Inner-Core Vacillation Cycles
of Hurricane Katrina in a Non-Hydrostatic Model

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Preface

This work rose from a research project as a part of an internship at Monash University in Melbourne. I want to acknowledge here that the basis of this thesis is given by previous work of Dr. Mai Nguyen and Univ. Prof. Dr. Michael Reeder who devotedly accompanied my work at Monash and continued to guide it until the end.
Abstract

Hydrostatic simulations of Hurricane Katrina (2005) conducted in a previous study with the Tropical Cyclone Limited Area Prediction System (TCLAPS) exhibit inner-core vacillations between highly asymmetric and nearly symmetric phases. The mean azimuthal low level potential vorticity (PV) structure shows the shape of approximately circular PV rings in symmetric phases which frequently break down to one or several monopolar PV anomalies that propagate centerwards. This process, named vacillation cycles (VCs), was so far only studied at a horizontal resolution of 5 km which is possibly near the limit of validity of hydrostatic balance, particularly in regions of deep convection. The objective of this study was to investigate how VCs are influenced by more realistically resolved vertical exchanges in a non-hydrostatic model. The PV fields are interpreted in terms of PV thinking, specifically the structure in low level PV and the relevant process that dictate its evolution. For this purpose multiple simulations of Katrina were conducted using the non-hydrostatic Weather and Research Forecast (WRF) model.

It was found that VCs are an important factor for the intensification of Katrina in the WRF model. In asymmetric phases the low level potential vorticity (PV) exhibits an asymmetric shape with one or several vortical hot towers (VHTs) that locally generate high magnitudes of PV and vertical updrafts. These PV anomalies detach from their parent convective VHTs and wrap themselves cyclonically into the center – a mechanism named cyclonic vortex Rossby wave (VRW) breaking. In symmetric phases the vortex appears to have a more symmetric shape with an azimuthally uniform shape in the low level PV and in the vertical velocity in the eyewall. In the transition from symmetric to asymmetric phases VHTs accelerate the mean circulation while the pressure is deepening rapidly. Cyclonic VRW breaking stirs PV from the radius of maximum wind (RMW) towards the center of the vortex. After maximum asymmetry the pressure decreases slower or is constant while the eyewall recovers its circular shape. Ensemble simulations demonstrate the low predictability of VCs: although the patterns of VCs appear in all simulations, the time of occurrence, duration and magnitude of the intensification during the individual phases are different.
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List of Abbreviations

ARW ........... Advanced Research WRF
CAPE .......... Convective Available Potential Energy
CISK .......... Conditional Instability of the Second Kind
CM5 .......... WRF run with GFS input data, initial time 00 UTC 26 AUG 2005 and constant SST
E5C ........... TCLAPS run presented by Nguyen et al. (2011)
ECMWF ........ European Centre for Medium-Range Weather Forecasts
EM5 .......... WRF run with ECMWF input data and initial time 00 UTC 26 AUG 2005
EN3 .......... WRF run with ECMWF input data, initial time 00 UTC 26 AUG 2005 and a finer mesh size of 3.3 km
EN5 .......... WRF run with ECMWF input data and initial time 00 UTC 26 AUG 2005
ERC .......... eyewall replacement cycles
FFT .......... Fast Fourier Transformation
GM5 .......... WRF run with GFS input data and initial time 00 UTC 26 AUG 2005
GN5 .......... WRF run with GFS input data and initial time 12 UTC 26 AUG 2005
LES .......... Large Eddy Simulation
LGE .......... Logistic Growth Equation
MPI .......... maximum potential intensity
N11 .......... Nguyen et al. (2011)
NCL .......... NCAR Command Language
NHC .......... National Hurricane Center
NMM .......... Nonhydrostatic Mesoscale Model
OLR .......... Outgoing Longwave Radiation
PBL .......... Planetary Boundary Layer
PV ............ Potential Vorticity
RM5 .......... WRF run with GFS input data, initial time 00 UTC 26 AUG 2005 and no radiation parameterization
RMW .......... Radius of Maximum Wind
RRTM .......... Rapid Radiative Transfer Model
RWB .......... Rossby Wave Breaking
SDPV .......... Standard Deviation of Potential Vorticity
SHIPS .......... Statistical Hurricane Intensity Prediction Scheme
SST ............ Sea Surface Temperature
TC .............. Tropical Cyclone
TCLAPS ......... Tropical Cyclone Limited Area Prediction System
TPF ............ Translating Pressure Fit
VC .............. Vacillation Cycle
VHT ............ Vortical Hot Towers
VRW ............ Vortex Rossby Wave
WISHE ......... Wind-Induced Surface Heat Exchange
WPS ............ WRF Preprocessing System
WRF ............ Weather Research and Forecasting Model
YSU ............ Yonsei University
Chapter 1

Introduction

1.1 Motivation

During the past 20 years track forecasting of tropical cyclones (TCs) has constantly been improved by development in research and computing power. However, there has been no corresponding significant improvement in forecasting TCs intensity (NHC 2008; DeMaria and Kaplan 1999; DeMaria and Gross 2003). A tiny step towards improvement in forecasting TCs intensity is the basic motivation of this thesis. In simulations of Hurricane Katrina Nguyen et al. (2011) (henceforth N11) observed vacillation cycles (VCs) between a symmetric and an asymmetric phase. The asymmetric phase shows a monopole like structure in low level potential vorticity (PV) and equivalent potential temperature $\theta_e$ which come along with a rapid deepening of the minimum surface pressure. Contrarily the symmetric phase is characterized by a low level ring of PV, $\theta_e$ and vertical velocity and also by a rapid acceleration of the maximum azimuthal-mean tangential wind. This process is observed in the model before the TC exhibits a mature stage, hence it might be a mechanism that is crucial to understand the intensification mechanisms Katrina experienced. In particular N11 concluded their paper with the comment, that further research with other models is necessary to validate VCs in the intensification process of Katrina (2005). So the main motivation for this thesis can be expressed in one question: Do VCs occur in non-hydrostatic WRF simulations of Hurricane Katrina?

A mature TC is in a steady state and can be described to a high extent by a simple axisymmetric model (Ooyama 1969; Emanuel 1986). While the details are stochastic or as described by Ooyama (1982) of probabilistic nature, their genesis and large scale evolution appear to be reasonably deterministic. Despite profound improvement in understanding the entity of a mature TC there is still a lack in predictability of intensification in developing stages of a TC. The coupling of intensification to the planetary boundary layer (PBL), convective turbulent processes and interrelated vorticity evolution needs further investigation. TCs are largely axisymmetric and much of their evolution can be described by axisymmetric theory. Nonetheless, as I will show in this thesis, real tropical cyclogene-
sis has a important asymmetric part to it. Despite on azimuthal average the TC seems to
be symmetric in early stages already, in the horizontal and vertical structure of the three dimensional TC the intensification process possesses a highly asymmetric and probabilistic manner. With the study of asymmetric features a more realistic understanding of TCs established in TC literature, namely vortex Rossby waves (VRWs), barotropic instability and vortical hot towers (VHTs). As a combination of these asymmetric mechanisms N11 examined VCs in a simulation of Hurricane Katrina. Those VCs are hypothesized to be an important feature in young stages of TCs and to have major influence on the intensification process.

1.2 A review on tropical cyclone research

1.2.1 Development in tropical cyclone intensity forecasting

The most important information for risk management and necessity of population evacuation is the track and the intensity of a TC. The lack of improvement in TC predictability motivated researchers in the past up to the present to find solutions for this problem. Hence the milestones in the development of intensity forecasting are presented herein chronologically.

The conceptual understanding of intensification was first formulated as a symmetric problem. A linear theory named ‘conditional instability of the second kind’ (CISK) was published by Charney and Eliassen (1964). The identical mechanism described with a non-linear theory independently by Ooyama (1963) remained unpublished. Two decades later he published a paper to clarify the current state on developing and mature TCs, with a focus on defining the CISK process which was misunderstood from several contemporary researchers back then (Ooyama 1982). As a sine qua non both theories describe a conditional instability, measurable with high CAPE, whereas the ’second kind’ refers to a coexistent vortical flow. Charney and Eliassen (1964) assumed that a cumulus cell and the pre hurricane depression of a vortex would help each other. The vortex is strengthened by the heat release of the cumulus cell. Consequential low level convergence of moisture feeds the cumulus cell which in turn intensifies the vortex again. Friction dissipates kinetic energy, but gives latent heat supply through frictional convergence. This balance between convergence and friction results in a feedback mechanism that was thought to develop from a tropical depression in a conditionally unstable environment into a TC.

Further details of differences and later criticism were discussed by Smith (1997), who also discussed the so called wind-induced surface heat exchange (WISHE) self-exciting process of Emanuel (1986). In contrast to CISK, Emanuel (1986) showed with his analytical model that the dominant stimulus of WISHE depends exclusively on self induced heat transfer from the ocean without ambient instability (hence low CAPE). He compared a TC to the conceptual model of an air parcel undergoing a Carnot cycle,
as illustrated in Fig. 1.1. First there is an isothermal expansion at the surface where

