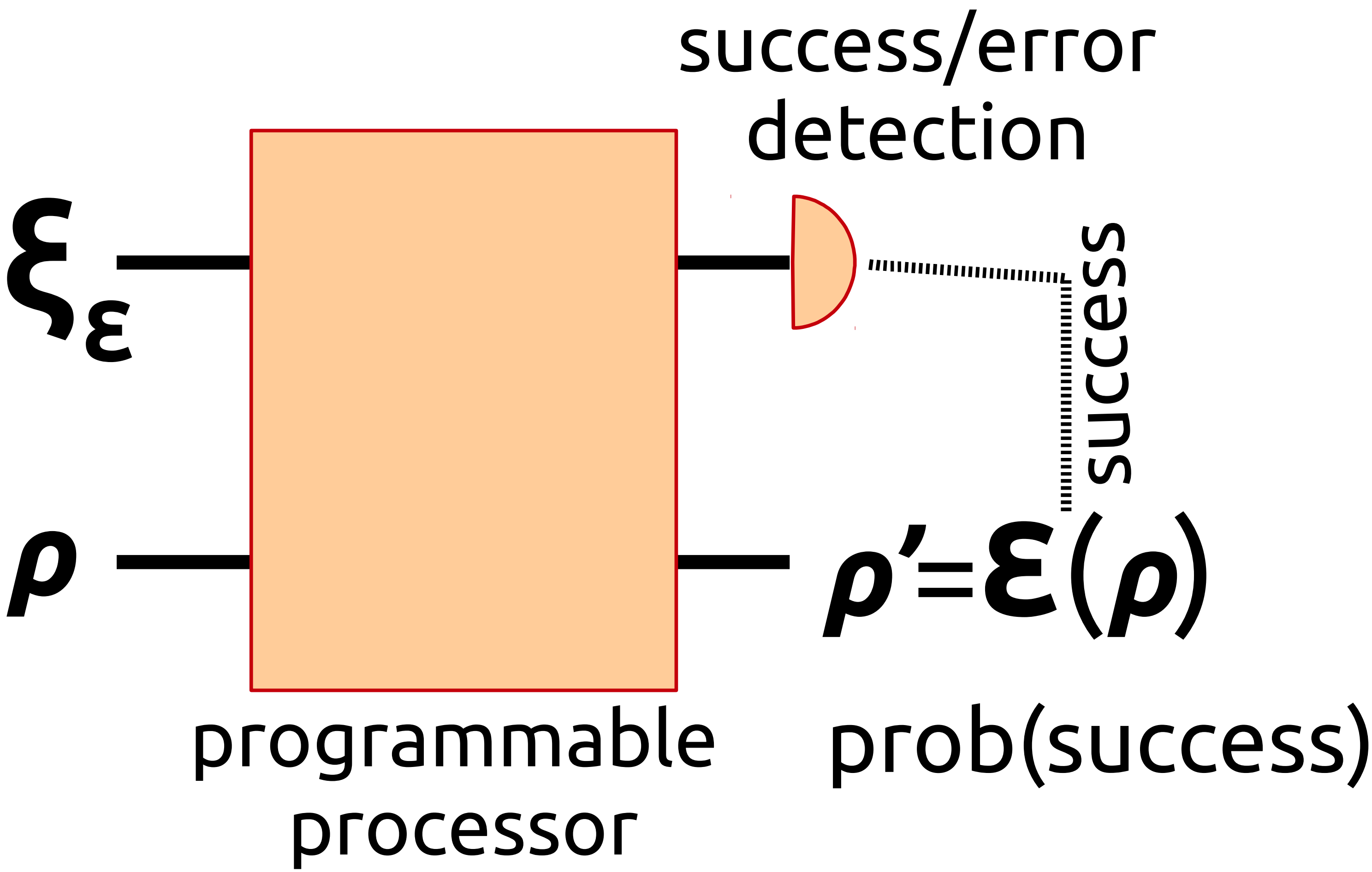
programmable processor

$\rho' = \mathcal{E}'(\rho)$

$\mathrm{dist}(\mathcal{E}', \mathcal{E}) \leq c$

PROGRAMMABILITY
BEYOND NO-GO THEOREM

→ **PROBABILISTIC PERFORMANCE**



APPROXIMATE Q LEARNING

Optimal strategy for unitary channels

MEASURE-AND-ROTATE

A. Bisio, Gi. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

APPROXIMATE Q LEARNING

Optimal strategy for unitary channels

MEASURE-AND-ROTATE

→ optimal learning = optimal estimation
→ storing is classical (but still QQ type!)