![Figure 1.1: The tropical cyclone as a Carnot heat engine. See text for explanation. Taken from Fig. 13 of Emanuel (1986).](image)

the warm reservoir, the sea \( (T_B) \), releases moist entropy to the parcel. While rising through the eyewall along angular momentum surfaces up to the tropopause and cooling down to \( T_{out} \), it expands moist adiabatically. Adiabatic expansion is justified with the assumption of constant saturated equivalent potential temperature along angular momentum surfaces, which means the atmosphere is locally neutral (in the vertical) to moist convection. Such an atmosphere has no CAPE and an air parcel can rise adiabatically. The anticyclonic circulation aloft cools the parcel down by radiating heat to space while being isothermally compressed. Finally the Carnot cycle is closed as the parcel warms up to the initial temperature by adiabatic compression as it descends to the ocean. Recently Emanuel himself investigated the weak point of his theory in a series of papers \((\text{Emanuel and Rotunno 2011; Emanuel 2011})\). With their convection-resolving axisymmetric numerical model they find the assumption that isentropic surfaces in the storm outflow have the same potential temperature as the air at large distances from the core to be untrue. Anyhow originally it was thought that the TC reaches a steady state where the net heating of the Carnot cycle is used to do work against frictional dissipation in the boundary layer. The wind-evaporation feedback at the sea surface is critical as it drives the heat engine. In the linear model of \(\text{Emanuel (1986)}\) the radial gradient of azimuthal-mean \( \theta_e \) is related to the vertical shear of the mean tangential velocity by the axisymmetric thermal wind equation. With the assumption of a balanced symmetrical vortex there is a negative radial gradient of the azimuthal mean mixing ratio and \( \theta_e \) towards the radius of maximum wind (RMW). Through a random increase of the mean mixing ratio in the core region near the surface, the feedback loop is initiated. As the mixing ratio is proportional to \( \theta_e \), the mean radial gradient of \( \theta_e \) increases congruently throughout the boundary layer. As a consequence, lifted \( \theta_e \) surfaces warm the core through a warmer inflow along angular momentum surfaces advected by the elevated radial inflow of the secondary circulation. The imbalance in the thermal wind relation subsequently accelerates the mean tangential wind at the top of the vortex boundary.
layer. Turbulent mixing processes carry the intensified wind speeds to the surface and hence raise the potential for sea to air water vapor flux and therefor also increase the near surface saturation mixing ratio, thereby leading to a further warming of the core and increase in the mean tangential wind and so on. In a second paper Rotunno and Emanuel (1987) show with a non-hydrostatic axisymmetric model that WISHE is fundamentally different from CISK. The ocean as heat engine increases circulation as long as \( \theta_e \) increases in the core. Moisture convergence alone would be insufficient as the necessity to elevate \( \theta_e \) from the boundary layer is demonstrated. So their model shows again that CAPE is not necessary at all for TC development and they argue that in models with CAPE as driving force the air sea interaction is underestimated. Emanuel (1986) and Rotunno and Emanuel (1987) argued that the theory of CISK needs a previous ambient instability which was later shown by Dengler and Reeder (1997) to be not true. They conducted TC simulations with the CISK parameterization of Ooyama (1969) and show that the development of a vortex to hurricane strength is possible with the CISK scheme even without convective instability in the initial state.

Arising from the motivation to predict the maximum potential intensity (MPI) of a developing TC different approaches were developed. In a review Camp and Montgomery (2001) discussed the two ways on calculation of MPI presented in Holland (1997) and Emanuel (1997). Particularly how close calculated MPI is to observations and why MPI theory fails for many hurricanes. The approach of Holland (1997) is unrealistic as it calculates minimum central pressures of down to unrealistically low 770 hPa for 304 K SST. The model of Emanuel (1997), which is based on the WHISE mechanism, matches TC observations much better but still lacks the requirements of an precise intensity forecast. His model determines MPI by only using SST and environmental relative humidity. The disequilibrium between the saturation of boundary layer and sea surface controls the flux of entropy which in turn allows a calculation of the maximum tangential wind. With the assumption of a solid body rotation this enables a calculation of minimum central pressure. Deficiencies are found in the parameterization of thermodynamic structure of the eye. Another major shortcoming is the assumption of a symmetric structure and accordingly neglected asymmetric mechanisms that could affect intensity. Camp and Montgomery (2001) conclude that neither of the MPI models represents the key mechanisms and they suspect that asymmetric vortex dynamics and the inclusion of the ocean mixed layer would produce a more representative MPI formulation.

Another approach by DeMaria and Kaplan (1994, 1999) is the Statistical Hurricane Intensity Prediction Scheme (SHIPS). The statistical-dynamical multiple regression scheme SHIPS combines synoptic with climatological and persistence predictors. Relative to climatology and persistence the SHIPS forecasts show statistically significant skill for the Atlantic and east Pacific at 36, 48, and 72h. DeMaria (2009) further improves his intensity prediction scheme based an a logistic growth equation (LGE). It uses statistically trained parameters to estimate the intensity of a storm, basically it uses the SST, vertical
shear of the forecast, convective instability and transitional speed. Both schemes cannot reliably predict intensity.

Studies on inner core asymmetric mechanisms (VRWs, barotropic instability, VHTs and VCs) improved the picture of a developing TC from an ideal rotating fluid in a steady state towards a predominantly asymmetric structure with a high degree of randomness. Research on VRWs, barotropic instability, VHTs and VCs require an own section and are hence discussed in detail in sections 1.2.2 - 1.2.5, respectively. In a series of three papers Nguyen et al. (2008), Montgomery et al. (2009) and Smith et al. (2009) give an integrated picture of the current state of the art in three-dimensional TC modelling. Montgomery et al. (2009) shows that, up to the present, there is a lack of understanding of the inner core dynamical processes in TCs; the in the literature entirely accepted WISHE mechanism is not mandatory for intensification: 'modest sea-air vapor fluxes are essential, the wind evaporation feedback is not.' In an experimental setup he shows that TCs can intensify even though heat fluxes in their model are velocity limited. Locally sufficient surface moisture fluxes are thought to get enough energy to feed VHTs, which dominatly intensify the vortex. They conclude that WISHE evaporation wind feedback is subdominant compared to the VHT intensification mechanism. Furthermore Nguyen et al. (2008) impose slight perturbations on initial conditions which strongly influence the TCs development. In ensemble model runs they find that inner core asymmetries are strongly dependent on the randomly perturbed initial boundary layer moisture distribution. Hence they find an intrinsic uncertainty of the upper bound for intensity as the single runs have different realization of the asymmetries, in particular the exact location and intensity of VHTs. Like in Montgomery et al. (2009) they argue that WHISE is not constraining the intensity, but that locally sufficient surface moisture fluxes get enough energy to feed VHTs, which intensify the vortex. Smith et al. (2009) conclude the series by reviewing and refining the conception of TC spin-up. Recently Montgomery and Smith (2011) discussed the different paradigms for TC intensification in a review paper. They separate the CISK descriptions of Charney and Eliassen (1964) and Ooyama (1982) into the CISK-paradigm and the cooperative-intensification paradigm, respectively, and as a third paradigm they also describe the WISHE paradigm of Emanuel (1986). A new asymmetric paradigm is proposed which focuses on asymmetric contributions of VHTs in the intensification in contrast to previous axisymmetric formulations. The work herein confirms their new paradigm as will be shown and discussed in chapter 4 and 5, respectively.

In summary, up to the present, predictions of TC intensity are not satisfying. This results from a still incomplete understanding of the physical processes and the inability to accurately model the formation and intensification of the mesoscale entity. Already Ooyama (1982) discussed the probabilistic nature of the intensity forecast problem. He concluded that the age-old problem of modelling turbulence and cloud microphysics explicitly and their stochastic nature in the time scale of a tropical cyclone create a variety
of possible model realizations.

1.2.2 Vortex Rossby waves

Rossby waves on a planetary scale are the most dominant form of waves that influence weather patterns. Rossby-like wave patterns occur in outward propagating spiral rain bands which are a feature of every TC. McDonald (1968) assumed these rain bands to be a consequence of Rossby-like waves. Later Guinn and Schubert (1993) returned to this problem and showed the major role of Rossby-like waves in spiral rain band formation.

Montgomery and Kallenbach (1997) first coined the term VRW and derived a dispersion relation, hence phase and group velocities, for linear barotropic VRWs. They obtain an increasing wavenumber with distance from the center, while the radial velocity decreases. VRWs reach a stagnation radius where wave energy is suspected to transform into kinetic energy of the mean flow. They suggested that this outward propagation also plays a role for forming secondary eyewalls. This is thought to explain the development of the so called eyewall replacement cycles (ERCs) observed and illustrated by Willoughby et al. (1982). Thereby the inner, old eyewall contracts to a minimum radius and weakens while an outer new eyewall forms at a greater radius and consumes, and accordingly caps, the energy input from the sea surface. In total an ERC can either weaken or strengthen a TC.

Möller and Montgomery (1999) investigate VRWs further and show the validity of the linear and quasi-linear theory of Montgomery and Kallenbach (1997) also for a barotropic model which includes strong nonlinear effects. In a shallow water model they show the symmetrizing mechanism of VRWs by imposing several PV asymmetries on a PV monopole. The asymmetries radiate outwards while the vortex develops a symmetric shape again. This process named ‘axisymmetrization’ is hypothesized to transfer energy to the basic state flow and thereby strengthen the vortex.

Chen et al. (2003) apply the theory of empirical normal modes to a TC model simulation which decomposes the wind and thermal fields to orthogonal modes of wave activity and find that 70-80% of statistical variance in a 24h period can be explained by VRWs. They show that VRW travel outwards at low levels and inwards at midlevels, up the inside of the eyewall and down at the outside. The accompanying eddy heat transport warms the eye, accelerates the mean tangential wind inside and outside the eyewall in low and midlevels and decelerates it above.

1.2.3 Barotropic instability

Asymmetries such as polygonal eyewalls and eyewall mesovortices have been documented in several studies (see e.g. Marks and Houze 1984; Lewis and Hawkins 1982). An analogy
is drawn by Schubert et al. (1999) who shows that barotropic instabilities cause similar patterns. In awareness of the incomplete dynamical representation of physical processes in their simple unforced barotropic model framework Schubert et al. (1999) examine the growth of barotropic instabilities. So by ignoring frictional and moist processes in their

Figure 1.2: Vorticity contour plots for the representative numerical experiment. The model domain is 600 km × 600 km, but only the inner 200 km × 200 km is shown. The contours begin at 0.0005 s$^{-1}$ and are incremented by 0.0005 s$^{-1}$. Low vorticity values are shaded blue and high vorticity values are shaded red. Vorticity from t = 0 h to 16 h. Adapted from Fig. 3 of Schubert et al. (1999).

barotropic model, they show that the breakdown of annular PV rings into polygonal shapes or mesovortices is initiated by barotropic instability. The initial conditions of
vorticity and the consequent evolution is illustrated in Fig. 1.2. In their model the ring of PV is thought to carry two relative to the flow counterpropagating PV waves along the strong PV gradients on its inner and outer side, hence they propagate on a positive and on a negative radial gradient. As these two waves phase lock when they have equal angular velocities relative to the surface initially small perturbations grow exponentially until they break the ring into rectangular eyewall shapes or mesovortices (see Fig. 1.2). Kossin and Schubert (2001) support these theoretical predictions with extended numerical simulations and comparison to radar and airborne measurements. Kossin and Eastin (2001) give further evidence to the barotropic framework with airborne measurements of two hurricanes and categorize observed structures that occur repeatedly into two regimes: relative vorticity that shows ring-like profiles, named Regime 1, and monopole structures of relative vorticity, named Regime 2 (see Fig. 1.3).