A. Bisio, Gi. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

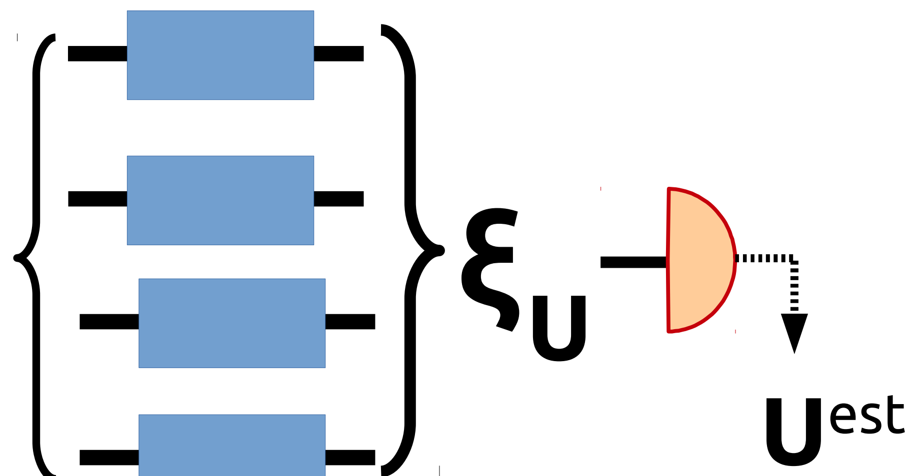
APPROXIMATE Q LEARNING

Optimal strategy for unitary channels

→ optimal state

$$|\Psi\rangle = \bigotimes_{j \in IRR} \sqrt{p_j / d_j}\, |I_j\rangle\rangle$$

$$|I_j\rangle\rangle \in H_j \otimes H_j$$



→ optimal POVM (continuous)

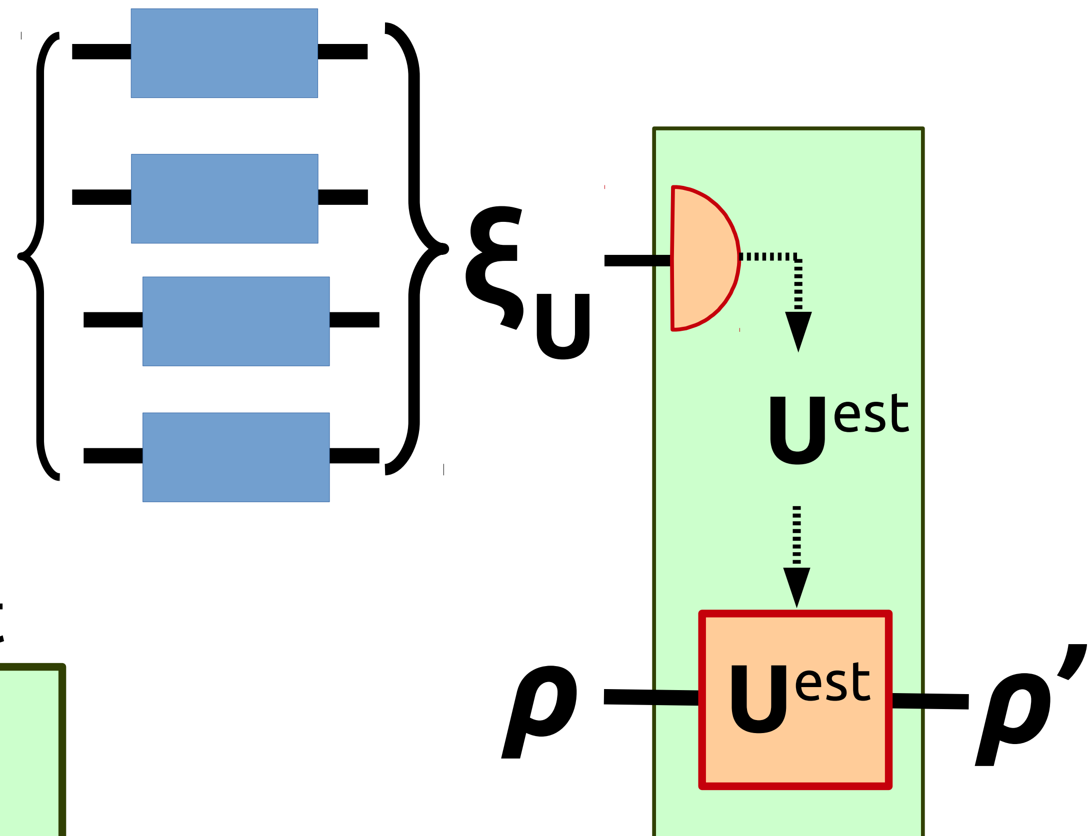
$$E_{U^{est}} = |\eta_{U^{est}}\rangle\langle\eta_{U^{est}}|$$

$$|\eta_{U^{est}}\rangle = \bigotimes_{j \in IRR} \sqrt{d_j}\, |U_j\rangle\rangle$$

A. Bisio, Gi. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

APPROXIMATE Q LEARNING

Optimal strategy for unitary channels



large N→1 for qubit

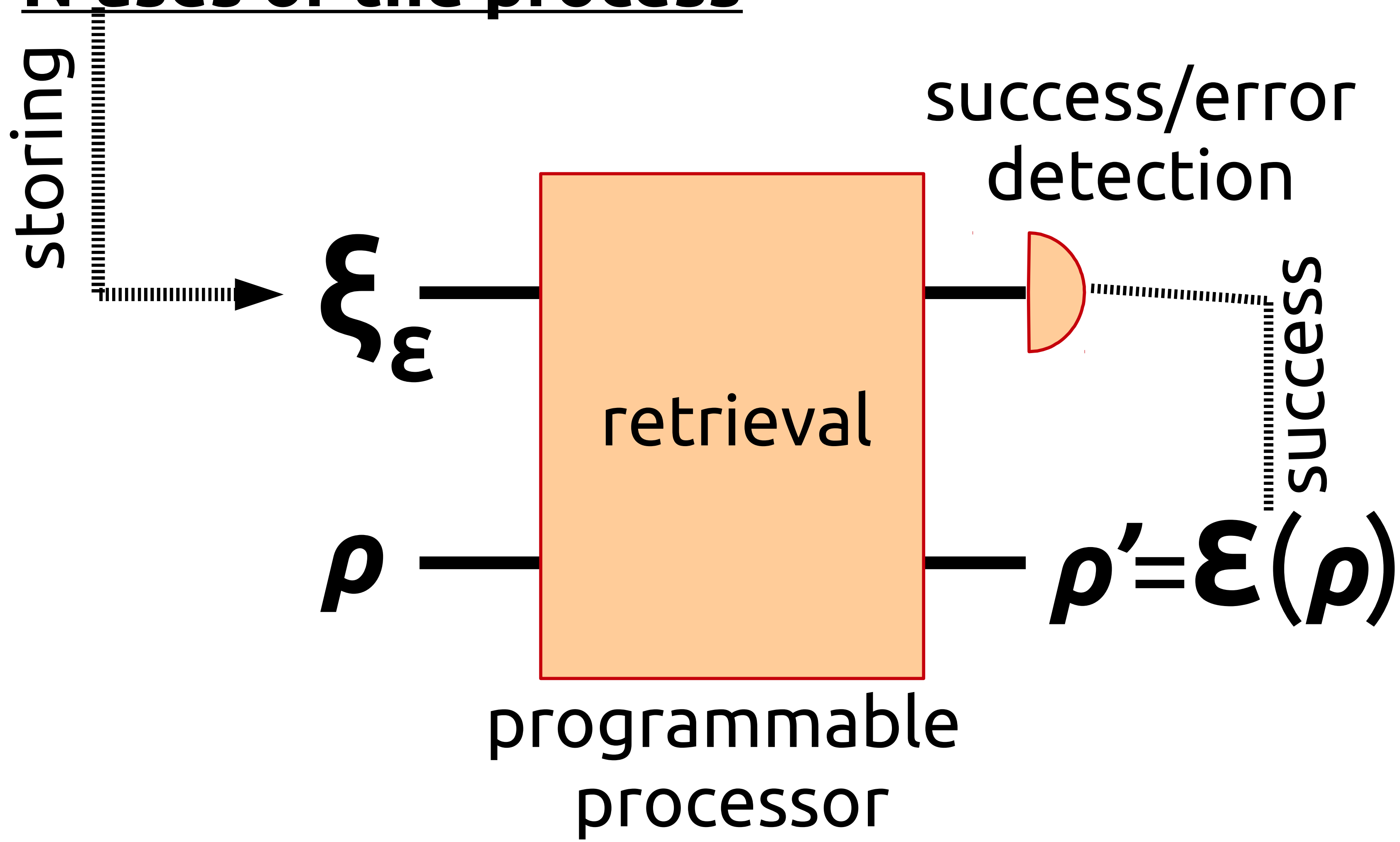
$$F \approx 1-\pi^2/4N^2$$

A. Bisio, Gi. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

PROBABILISTIC Q LEARNING

→ **PROBABILISTIC PERFORMANCE**

→ **N uses of the process**

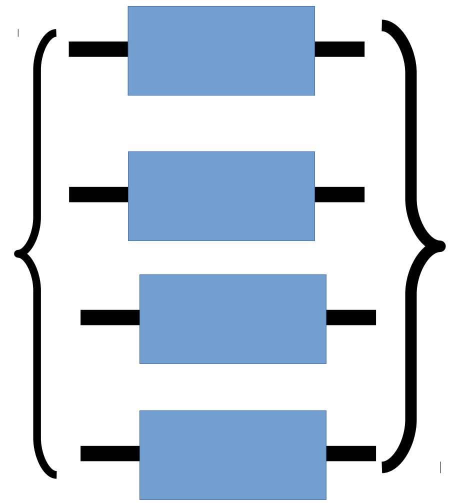


PROBABILISTIC Q LEARNING

Optimal strategy for unitary channels

→ optimal storing