**Figure 1.3:** Horizontal profiles of relative vorticity $\zeta$ ($10^{-4} \text{s}^{-1}$) and tangential wind $v$ ($\text{m s}^{-1}$), azimuthally averaged with respect to distance from RMW, within regime 1 and regime 2 at 850 mb in Hurricane Diana (1984). Average profiles indicated as regime 1 are based on 22 radial legs flown during 1100 UTC 11 Sep to 0000 UTC 12 Sep. Average profiles indicated as regime 2 are based on 18 radial legs flown during 00001200 UTC 12 Sep. Adapted from Fig. 7 of Kossin and Eastin (2001).

### 1.2.4 Vortical hot towers

The third asymmetric feature in TCs, VHTs, were first examined by Hendricks et al. (2004) followed two years later by Montgomery et al. (2006), who both showed their major organizational role in TCs by modelling early stages of TC evolution. Montgomery et al. (2006) describe VHTs as cores of deep cumulonimbus convection that favor the development from a mesoscale convective system to a tropical depression in two steps. In the first step multiple single intense small scale VHTs create strong PV anomalies due
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to diabatic heating. As they are rotating they convert latent heat efficiently into kinetic, rotational energy of horizontal winds. In the second step by fusing together they intensify the mean circulation of the larger scale vortex. As an important step to initiate the WISHE mechanism Montgomery et al. (2006) suggest VHTs to have a dominant effect in the formation of TCs. The vertical extend of vertical wind and absolute vorticity of a modelled VHT is illustrated in Fig. 1.4 after 40 minutes integration time on 5 levels. The vertical updraft is extending through the whole troposphere. The strong vorticity dipoles in the upper 4 horizontal cross sections on the right hand side of Fig. 1.4 are suggested to be a result of tilted ambient vorticity. As mentioned before Nguyen et al. (2008) and

Montgomery et al. (2009) later suppose predominance of intensification by VHTs to the WISHE mechanism. Nguyen et al. (2008) examine the sensitivity to the intensification process of TCs to VHTs. By perturbing the moisture distribution in the PBL the major heat source and hence driving force is alternated which results in a strong variability in intensity. They conclude that VHTs strongly affect the intensification process, their exact inner-core flow and their vertical redistribution of latent heat through convection itself is concluded to be ‘random and intrinsically unpredictable’.

1.2.5 Vacillation cycles

N11 connected the processes of rapid intensification mentioned above (VRWs, barotropic instability and VHTs) and hypothesize that rapid intensification is reached by going through VCs. They used the modelled evolution of Hurricane Katrina (2005) as an example. They analyze the inner core physical mechanisms in their hydrostatic, high-

Figure 1.4: Example vertical velocity $w$ (m s$^{-1}$) and absolute vertical vorticity $\eta$ ($\times 10^{-4}$ s$^{-1}$) signatures at $t = 40$ min. Horizontal cross-sections are 20 km $\times$ 20 km subdomains centered at $x = 50$ km, $y = 6$ km. Taken from Fig. 9 b) of Montgomery et al. (2006).
resolution version of the Tropical Cyclone Limited Area Prediction System (TCLAPS) model (Davidson and Weber 2000). They show cyclic repetitions between symmetric and asymmetric phases in Hurricane Katrina, corresponding to Regime 1 (symmetric) and Regime 2 (asymmetric) of observations shown in Kossin and Eastin (2001). In Regime 1 of Kossin and Eastin (2001) there is a ring like structure of relative vorticity $\zeta$ just inside of the RMW and a constant steep increase of windspeed towards the RMW. In contrast Regime 2 is characterized by a mono-polar $\zeta$ profile and higher tangential winds in the center and a lower maximum at the RMW (see Fig. 1.3). N11 are the first to examine this cyclic repetition of symmetric and asymmetric phases in a numerical model and coin them VCs. To illustrate the distribution of PV on 850hPa in VCs, typical profiles of symmetric and asymmetric phases are shown in Fig. 1.5a and b, respectively. The temporal evolution of the asymmetries is illustrated in the top graph of Fig. 1.5c with the maximum of the standard deviation of PV (SDPV$_{\text{max}}$) along constant radii. High values of SDPV$_{\text{max}}$ are associated with asymmetric phases, whereas SDPV$_{\text{max}}$ is small in symmetric phases. Additionally the amplitudes of azimuthal wave numbers of the Fourier transformed PV shown in the bottom graph indicate VCs; with higher amplitudes of wavenumber 3 and 4 in asymmetric and higher wavenumber 0 values in symmetric phases. As Hurricane Katrina develops a symmetric PV ring by diabatic heating in the eyewall, the necessary conditions for barotropic instability, consistent with the work of Schubert et al. (1999) are explained to promote slowly growing perturbations which are boosted by convective instabilities to grow into VHTs. In particular the energy budget equations of Kwon and Frank (2008) and the stability analysis of Weber and Smith (1993) are applied to strengthen their arguments. So they describe a VC starting from an symmetric state that breaks down by convective-barotropic instability which triggers VHTs that dominate the vortex. As VHTs travel centerwards in the asymmetric phase the vortex axisymmetrizes again by VRWs so that a symmetric eyewall can establish again. This symmetric state in turn breaks down again by convective-barotropic instability and so on. The repetition of these processes increases the circulation and decreases the pressure in a stepwise manner until a mature vortex established that keeps its symmetric shape as the high stability of the warm core prevents a further breakdown.

The strong diabatic heating rate in VHTs produces positive low-level PV anomalies which detach from their parent convective entities and move as local PV maxima cyclonically to the center. The ”$\beta$-effect” can be explained with barotropic dynamics as an air parcel conserves its absolute vorticity in a barotropic framework. By moving polewards on the eastern side of a vortex its relative vorticity becomes more anticyclonic whereas air parcels that move equatorwards on the western side become more cyclonic. This effect causes a formation of a vorticity dipole asymmetry on a $\beta$-plane and arising from this theory intense cyclonic vorticity propagates upwards a negative vorticity gradient of a vorticity monopole (see Smith and Ulrich 1990, 1993; Montgomery and Smith 2011). In a baroclinic vortex PV is conserved in a frictionless adiabatic flow (analogous to absolute vorticity in a barotropic vortex). Following this N11 argue that
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(a) Symmetric Phase

(b) Asymmetric Phase

(c) Evolution

Figure 1.5: Vortex structure of PV (PVU = $10^{-6} \times m^2 s^{-1} K kg^{-1}$) color shaded at 850 hPa during symmetric phase S1 in subfigure a at 39 h simulation time and asymmetric phase A2 in subfigure b at 44 h. The contour lines are vertical velocity (m s$^{-1}$). Top Graph in subfigure c: Evolution of PV asymmetries at 850 hPa with the dotted red line with circles as SDPV$_{\text{max}}$ (PVU) and maximum mean tangential wind $\tau_{\text{max}}$ as solid black line (m s$^{-1}$) and its tendency $\frac{\partial v_{\text{max}}}{\partial t}$ as dashed blue line with triangles (m s$^{-1}$ h$^{-1}$)). Bottom Graph in subfigure c shows amplitudes of azimuthal wave numbers of PV (PVU) from 0-6 at 50 km radius. The horizontal axis shows hours elapsed from 00 UTC on 26 August 2005. Adapted from Fig. 1 and 3 c and d of N11.
PV maxima are embedded in a negative radial gradient of mean-azimuthal PV and are thought to move, similar to a vortex on a $\beta$-plane, towards the center with higher PV. N11 illustrate further how after a maximum asymmetry VHTs weaken as they consumed CAPE and are tilted by local vertical wind shear.

VCs are hypothesized to play a major role in the intensification process of Hurricane Katrina and are found to be clearly different to ERCs occurring in mature hurricanes. N11 argue that ERCs are characterized by a capped energy input in the inner eyewall as a formation of a new eyewall at a greater radius occurs. This happens uniformly in azimuth and does not incorporate asymmetries in the flow whereas VCs happen in the eyewall itself and show a strongly asymmetric shape.

Finally, it is concluded that further sensitivity studies on VCs with more sophisticated models are necessary to estimate their influence on the intensification of TCs. This task is investigated in chapter 4 with the non-hydrostatic WRF model. N11’s argumentation of the breakdown from symmetric to asymmetric phase by convective-barotropic instability is criticized in chapter 5, as well as their $\beta$ plane explanation on centerward movement of PV maxima.

### 1.2.6 Sensitivity of intensity on model parameterization, initial conditions and resolution

This section summarizes papers on the sensitivity of TC simulations to model parameterization, initial conditions and resolution of full physics numerical models.

As mentioned in section 1.2.1 slight perturbations in initial conditions can diversify the TC intensity (Nguyen et al. 2008). There are several sensitivity studies on TCs modelled with WRF. Rotunno et al. (2009) examine sensitivity to resolution of an idealized TC with a horizontal grid size down to <100 m using large eddy simulations (LES). They find intense three dimensional turbulence along the inner edge of the eyewall of strong hurricanes and that the transition from parametrized diffusion to partially resolved modelling of turbulence with LES strongly changes the dynamical structure in the model. Rotunno et al. (2009) state that increasing the resolution in an LES model directly increases the ability of resolving eddies explicitly whereas increasing the resolution in a mesoscale model simply reduces the horizontal diffusion coefficient. The intensity of their vortex increases with decreasing diffusion coefficient therefore they conclude that intensity is strongly dependent on it. Furthermore, they argue that, due to insufficient observations in hurricane PBLs it is not clear how LES and mesoscale models should handle subgridscale turbulence, hence how to determine exactly how they should be parameterized.

In another sensitivity test Gentry and Lackmann (2008) illustrate that their 1 km
resolution WRF simulation has weaker eyewall updrafts but is still the most intense simulation compared to coarser resolutions. Additionally the structure of PV changes dramatically compared to coarser resolutions. With increasing resolution more PV enters the center of the vortex, which results in stronger ventilation of the core by VRW breaking and the connected stronger horizontal mixing into the eyewall. Finally they emphasize that a resolution of 1 km is necessary to resolve both up and downdrafts in the eyewall.