$$|\psi\rangle = \bigotimes_{j\in IRR} \sqrt{p_j/d_j}\,|I_j\rangle\rangle$$



ξ_U

PROBABILISTIC Q LEARNING

Optimal strategy for unitary channels

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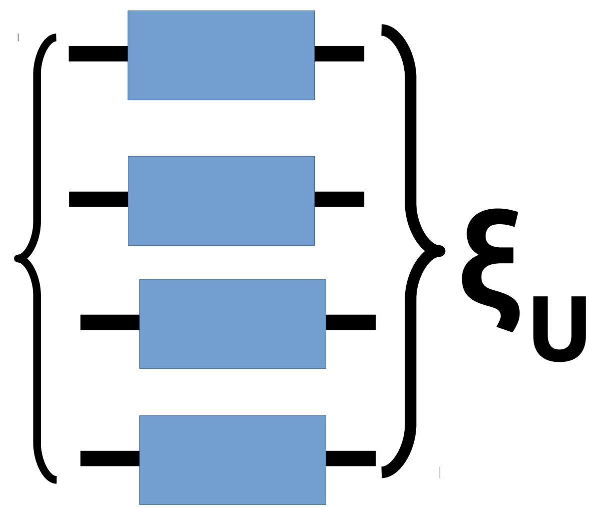
$$|\Psi\rangle = \bigotimes_{j\in IRR} \sqrt{p_j/d_j}\,|I_j\rangle\rangle$$



→ optimal retrieval

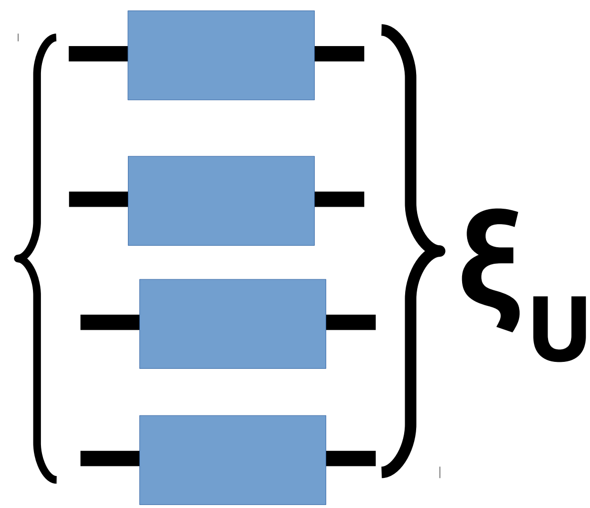
~~MEASURE-AND-ROTATE~~

PROBABILISTIC Q LEARNING

Optimal strategy for unitary channels

→ optimal storing

$$|\boldsymbol{\Psi}\rangle = \bigotimes_{j \in IRR} \sqrt{p_j / d_j}\, |\mathbf{I}_j\rangle\rangle$$

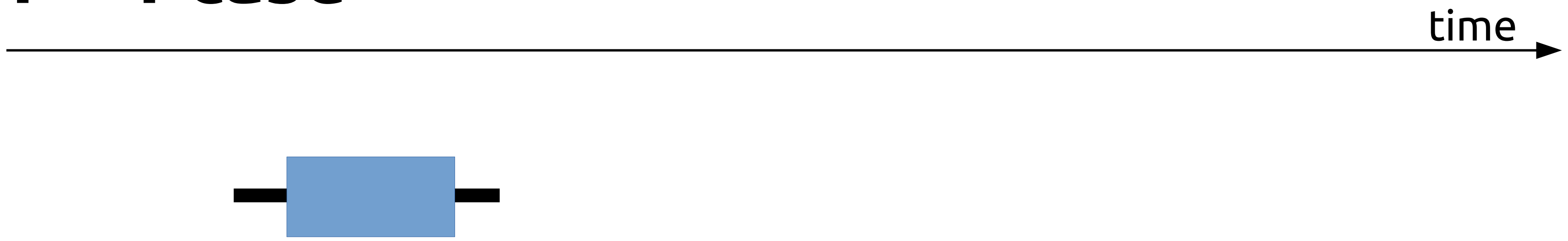


→ optimal retrieval

QUANTUM

PROBABILISTIC Q LEARNING

1→ 1 case

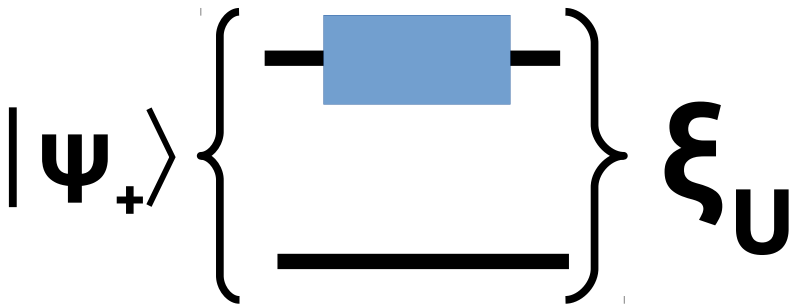


ρ →

PROBABILISTIC Q LEARNING

1→ 1 case

time

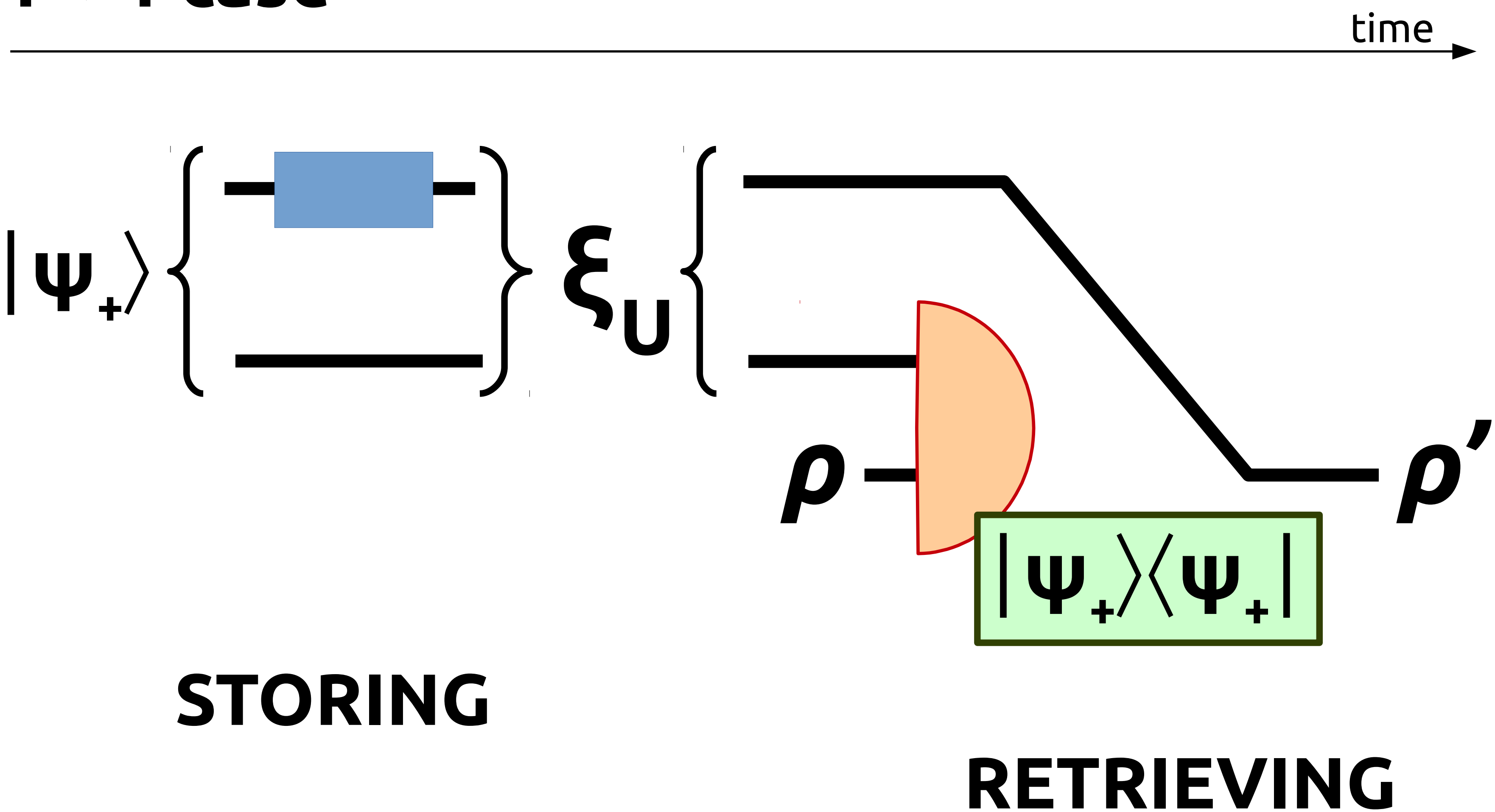


ρ →

STORING

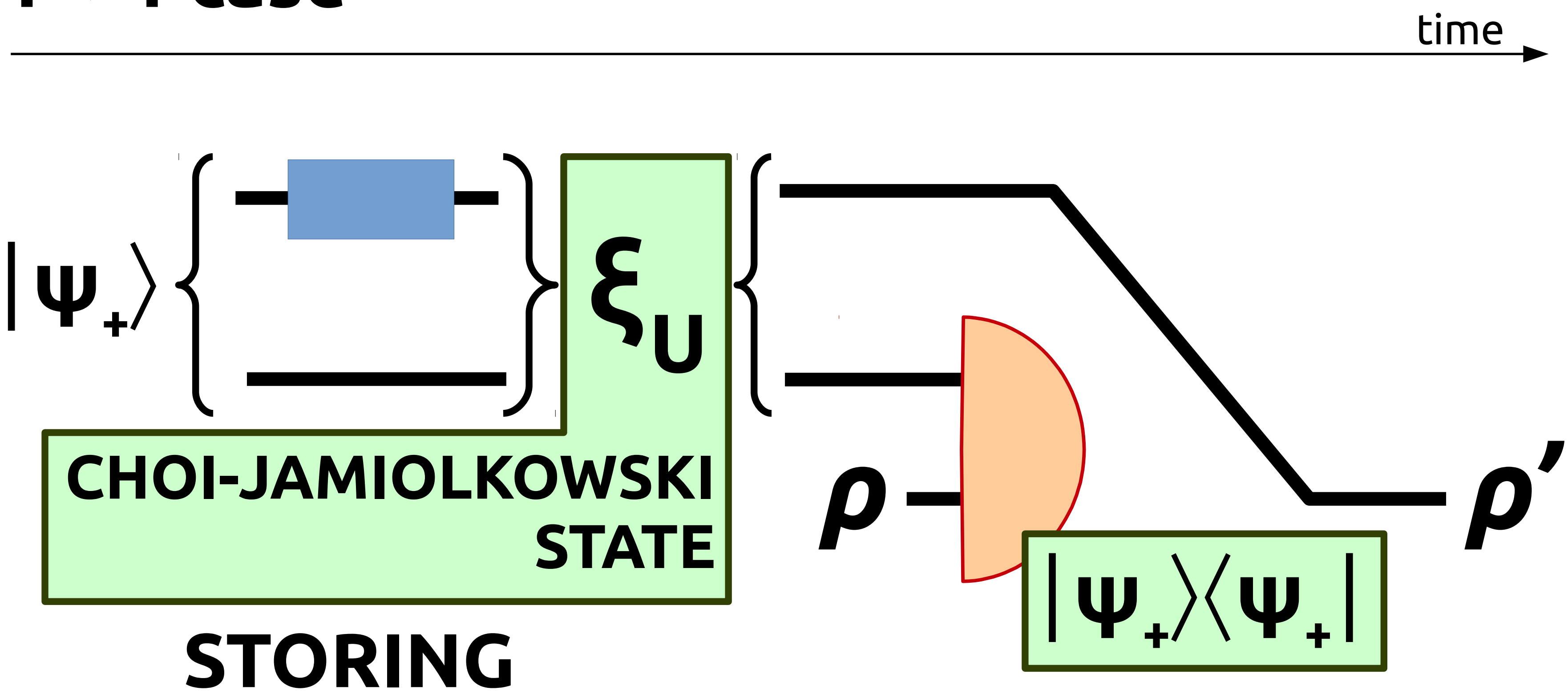
PROBABILISTIC Q LEARNING

1→ 1 case



PROBABILISTIC Q LEARNING

1→ 1 case

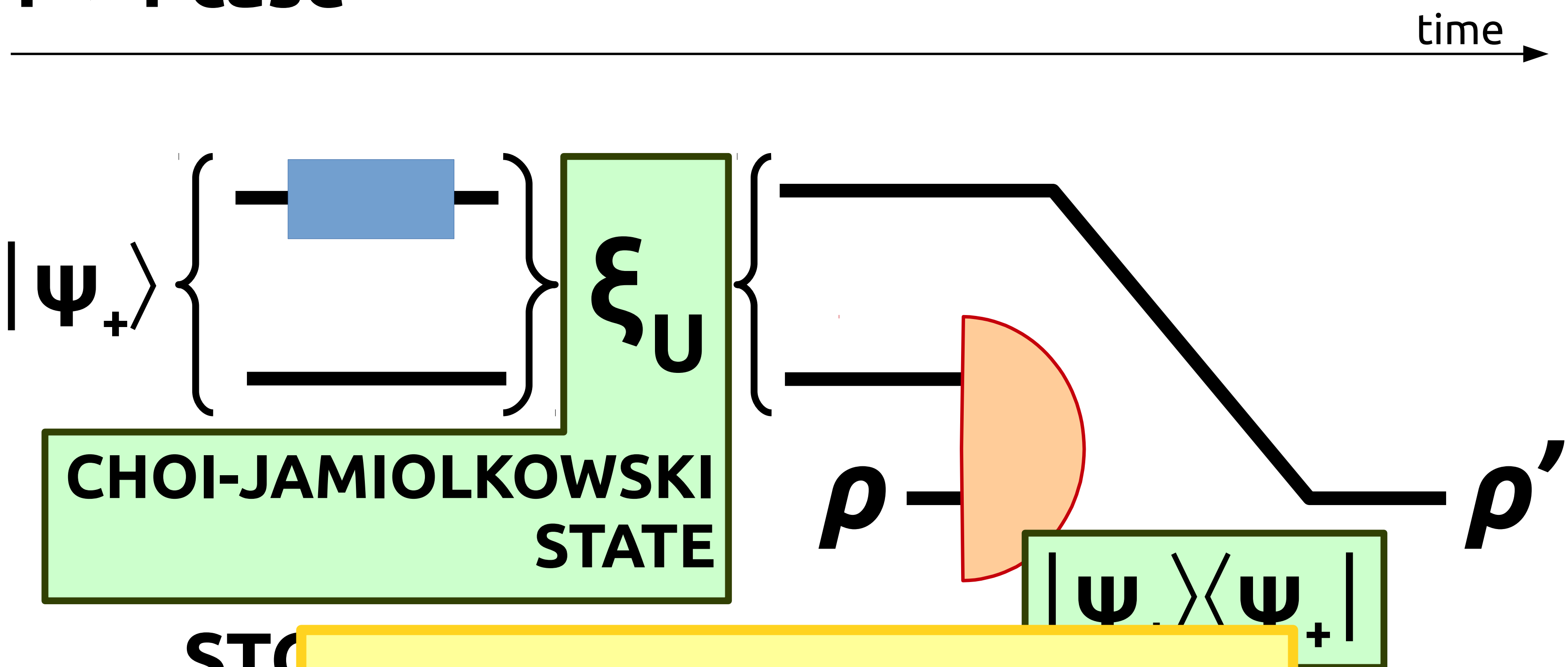


INCOMPLETE

TELEPORTATION

PROBABILISTIC Q LEARNING

1→ 1 case



STO

$$P_{success}=1/d^2$$

INCO

VING

TELEPORTATION

PROBABILISTIC Q LEARNING