Li and Pu (2008) examine the dependence of intensity in TC simulations on cumulus parameterization and PBL parameterization. Their modelled TCs are strongly dependent on surface latent heat fluxes and convective heating rates. Using a 9 km mesh size, the vortex does not intensify without cumulus parameterization as the resulting small vertical velocities imply small vertical energy fluxes. In contrast at horizontal resolutions less than 3 km, the cumulus scheme does not influence the track and intensity forecast but only the precipitation structure. Conversely the model is sensitive to differences in PBL parameterization at all resolutions (see also Nolan et al. 2009). The sensitivity of PBL parameterization is also discussed in section 8.2 of Montgomery and Smith (2011), they list studies that investigated this problem but conclude that the choice of the optimum scheme for TC intensity prediction is still to be determined.
1.3 Goals and outline

In the light of the motivation above, in section 1.1, this thesis addresses the central question: Does hurricane Katrina go through VCs in the non-hydrostatic WRF model? In particular the results of the hydrostatic model results of TCLAPS in N11 will be tested for their robustness with a focus on the low level PV evolution of the WRF model in direct comparison to TCLAPS. To achieve a maximal comparability a resolution similar to the TCLAPS run of 5 km mesh size is chosen.

More specific questions that shall be adressed are:

- Do observations show patterns of VCs?
- Does the vortex satisfy the necessary conditions for a barotropic instability?
- The $\beta$ effect explanation is not satisfactory. What causes mesovortices in asymmetric phases to enter the center?
- What role do VRWs play in the intensity evolution?
- How sensitive is intensification to different resolution, initial and boundary conditions?
- What are the differences that arise from WRFs non-hydrostatic?
- Are VCs a result of inhomogeneities in SST or do they occur also for spatially constant SST?

This thesis contains the following: the introduction in this chapter contains a statement on the motivation for this thesis followed by background information on TC literature that shall introduce the reader into the still unsolved problem of accurate intensity forecasts for TCs. The development in intensity forecasting is presented historically followed by a description of several sensitivity studies and connected problems in modelling TCs. Moreover literature on several asymmetric inner core mechanisms is reviewed which are thought to be of importance for TC intensification. Chapter 2 presents fundamentals of PV to give a common basis to better understand the asymmetric mechanisms described later on. The principles of PV invertibility and finally its usefulness for understanding dynamical processes are investigated. Advanced readers might skip this chapter as it is mainly written to introduce readers that are new to PV thinking ideas. The methods implemented for this thesis are treated in chapter 3. The model description of WRF and its comparison to TCLAPS is followed by definitions on the vortex center finding, a symmetry parameter for PV and the fast Fourier transformation (FFT) for wavenumber analysis. Results are presented in chapter 4 with a focus on one simulation with the best simulation of central pressure followed by sensitivity tests and an analysis of cycloonic VRW breaking patterns in the WRF output. The results are discussed in chapter 5 and are finally concluded in chapter 6.
Chapter 2

Potential vorticity fundamentals

This chapter introduces into the PV as a powerful tool to understand dynamical atmospheric processes. For readers that are familiar with PV thinking ideas this chapter might be trivial and they may skip it and jump straight to chapter 3. First a derivation of barotropic PV introduces into ideas on conservation of vorticity followed by a derivation of Ertel’s isentropic PV. A simple rotation-symmetric example serves as the basis of later argumentations and finally a section on non-conservative processes presents theoretical considerations that are as well of importance for the reasoning in the following chapters.

2.1 Barotropic potential vorticity

The PV perspective is a very practical way to observe and understand the dynamical behavior of the atmosphere. The following short summary of the derivation of PV and PV equations on isentropic surfaces is based on sections on PV in the books of Wallace and Hobbs (2006), Etling (2008) and Pichler (1997), the theorems of Haynes and McIntyre (1987) and on the detailed description of PV from Hoskins et al. (1985). It shall remind the reader of the basic principles of PV and its potential in simplifying and unifying dynamical processes into one quantity.

To illustrate the conservation principle of PV we first follow Wallace and Hobbs (2006) who derive the simplest form of PV, the barotropic PV. Imagine a layer of an incompressible fluid of depth $H(x, y)$ moving with velocity $\mathbf{v}(x, y)$ and $A$ as the area of an imaginary block consisting of a set of fluid parcels. Considering mass conservation we can then write

$$\frac{d}{dt}(HA) = 0. \quad (2.1)$$

Using the identity of divergence

$$\frac{1}{A} \frac{dA}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nabla_h \cdot \mathbf{v} \quad (2.2)$$

one can derive

$$\frac{1}{H} \frac{dH}{dt} = -\frac{1}{A} \frac{dA}{dt} = -\nabla_h \cdot \mathbf{v} \quad (2.3)$$
So there is a direct relation of layer depth to the divergence: a thinning (thickening) of the layer coincides with divergence (convergence). Now we remember that many aspects of large scale extratropical weather systems can be understood with conservation of vorticity of the horizontal wind field. Mathematically a close approximation to the real atmosphere, neglecting friction and the vertical component of advection, is the vorticity equation in Lagrangian form

\[
\frac{d\eta}{dt} = -\eta \nabla h \cdot \mathbf{v}
\]  

(2.4)

with \( \eta = f + \zeta \) vertical component of absolute vorticity and \( f \) and \( \zeta = \mathbf{k} \cdot \nabla \times \mathbf{v} \) the vertical component of planetary and relative vorticity, respectively (\( \mathbf{k} \) is the unit vector along the z axis and \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \)). As the divergence term on the right hand side in slowly developing extratropical weather systems is relatively small, we derive the non-divergent form of the vorticity equation

\[
\frac{d\eta}{dt} \approx 0.
\]

(2.5)

This simplified form allows coarse predictions into the future as numerical values are transported by the horizontal wind field. It can be thought of as a substance (conservative tracer) that is advected by the mean flow. If one now substitutes the divergence of (2.4) into (2.3) one obtains

\[
\frac{d\eta}{dt} = \frac{\eta}{H} \frac{dH}{dt}
\]

(2.6)

which can be rearranged to

\[
\frac{d}{dt} \ln \eta = \frac{d\ln H}{dt}
\]

(2.7)

and

\[
\frac{d}{dt} [\ln \eta - \ln H] = 0
\]

(2.8)

which is equivalent to the conservation equation

\[
\frac{d}{dt} \left( \frac{\eta}{H} \right) = 0,
\]

(2.9)

where \( \frac{\eta}{H} \) is the barotropic PV. So now if there is again a thinning (thickening) and divergence (convergence) of a layer simultaneously the absolute vorticity \( \eta \) is decreasing (increasing). Hence the layer thickness is proportional to the absolute vorticity.

Assuming adiabatic flow, the air parcels cannot pass through isentropic surfaces, i.e. surfaces of constant potential temperature. It follows that the mass of air parcels bounded by two isentropic surfaces is conserved following the the flow:

\[
\frac{A \delta p}{g} = \text{constant},
\]

(2.10)

where \( \delta p \) is the pressure difference between two isentropic surfaces that have the constant potential temperature difference \( \delta \theta \) as shown in Fig. 2.1.
2.2 Ertel’s isentropic potential vorticity

The following derivation of the PV, inspired by Etling (2008) and Pichler (1997), illustrates that also in a baroclinic, frictionless and divergent atmosphere the PV is a conservative quantity. Therefore one can use the equation for absolute vorticity in a \( p \)-system

\[
\frac{\partial \eta}{\partial t} + \mathbf{v} \cdot \nabla \eta = \eta \frac{\partial \omega}{\partial p},
\]  
(2.11)

and the first law of thermodynamics for adiabatic processes:

\[
\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = 0.
\]  
(2.12)

By differentiating (2.12) with respect to \( p \) one obtains

\[
\frac{\partial}{\partial t} \frac{\partial \theta}{\partial p} + \mathbf{v} \cdot \nabla \frac{\partial \theta}{\partial p} + \frac{\partial \mathbf{v}}{\partial p} \cdot \nabla \theta = 0.
\]  
(2.13)

Where the third term can be separated into its horizontal and vertical components with \( \nabla_p \) and \( \mathbf{v}_h \) referring to the horizontal gradient and velocity vectors along pressure surfaces, respectively.

\[
\frac{\partial \mathbf{v}_h}{\partial p} \cdot \nabla_p \theta + \frac{\partial \omega}{\partial p} \frac{\partial \theta}{\partial p} = 0.
\]  
(2.14)

The left term of (2.14) cancels to zero by using the thermal wind relation

\[
\frac{\partial \mathbf{v}_h}{\partial p} = -\frac{R}{f_p} \mathbf{k} \times \nabla_p T
\]  
(2.15)

and the potential temperature \( \theta = T(\frac{\mathbf{v}_p}{p}) \frac{R}{\gamma_p} \).

\[
\frac{\partial \mathbf{v}_h}{\partial p} \cdot \nabla_p \theta = -\frac{R}{f_p} (\mathbf{k} \times \nabla_p T) \cdot \nabla_p \theta = -\left( \frac{p}{p_0} \right) \frac{R}{f_p} \mathbf{k} \cdot (\nabla_p \theta \times \nabla_p \theta) = 0,
\]  
(2.16)
Potential vorticity fundamentals

\[ (\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c}) \] and \( \vec{a} \times \vec{a} = 0 \). Now before adding them we multiply (2.11) with \( \frac{\partial \theta}{\partial p} \) and (2.13) with \( \eta \)

\[
\left. \begin{array}{c}
\frac{\partial \theta}{\partial p} \frac{\partial \eta}{\partial t} + \frac{\partial \theta}{\partial p} \nabla \cdot \eta \nabla \eta = \eta \frac{\partial \theta}{\partial p} \frac{\partial \omega}{\partial p} \\
\frac{\partial \omega}{\partial p} + \eta \nabla \cdot \nabla \theta = -\eta \frac{\partial \omega}{\partial p} \frac{\partial \theta}{\partial p}
\end{array} \right\} + (2.17)
\]

and finally obtain the conservation equation of PV for adiabatic processes:

\[
\frac{\partial}{\partial t} \left( \eta \frac{\partial \theta}{\partial p} \right) + \nabla \cdot \left( \eta \frac{\partial \theta}{\partial p} \right) = \frac{d}{dt} \left( \eta \frac{\partial \theta}{\partial p} \right) = 0 . \tag{2.18}
\]