Optimal N→ 1 for unitary channels

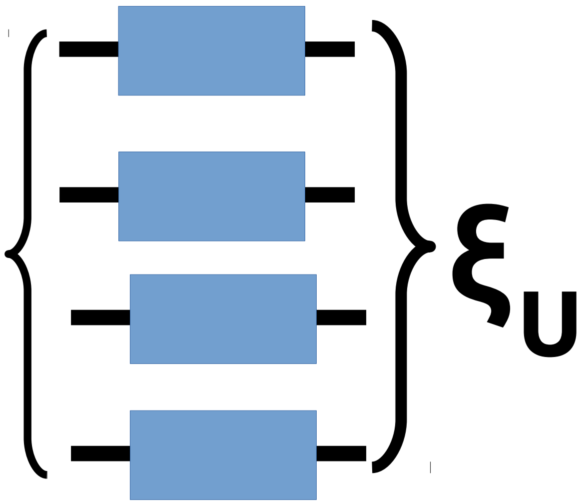
$$P_{success} = \frac{N}{N-1+d^2}$$

PROBABILISTIC Q LEARNING

Optimal N→ 1 for unitary channels

→ <u>optimal storing</u>

$$|\psi\rangle = \bigotimes_{j\in IRR} \sqrt{p_j/d_j}\,|I_j\rangle\rangle$$



PROBABILISTIC Q LEARNING*

Optimal N→ 1 for unitary channels

→ <u>optimal retrieval</u> (quantum comb formalism)

→ reduction to linear programming problem

$$\begin{aligned} \underset{\mu_J, p_j}{\text{maximize}} \quad & \lambda = \sum_J d_J^3 \mu_J, \\ \text{subject