In a \( \theta \)-System one can obtain the equal result with respect to surfaces of potential temperature (Pichler 1997)

\[
\frac{d}{dt} \left( \eta \frac{\partial \theta}{\partial p} \right) = 0 \tag{2.19}
\]

with the isentropic absolute vorticity

\[
\eta_\theta = \zeta + f = k \cdot \nabla \theta \times \vec{v}_h + f \tag{2.20}
\]

The conserved quantity, the PV, is often defined by multiplication with \(-g\), the negative gravitational acceleration,

\[
PV = -g \eta \frac{\partial \theta}{\partial p} \tag{2.21}
\]

to get positive values of PV for cyclonic circulations in the northern hemisphere in the potential vorticity unit PVU = \(10^{-6} \, m^2 \, s^{-1} \, K \, kg^{-1}\). Equation (2.21) is the famous theorem for adiabatic, frictionless motion of Ertel (1942), where the PV is conserved even for fully three-dimensional non-hydrostatic motion. As in detail presented in Hoskins et al. (1985) PV is a very powerful tool to diagnose the dynamical structure of meteorological fields. It is commonly used to locate the troposphere on isentropic surfaces and also cyclonic storms associated with PV minima. Additionally areas of possible Rossby wave propagation can be identified along strong gradients of PV. Beyond that the concept of PV inversion permits a better understanding of cyclogenetic processes. Analogous to the barotropic case where the absolute vorticity defines the corresponding large scale wind field, in the baroclinic case under the assumptions of a reference state, a balance condition and boundary conditions the potential vorticity determines the entire wind and mass field.

### 2.3 A simple rotation-symmetric example

The non-localness of PV shall be illustrated with a simple rotation-symmetric example of Pichler (1997), assuming a balanced, rotational, adiabatic and frictionless motion. The velocity has hence only a tangential component \( v(r, \theta) \) which is a function of radius \( r \) and potential temperature \( \theta \) (the height coordinate in a \( \theta \)-system). The azimuthal balance
of the so called Montgomery potential $M$ or isentropic stream function ($M \equiv c_p T + \phi$, with $\phi \equiv gz$) is expressed through
\[
\frac{v^2}{r} + f v = \frac{\partial M}{\partial r}.
\]
(2.22)

The hydrostatic equation in a $\theta$-system can be written as
\[
\frac{\partial M}{\partial \theta} = \frac{c_p T}{\theta} = \Pi(p)
\]
(2.23)

with the Exner function $\Pi(p) = c_p \left( \frac{p}{p_0} \right)^{\frac{R}{c_p}}$, where $p_0 = 1000$ hPa, $R$ is the gas constant for dry air and $c_p$ is the specific heat of dry air at constant pressure. Differentiating (2.22) by $\theta$ and (2.23) by $r$ and subtracting them, one obtains
\[
\frac{\partial v}{\partial \theta} \left( f + \frac{2v}{r} \right) = \frac{\partial \Pi}{\partial p} \frac{\partial p}{\partial r}.
\]
(2.24)

By differentiating the equation again with respect to $\theta$ one gets
\[
\frac{\partial^2 v}{\partial \theta^2} \left( f + \frac{2v}{r} \right) + \frac{\partial}{\partial \theta} \left( f + \frac{2v}{r} \right) \frac{\partial v}{\partial \theta} = \frac{\partial p}{\partial r} \frac{\partial^2 \Pi}{\partial \theta \partial p} + \frac{\partial \Pi}{\partial p} \frac{\partial^2 p}{\partial \theta \partial r}.
\]
(2.25)

Now Pichler (1997) neglects the first term with the second derivative of $v$ and as
\[
\frac{\partial}{\partial r} \left( \frac{\partial \Pi}{\partial p} \right) = \frac{\partial^2 p}{\partial \theta \partial r}
\]
(2.26)

and hence
\[
\frac{\partial}{\partial r} \left( \frac{\partial \Pi}{\partial p} \right)^{-1} = \frac{\partial^2 p}{\partial \theta \partial r}
\]
(2.27)

one can rearrange (2.25) with $\frac{\partial \Pi}{\partial p} \left( \frac{\partial \Pi}{\partial p} \right)^{-1} = 1$ to
\[
\frac{\partial}{\partial r} \left( \frac{\partial \Pi}{\partial p} \right)^{-1} = \frac{\partial^2 p}{\partial \theta \partial r}
\]
(2.28)

Obviously, in this rotation-symmetric case (in cylindrical coordinates without azimuthal change in the radial velocity $u$) the absolute vorticity can be written as
\[
\eta_{\theta \text{cy}} = k \cdot \nabla \times v = f + \frac{1}{r} \frac{\partial (vr)}{\partial r} - \frac{1}{r} \frac{\partial u}{\partial \lambda} + f = \frac{1}{r} \frac{\partial (vr)}{\partial r} + f.
\]
(2.29)

Now differentiating (2.21) with $\eta_{\theta \text{cy}}$ by $r$ one gets
\[
\left( \frac{g}{g} \frac{\partial \eta_{\theta \text{cy}}}{\partial \theta} \right)^{-1} \frac{\partial PV}{\partial r} + PV \frac{\partial}{\partial r} \left( \frac{g}{g} \frac{\partial \eta_{\theta \text{cy}}}{\partial \theta} \right)^{-1} = \frac{1}{r} \frac{\partial (vr)}{\partial r}.
\]
(2.30)

By combining (2.30) and (2.28) in a final step one retrieves a nonlinear partial differential equation that describes the connection between the potential vorticity and the rotation-symmetric balanced flow:
\[
\frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial (vr)}{\partial r} \right] + \frac{PV}{g} \frac{\partial}{\partial \theta} \left( \frac{\partial \Pi}{\partial p} \right)^{-1} \left( f + \frac{2v}{r} \right) \frac{\partial v}{\partial \theta} = - \left( g \frac{\partial \eta_{\theta \text{cy}}}{\partial \theta} \right)^{-1} \frac{\partial PV}{\partial r}.
\]
(2.31)
With proper boundary conditions (e.g. $v \to 0$ for $r \to \infty$) and static stability of a reference atmosphere one could derive a linear form of (2.31), which could give a first guess to solve it (see Pichler 1997). Equation (2.31) describes the distribution of radial wind and potential temperature for a given PV field and is independent of time. However, the given distribution of PV does not only influence a certain level but also the velocity fields of the layers below and above. The radial derivative of the relative vorticity in term (a) together with (b), which includes a first and second vertical derivative of the tangential velocity, equate the negative radial gradient of the PV multiplied with the inverse stability. So this simple example illustrates the dependency between the vertical velocity profile, the vertical stability and the radial PV gradient in the horizontal plane.

Now if one imagines a solid body rotation where the velocity increases linear with increasing radius (a) will cancel. One then can rearrange (2.31):

$$\frac{\partial \theta}{\partial p} \frac{\partial}{\partial \theta} \left( \frac{\partial \Pi}{\partial p} \right)^{-1} \left( f + \frac{2v}{r} \right) \frac{\partial v}{\partial \theta} = -\frac{1}{PV} \frac{\partial PV}{\partial r}. \quad (2.32)$$

Hence the radial gradient of PV equates the lhs, a function of stability and velocity. A linear increase of tangential velocity is typically found inside of the RMW in mature TCs. The applicability of this equation 2.32 to a real hurricane is questionable, but it shows for this example how the radial PV gradient is in balance with the stability and the vertical velocity profile.

In general the fact that the horizontal PV field has a remote action is important to understand cyclogenetic and anti-cyclogenetic processes in the atmosphere. As for instance an interaction of phase coupled PV anomalies in initially upper and lower atmospheric layers can generate a cyclonic circulation throughout the whole troposphere, a low pressure system. In particular for a TC we shall keep in mind for the later analysis in section 4.2 that single isentropic slices of PV in the WRF model output are not independent from each other. In summary the derivations above shall illustrate the non localness of PV which is, in chapter 5, assumed to be crucial for inner core dynamics in real hurricanes.

### 2.4 Non-conservative processes

The importance of PV invertibility, also known as 'PV thinking', is discussed extensively by Hoskins et al. (1985). They derive Ertel’s theorem for adiabatic, frictionless motion on a different pathway with the three-dimensional absolute vorticity $\eta = 2\Omega + \nabla \times \mathbf{v}$. Using the Bjerknes circulation theorem they obtain the more general form

$$Q = \frac{1}{\rho} \eta \cdot \nabla \theta. \quad (2.33)$$

Following Hoskins et al. (1985) the PV equation in height coordinates with frictional force $\mathbf{K}$ and a diabatic potential temperature source $\dot{\theta}$ is:

$$\frac{DQ}{Dt} = \frac{1}{\rho} \eta \cdot \nabla \dot{\theta} + \frac{1}{\rho} \mathbf{K} \cdot \nabla \theta \quad (2.34)$$
Hence the so far simple conservation of PV is complicated by diabatic heating and friction, which shall hereafter be referred to as nonconservative processes. The generation of PV from nonconservative processes is according to (2.34) possible through a gradient in the diabatic heating rate in the direction of the absolute vorticity or by frictional generation of vorticity in the direction of the $\theta$ gradient. By multiplying equation (2.34) with $\rho$ and integrating it over the material volume $\tau$ with surface $S$, the vector $n$ normal to this surface and the identities $\nabla \cdot \eta = \nabla \cdot K = 0$, $\rho Q = \nabla \cdot (\eta \theta)$, ect., one can derive the equation

$$\frac{\partial}{\partial t} \iiint_{\tau} \rho Q d\tau = \iint_{S} (\dot{\theta} \eta + \theta K) \cdot ndS$$  \hspace{1cm} (2.35)

Hence the mass-integrated PV over $\tau$ can only change if there is a source of $\theta$ or a frictional force on its boundary. Inside the volume diabatic and frictional sources can only redistribute PV. Hoskins et al. (1985) postulate two principles: 1) the conservation principle that holds approximately when adiabatic flow dominates diabatic and frictional forces (applies when rhs of (2.34) is negligible). And 2) the principle of invertibility which is valid under the assumption of a reference state, balance conditions and proper boundary conditions. So under those three conditions the mass and wind field can be inverted from a given PV distribution.