to} \quad & 0 \leq d_J \mu_J \leq \frac{p_j}{d_j^2} \quad \forall j \in \mathrm{j}_{JJ} \quad \forall J \\ & p_j \geq 0 \quad \sum_j p_j = 1, \end{aligned}$$

→ combinatorial identity for permutation group

$$\sum_j (c_j - r_j)^2 \frac{h_J}{h_j} = N - 1$$

→ result is analytical

RELATED RESULTS

Retrieval of inverse of U (undo) for qubits

→ the same success probability

→ difference only in retrieval

→ probabilistic alignment of reference frame

RELATED RESULTS

Trade-off for probabilistic processors

→ retrieval part provides best known bounds (exponentially better than before) on the memory size as a function of failure probability f

$$\dim H_{\mathrm{mem}} = \sum_{j \in \boldsymbol{IRR}} d_j^2 = \binom{N-1+d^2}{N}$$

$$\propto (1/f)^{(d^2-1)}$$

SUMMARY

$$P_{success} = \frac{N}{N-1+d^2}$$

→ probabilistic quantum storage and retrieval
→ learning U without knowing

Approximate learning coincide with estimation, hence not really quantum.

Probabilistic learning is teleportation, hence, really quantum.

THANK YOU FOR YOUR ATTENTION

Joint work with
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