Supplementary to this derivation Haynes and McIntyre (1987) prove mathematically that even when diabatic and frictional forces are acting: (i) there ‘can be no net transport of PV across any isentropic surface’ and (ii) ‘PV can neither be created nor destroyed within a layer bounded by two isentropic surfaces’. Accordingly PV can just be generated on intersections of $\theta$ levels with ground or the PBL - the total integrated PV between two isentropic surfaces does not change.

In a paper on the role of diabatic heating and friction in a PV based study of a baroclinic cyclone Stoelinga (1996) investigated the contribution of nonconservative processes to cyclogenesis. He concluded that the strength of the mature midlatitude surface cyclone he observed was generated in particular by latent heating which contributed about 70% to the PV. His illustration of the PV generation in the frontal zone is shown in Fig. 2.2. Interestingly they use methods to split the PV generation into the conserved advective part by a piecewise inversion method and into parts of diabatic and frictional processes by partitioned integration. They clearly show the importance of nonconservative processes in midlatitude cyclogenesis which in contrast was historically described with a focus on the dry, inviscid baroclinic dynamics. They conclude that not only the dry, inviscid baroclinic dynamics prevail but the combination of or rather interaction between low-level generated diabatic PV and upper level PV anomalies.

The overall picture of PV that emerges can be used for TCs. Here diabatic processes in low levels generate PV anomalies that feed the cyclonic circulation. The PV generation in a cold front has similarities to the boundary layer of a TC. As it
Figure 2.2: Generation of PV by latent heating in an idealized 2D frontal zone. Thin solid lines are contours of absolute momentum. Arrowheads on thin solid lines indicate that they are also streamlines. White arrows are absolute vorticity vectors. Heavy solid lines are contours of PV generation due to latent heating, with maxima of generation and depletion indicated by ‘+’ and ‘−’ symbols, respectively. Intensity of gray shading is proportional to upward vertical velocity, latent heating and PV. Figure 9 of Stoelinga (1996)

will be shown in section 4.2 isolines of the azimuthally averaged potential temperature of a hurricane, which are also isolines of angular momentum, slope upwards similar to Fig. 2.2. With the theorems (i) and (ii) of Haynes and McIntyre (1987) in mind, it follows that PV is generated mainly in the boundary layer by diabatic heating and friction.

The PV inversion for a TCs eyewall region is possible when presuming a reference state, balance conditions and boundary conditions. Wang and Zhang (2003) developed such a system and can invert a given 3D PV field with given boundary conditions and physics forcing while assuming an axisymmetric reference state. The system can reproduce the major features that characterize a TC the rotational winds, organized eyewall updrafts, subsidence in the eye, cyclonic inflow in the PBL and upper level anticyclonic outflow. It is to note that Wang and Zhang (2003) apply this inversion to a highly symmetric cyclone and they conclude that it would be interesting to apply this method to an asymmetric one.

Hausman et al. (2006) also apply the PV invertibility principle on an axisymmetric, non-hydrostatic tropical cyclone model and use the PV field to understand the dynamical evolution in the hurricanes intensification process. They find that the difference of the dry Ertel PV distribution and their moist PV is negligibly small in their model. A great deficiency of the PV invertibility applied to a TC is the necessity of an axisymmetric reference state, hence the asymmetric evolution of a TC cannot be reconstructed. Anyhow it would exceed the framework of this thesis to execute such an inversion, but it shall be clarified that the PV is a very practical tool to illustrate both the adiabatic inviscid
cyclogenesis as well as diabatic and frictional processes.

The present chapter developed PV thinking ideas with increasing complexity from a barotropic framework followed by introduction of nonconservative processes, sources diabatic heating and friction, and finally concluded with a description on PV inversion. The non-localness of PV in an adiabatic rotation-symmetric example is illustrated with equation (2.31). Furthermore, equation (2.34) showed the additive effect of nonconservative processes whereas (2.35) clarified that the PV, that enters the TC from the boundary layer, can only redistribute itself. Finally papers on PV inversion partially revealed the balanced and unbalanced components that exist in a TC. Generally, a PV inversion increases the dynamical understanding but at the same time the assumptions of a PV inversion limit the degree of freedom. As stated by Stoelinga (1996) it is impossible to clarify the sources of all of the interactions between various conservative and nonconservative processes, in the nonlinearity of a full physics numerical model it is impossible to distinguish the PV anomalies and the PV background they create unambiguously. However, all the ideas presented just above shall improve the understanding of PV thinking and prepare for the analysis and discussion of the WRF model output in chapter 4 and chapter 5, respectively.
Chapter 3

Methods

This chapter briefly summarizes the methods used to such an extent that all the results could be reproduced by any devoted researcher. A central component of this thesis is to test the results of N11 for their robustness using the same analytical methods but a non-hydrostatic model. Details on both models are followed by descriptions of the analytical methods used.

Section 3.1 deals with a general description of the WRF model and the direct model comparison of WRF and TCLAPS, specifically of the chosen parameterization schemes. Section 3.2 describes the vortex center finding algorithm followed by the definition of a symmetry parameter in section 3.3. Finally the FFT used for the wavenumber analysis is presented in section 3.4.

3.1 Model description and set-up

3.1.1 Advanced Research WRF

The weather research and forecast model (WRF) is a state-of-the-art atmospheric simulation system which is widely used in the scientific community, including TC research (Rotunno et al. 2009; Li and Pu 2008; Nolan et al. 2009; Gentry and Lackmann 2008; Fierro et al. 2009). It is suitable for a wide range of scales from meters of horizontal mesh-sizes for LES to kilometers for simulations on a global scale. It is applicable for idealized cases as well as for case studies of real cases like for Hurricane Katrina herein. The scientific and algorithmic approaches in WRF, including the solver, physics options, possibilities for initialization and boundary conditions as well as grid-nesting techniques are documented in Skamarock (2008). The ARW Users Guide (Wang et al. 2008) provides information on the whole process of running WRF from installation over running cases to graphical illustration. There are two solvers available in the WRF software framework: the Nonhydrostatic Mesoscale Model (NMM) solver and the Advanced Research WRF (ARW) solver. In this thesis the ARW dynamic core is chosen which uses the non-
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hydrostatic perturbation form of the flux-form Euler equations (Skamarock 2008). For the vertical coordinate in WRF the \( \sigma \) coordinate system of Laprise (1992) is used

\[
\sigma = \frac{p_h - p_{ht}}{p_{hs} - p_{ht}}
\]

with the hydrostatic pressure component \( p_h \) and the pressure values along the surface and top boundaries \( p_{hs} \) and \( p_{ht} \), respectively. This results in \( \sigma \) values of 1 at the surface and 0 at the upper model boundary.

Several WRF runs of Katrina that differ only in initial and boundary conditions, apart from one run with finer horizontal mesh size, were conducted. The runs are named EM5, EN5, GM5, GN5, CM5, RM5 and EN3; here the number indicates the horizontal mesh size whereas the letters give details about initial data and additional information. The runs EM5, EN5 and EN3 are initialized with ECMWF data at 0000 UTC 26 Aug 2005 for EM5 and at 1200 UTC of the same day for EN5 and EN3. GM5, GN5, CM5 and RM5 are initialized with GFS data also at 0000 UTC 26 Aug 2005 for GM5, CM5 and RM5 and again 12 hours later for GN5. CM5 only differs from GM5 in the sea surface temperature (SST) which is held constant at 304.5 K in CM5. In RM5 the parameterization of shortwave and longwave radiation is switched off. Note that throughout this report the hours for every single run are counted from these initial times. All runs are conducted with two-way interactive nested domains and except EN3 use an innermost domain with 301 \( \times \) 301 grid points that has the southwest corner located at 17.5°N and 267.5°E on a mercator projection with 5.5 km horizontal mesh size (corresponds to about \( \sim 0.05° \) in TCLAPS). In EN3 432 \( \times \) 414 grid points are used with a 3.3 km mesh size and the southwest corner at 20.5°N and 266.8°E on a mercator projection. The GFS input data provides 27 vertical levels, whereas the ECMWF input data possesses 61 levels. For WRF runs using GFS the vertical resolution is increased to 45 vertical levels with the automatically produces standard \( \sigma \) levels of WRF. In contrast the vertical resolution of ECMWF is kept on 61 levels but because of large distances in low and high altitudes the density of \( \sigma \) levels is redistributed to better resolve the PBL and the upper troposphere. This is realized with a program for the conversion of ECMWF’s \( \sigma \) levels to the user defined cartesian vertical z-coordinates (see ‘Eta level programme’ in Appendix A of Wagner 2011). An overview on the differences and commonalities of the single runs is listed in table 3.1. To be able to reproduce the results presented herein the complete namelists for WRF run GM5, including all settings described above for the WRF preprocessing system (WPS) and for the WRF model itself, are listed in Appendix A.2 and A.1 (namelist.wps and namelist.input, respectively).

To represent the effect of subgridscale convection, a cumulus parameterization is used. It is an adjustment or mass-flux scheme that accounts for unresolved vertical fluxes like updrafts, downdrafts and compensation motion on the outside of convective cells. As discussed in Molinari and Dudek (1992) there are three approaches to trigger cumulus convection: traditional, explicit and hybrid. The appropriate scales for simulation with
the three different approaches range from 50-60 km for the traditional approach (parameterizes convectively unstable gridpoints) to less than 3 km for the explicit approach (convection is mainly resolved, i.e. explicit calculation) over the grey zone between 3 and 50 km handled by the hybrid approach (partially parameterizes subgridscale convection). In particular, for TC simulations, the explicit approach is possible in the dynamically balanced system of a TC as modest large scale instability and very strong vertical forcing are apparent (Molinari and Dudek 1992). Here the cumulus parameterization chosen is based on a cloud model of Kain and Fritsch (1993) which uses moist up and downdrafts and includes the effects of detrainment and entrainment and uses relatively simple microphysics (\texttt{cu.physics} = 1). For the finest domain the cumulus scheme is switched off for all runs, respectively (\texttt{cu.physics} = 0).

The cloud microphysics of the Purdue Lin scheme of Chen and Sun (2004) are based on the bulk water microphysical parameterization technique of Lin et al. (1983). This bulk parameterization includes six classes of hydrometeors: vapor, cloud water, rain, cloud ice, snow, and graupel. It is a relatively sophisticated microphysics scheme of WRF suitable for real-data high-resolution simulations. It models mass mixing ratios for the single hydrometeors in all interactions between them: collision-coalescence, collision-aggregation, accretion, transformations from ice to snow, Bergeron processes, evaporation outside the cloud, genesis of hail by probabilistic freezing of raindrops and melting of hail and snow (\texttt{mp.physics} = 2).

Surface layer schemes range from sophisticated multilayer vegetation and soil moisture models to simple thermal models. These models calculate the surface heat and moisture fluxes as well as the surface stress. The obtained information on surface variables subsequently initializes land-surface models to calculate the boundary conditions for the planetary boundary layer (PBL). For the surface layer the Monin-Obukhov scheme is used herein (\texttt{sf.sfclay.physics} = 1). The land surface model provides heat and moisture fluxes over land and sea points. It calculates land surface fluxes from deeper layers to the surface for each grid point seperately with a thermal diffusion scheme (\texttt{sf.surface.physics} = 1). Heat moisture fluxes from surface are then calculated based on similarity theory (\texttt{isfflx} = 2).

The PBL parameterization is not only responsible for the boundary layer but, instead, handles sub-grid-scale vertical mixing throughout the whole troposphere. Hence if activated it replaces the explicit vertical diffusion with the assumptions included in the PBL scheme. Here the sub-grid-scale processes in Katrina’s boundary layer are represented using the Yonsei University (YSU) PBL scheme (based on Hong et al. (2006), \texttt{bl.pbl.physics} = 1). As in prior versions the YSU scheme calculated a profile of the vertical diffusion coefficient $K$ for the entire depth of the PBL, the main improvement in the updated YSU scheme is a revised vertical diffusion algorithm with an explicit extraction of entrainment processes from the buoyancy profile at the top of the PBL.
The YSU scheme need to be used in conjunction with Monin-Obukhov scheme for the surface layer. Additionally the diffusion option that evaluates second order diffusion terms on coordinate surfaces ($\text{km}_{\text{opt}} = 4$) as well as the horizontal Smagorinsky first order closure ($\text{diff}_{\text{opt}} = 1$), which are used herein, are recommended for simulations with real case data.

For longwave radiation the Rapid Radiative Transfer Model (RRTM) based on Mlawer et al. (1997) is used herein. It is an accurate spectral-band scheme using look-up tables which account for longwave processes in multiple bands of greenhouse gases (CO$_2$, O$_3$), microphysics species of clouds and optionally trace gases ($\text{ra}_{\text{lw}}\text{.physics} = 1$). The shortwave radiation from the sun, including visible and surrounding wavelengths, is parameterized herein by a scheme based on Dudhia (1989) ($\text{ra}_{\text{sw}}\text{.physics} = 1$). It includes processes like absorption, reflection and scattering in the atmosphere and at surfaces. It uses a downward integration to calculate absorption and scattering in cloud and clear sky-conditions. For both the longwave and the shortwave radiation scheme the time step for updating the radiation fluxes can be specified for each of the domains separately ($\text{radt} = 10,3$).

Generally, the physics parameterizations use the dynamical solver output and calculate the parameterized sub-grid scale processes. In particular, surface and PBL schemes are updated after every time step, as this provides best results, whereas for cumulus and radiation schemes a lower update frequency is cost-saving while still sufficient. All the options chosen here and other possibilities are described in the ARW Users Guide (Wang et al. 2008).

### Table 3.1: Settings of conducted WRF runs

<table>
<thead>
<tr>
<th>run</th>
<th>EM5</th>
<th>EN5</th>
<th>GM5</th>
<th>GN5</th>
<th>CM5</th>
<th>RM5</th>
<th>EN3</th>
</tr>
</thead>
<tbody>
<tr>
<td>input data</td>
<td>ECMWF</td>
<td>ECMWF</td>
<td>GFS</td>
<td>GFS</td>
<td>GFS</td>
<td>GFS</td>
<td>ECMWF</td>
</tr>
<tr>
<td>hor. grid size</td>
<td>5.5 km</td>
<td>5.5 km</td>
<td>5.5 km</td>
<td>5.5 km</td>
<td>5.5 km</td>
<td>5.5 km</td>
<td>3.3 km</td>
</tr>
<tr>
<td>vertical levels</td>
<td>61</td>
<td>61</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>61</td>
</tr>
<tr>
<td>pecularity</td>
<td>constant SST</td>
<td>no radiation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 3.1.2 Comparison between the WRF model and TCLAPS

The differences and commonalities, as well as weak points of WRF and TCLAPS, are discussed now. To focus on the impact of the main difference - hydrostatic vs. non-hydrostatic model - most WRF runs are set up as close as possible to N11’s TCLAPS implementation named E5C with the similar horizontal grid size of 5 km. The run presented in N11 originates from TCLAPS run E5C which is presented in detail in Nguyen (2010) and is used for comparison to WRF runs herein. As shown by Davidson and Weber (2000) TCLAPS shows good skill in operational forecasting of TCs. However, it still has
two major weak points. First, in TCLAPS the simplification of a hydrostatic atmosphere is a constraint in the prognostic equations of McDonald (1968). In contrast the equations of WRF described in Skamarock (2008) are non-hydrostatic. Thunis and Bornstein (1996) discuss classifications of mesoscale flow and applicability of simplifications of the governing equation by use of a scale analysis. As shown in a flow classification in their Fig. 2 only a part of the updrafts in deep convection can be modeled by hydrostatic equations. This is intuitively clear as the derivation of the hydrostatic approximation \( \frac{dp}{dz} = -\rho g \) arises from the assumption that the magnitude of the vertical acceleration is much smaller that the magnitude of the pressure gradient force \( \left| \frac{dw}{dt} \right| \ll \left| \alpha \frac{dp}{dz} \right| \) with \( \alpha = 1/\rho \) as shown e.g. in the scale analysis of Pielke (2002) in his section 3.3.1. In convective clouds the vertical acceleration can reach similar magnitudes as the horizontal wind so this assumption is inappropriate for the eyewall region. Nevertheless, the relatively high resolution despite hydrostatic approximation has been justified with the good performance in track forecasting of TCLAPS (Nguyen 2010; Davidson and Weber 2000).

Calculating cumulus convection explicitly at a resolution of 5 km can be problematic. WRF runs at 5 km in here as well as TCLAPS run E5C are conducted with the explicit approach despite their resolution is actually too coarse to resolve convection accurately, as discussed just above in section 3.1.1. Nguyen (2010) exercise ensemble simulations in which the model run named CB calculated with cumulus parameterization (Tiedtke 1989) develops less intensity as vertical updrafts are suppressed. To make fine structured convective effects clearly evident, a run at 3.3 km resolution is presented in chapter 4.

Furthermore, TCLAPS is used as an operational model, so it is implemented with relatively high diffusion to stay stable, whereas WRF as a research model is in the setup chosen herein less diffusive and hence allows more high frequency variability. In particular there is a divergence damping implemented in TCLAPS that selectively damps the regions of intense divergence. This damping is fitted to application in TCs and proves its validity in forecast skill. A short wave noise filter in WRF (\texttt{diff\_6th\_opt}) could damp high frequency undulations and generate a similar effect but it is of interest here to keep wave dynamics that possibly play a role but would be damped out. The methods of calculating diffusion used herein are gradients along the surface coordinates from which horizontal diffusion coefficients are diagnosed from horizontal deformation, while the vertical diffusion is calculated by the PBL scheme (\texttt{diff\_opt = 1, km\_opt = 4}). As a result of higher diffusion and the hydrostatic approach the model output of TCLAPS is much smoother compared to WRF and as we will see in chapter 4 the WRF fields are more noisy and contain more high frequency undulations.

Another important difference between the models is the sophisticated initialization procedure of TCLAPS which develops real vortex dynamics of primary and secondary circulation that global models cannot resolve (Davidson and Weber 2000). In the first
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24 hours TCLAPS nudges initial and boundary conditions from the Australian Bureau of Meteorology’s operational global assimilation and prediction system to a synthetic vortex based on TC advisories, which define the location, size, and past motion of the storm (see also, Nguyen 2010). In contrast WRF has to develop these vortex dynamics by itself. There is an option in WRF to insert a bogus vortex, but the complexity is relatively poor and a WRF simulation conducted with this option yielded a bad representation of Katrina’s evolution (not shown).

Table 3.2: Comparison of parameterizations used in WRF and TCLAPS

<table>
<thead>
<tr>
<th>parameterization</th>
<th>WRF</th>
<th>TCLAPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>longwave radiation</td>
<td>RRTM of Mlawer et al. (1997)</td>
<td>Longwave scheme of Fels and Schwarzkopf (1975)</td>
</tr>
<tr>
<td>microphysics</td>
<td>Explicit calculation of six classes of water species with Purdue Lin scheme (Chen and Sun 2004) based on Lin et al. (1983)</td>
<td>Explicit calculation of six classes of water species with bulk explicit microphysics by Dare (2004), similar to Lin et al. (1983)</td>
</tr>
<tr>
<td>surface layer</td>
<td>similarity theory (Monin and Obukhov 1954)</td>
<td>similarity theory (Monin and Obukhov 1954)</td>
</tr>
<tr>
<td>PBL</td>
<td>K-profile in YSU scheme (Hong et al. 2006)</td>
<td>K-profile in ECMWF’s scheme (Beljaars and Betts 1992)</td>
</tr>
<tr>
<td>Grid</td>
<td>5/3.3 km on 45/60 levels</td>
<td>5 km on 27 levels</td>
</tr>
</tbody>
</table>

The comparison of parameterization schemes implemented in WRF and TCLAPS are listed in table 3.2. Generally the parameterizations of the two models are by purpose quite similar and should not cause major differences despite the ones mentioned above. The radiation scheme for longwave radiation is more sophisticated in WRF (Mlawer et al. 1997) but based on similar approaches as in TCLAPS’ longwave scheme (Fels and Schwarzkopf 1975). Microphysics are both based on the bulk water microphysical parameterization technique of Lin et al. (1983). The cumulus scheme only influences the coarser domains of WRF, as it is two way nested followingly there is a negligible influence of those schemes onto the finest domain. In both models of the Monin-Obukhov similarity theory to calculate the surface fluxes between the model surface and the first atmospheric model layer (Monin and Obukhov 1954). The different realizations of PBL schemes both determine vertical diffusion by means of a K-profile. In particular WRF determines the eddy viscosities $K_h$ and $K_v$ from horizontal deformation (see section 4.2.3 of Skamarock 2008) whereas in TCLAPS a diffusion equation is solved to obtain
3.2 Vortex center finding

horizontal diffusion (see section 2.2.5 of Nguyen 2010). The most important difference in the PBL schemes is the divergence diffusion that is applied in TCLAPS, it considerably smoothes intense divergence (complete descriptions of the PBL schemes can be found in Hong et al. (2006) and Beljaars and Betts (1992)). Another commonality is that both models only use the input from global models for SST, there is no coupled ocean mixed layer model. Also the horizontal resolution with 5 km of WRF is similar to TCLAPS in most model runs except for run EN3 with 3.3 km whereas the vertical resolution in WRF is higher, with 45 vertical levels for GFS and 61 vertical levels for ECMWF input data, compared to TCLAPS 27 levels.

In summary for the purpose herein the two models differ most relevantly in the diffusion parameterization and in the basic equations - hydrostatic vs. non-hydrostatic. The WRF runs are kept as close to TCLAPS as possible to condense the effects of these major differences. The results in chapter 4 will illustrate the outcome. It is obvious that more high frequency undulations and hence a more noisy fine scale structure is inevitable in the WRF model output.

3.2 Vortex center finding

An exact vortex center finding is indispensable for the application of radial and azimuthal analysis of the model output. Small errors in the determination of the center introduce large errors in azimuthal analysis. For instance the maximum of azimuthal wind would be lowered by a shifted center as the circle along which is averaged would not accord with a perfectly symmetric RMW but intersect it. Similarly a spectral decomposition with a FFT along a symmetric RMW would result in a maximum amplitude of wavenumber zero, whereas a shifted center would intersect the RMW twice. Followingly the FFT would result in a maximum amplitude of wavenumber two.

The center is determined for the surface and each pressure level separately. The first guess center is defined by the average of the location of minimum wind and the location of minimum sea level pressure for the surface level and accordingly the minimum of geopotential height for all pressure levels above. As indicated by the red dots in Fig. 3.1 the track of simulation CM5 undulates around the more precise, smoother translating pressure fit (TPF) track, indicated by the black line with asterisk signs. The used TPF technique has been developed by Kepert (2005) for objectively locating the pressure center of TCs from observations. The code of Kepert (2005) determines the center of minimum pressure and minimum height by randomly perturbing parametric TC pressure profiles of Holland (1980) and minimizing the cost function:

\[ J_p(\beta, \mathbf{a}) = \sum_{i=1}^{n_{\text{obs}}} \frac{(p_i - p_{\text{hol}}([x_i, t_i; \mathbf{a}], \beta))^2}{\sigma_{P_i}^2}. \]  \hspace{1cm} (3.2)
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Figure 3.1: Blue solid line with plus signs indicates observational ‘best track’ data in 6 h intervals from Knabb et al. (2006) from 00UTC 26 Aug 2005 to 00UTC 29 Aug 2005. The hourly location of the first guess center of the WRF tracks of run CM5 are indicated with red dots and the TPF center by a black solid line with asterisks every 6 h. The TPF center algorithm could not determine a center at 0 - 4 h.

The parametric profile $p_{hol}$ is defined by the shape and intensity of $\beta = (v_m, r_m, b, p_c)$, where $v_m$ is the maximum cyclostrophic wind at $r_m$ the RMW with the profile shape parameter $b$ and the central pressure $p_c$. In the denominator $\sigma^2_{P_i}$ is the error variance of the $i$th pressure data point at location $x_i$ and time $t_i$ which is relative to the time of center determination. Additionally the cost function $J_p$ is a function of $\mathbf{a}$, as the TPF technique is built for observations in a temporal window of a couple of hours it includes the assumption of a linear track in this variable $\mathbf{a}$. For the application herein at a certain point in time in the model the cost function simplifies to:

$$J_p(\beta) = \sum_{i=1}^{n_{obs}} \frac{(p_i - p_{hol}(r[(x_i), \beta]))^2}{\sigma^2_{P_i}}.$$  \quad (3.3)

For determination of the center $J_p$ is minimized over $\beta$. In other words the actual pressure profile is fitted to a range of typical TC profiles and the TPF center is hence determined by the large scale vortex and not by mesoscale undulations in the center. Primarily the technique was developed for application on observations near the core. The data density of model output is of considerably larger density as dropsonde or aircraft observations, hence the TPF code generates robust results. The code diverges in cases where the pressure anomalies are weak or if the first guess center is far away from the actual center (as for 0-4 h.
3.3 Symmetry parameter of potential vorticity

A symmetry parameter is defined here to condense the temporal evolution of the degree of vortex symmetry of the modeled TCs into one variable. For this purpose N11 define the maximum standard deviation of potential vorticity (SDPV$_{\text{max}}$) as described below.

First one needs the PV itself which is calculated on WRFs model levels with a NCAR command language (NCL) routine as:

$$PV_\sigma = -g \left[ -\frac{\partial v}{\partial p} \frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial p} \frac{\partial \theta}{\partial y} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right) \frac{\partial \theta}{\partial p} \right].$$

(3.4)

This PV$_\sigma$ is calculated on the model’s σ-surfaces as denoted by the subscript. It is an approximation to the PV in equation (2.33) wherein the horizontal derivatives of the vertical velocity $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ as well as the horizontal components of the Coriolis force are neglected; it is the hydrostatic form of Ertel’s PV (detailed derivation in Appendix A.3).

In a stably stratified atmosphere $\nabla \theta$ dominantly points upward and as the PV is maximal when $\eta$ is parallel to $\nabla \theta$ it is dominantly vertical in such conditions. In the PBL of a TC the isentropic surfaces can be significantly tilted as in fronts, hence in those areas the horizontal contribution to the PV in 3.4 is considerable.

However, this PV$_\sigma$ as well as other variables are averaged every 5 km around the TPF center by

$$\overline{X}_R = \frac{1}{n} \sum_{n=1}^{n} X(d_i), \quad (R - \frac{\Delta r}{2}) \leq d_i \leq (R - \frac{\Delta r}{2})$$

(3.5)

and standard deviations along these circles are calculated followingly. Here $X(d_i)$ is the value of any variable $X$ at point $i$, that has the distance $d_i$ to the TC center, $\Delta R$ is the thickness of a ring (here 5km), and $n$ ist the number of grid points inside that ring. As mentioned above in section 3.2 this azimuthal analysis is very sensitive to the center finding.

The standard deviation of PV$_\sigma$ is defined as

$$SDPV_R = \sqrt{\frac{1}{n} \sum_{n=1}^{n} (\overline{PV}_\sigma(d_i) - \overline{PV}_\sigma R)^2}, \quad (R - \frac{\Delta r}{2}) \leq d_i \leq (R - \frac{\Delta r}{2}).$$

(3.6)

SDPV$_{\text{max}}$ is the maximum value of SDPV at all radii inside of 300 km. It is a variable used by N11 to condense the information on the degree of symmetry (low SDPV$_{\text{max}}$)
or asymmetry (high SDPV\textsubscript{max}) of a TC into a single number. This variable illustrates a maximal variation of PV around the center. The PV amplitude and variability along a ring around the vortex center determine the magnitude of SDPV\textsubscript{max}. Symmetric PV profiles with small variations will result in a small SDPV\textsubscript{max} whereas intense single PV maxima will result in high values of SDPV\textsubscript{max}.

### 3.4 Wavenumber analysis with fast Fourier transformation

The FFT is used for wavenumber analysis of the PV from WRF output. The WRF output is therefore interpolated equidistantly onto \( N = 36 \) points on circles every 10 degree around the center. This is done for an increasing radius of the single circles every \( dr = 10 \) km. The complex, discrete fourier transform

\[
F(j)_r = \frac{1}{N} \sum_{k=0}^{N-1} x_k \exp \left[ -i \frac{2\pi j k}{N} \right], \quad j = 0, 1, \ldots, N - 1, \quad r = 10, 20, \ldots, 100 \tag{3.7}
\]

with \( j \) as the wavenumber, can be computed efficiently with an algorithm of Cooley and Tukey (1965). The real part of the resulting fourier transform gives amplitudes decomposed into a power spectrum of frequencies or congruently wavenumbers. For example if a PV circle \( x_{50} \) would have three equally distributed maxima at 50 km radius the fourier transform with wavenumber three would be dominant, it would have its maximal amplitude there. For a constant PV in \( x_{50} \) there would be a maximum at wavenumber zero and no amplitude for higher wavenumbers. Again, as mentioned above in section 3.2, this wavenumber analysis of the fourier transform is very sensitive to the center finding. A perfectly circular PV profile would have a maximum at wavenumber two if the PV would be interpolated on a circle with a shifted center.
Chapter 4

Results of WRF simulations

The WRF simulation GM5 is presented in detail in section 4.1 to generally illustrate the evolution and dynamical structure of the modelled TC Katrina. Section 4.2 then focuses on the sensitivity of the simulation to the initial and boundary conditions and the radiation parameterization. Finally section 4.3 concludes chapter 4 with a description of vortex Rossby wave breaking patterns in the model output with a separation into cyclonic and anticyclonic wave breaking.

4.1 The GM5 simulation

4.1.1 Vortex structure and evolution

The purpose of this thesis is to investigate if the VCs proposed by N11 occur in a non-hydrostatic model. Therefore, the WRF model was chosen and run GM5, which is set up as close as possible to TCLAPS run E5C, is presented now. The modeled vortex is initialized at 00 UTC 26 Aug 2005 when the storm was located over Florida and is simulated until 00 UTC 29 Aug 2005, which is when the real Katrina reached its maximum intensity eleven hours before landfall. For the initialization of the GM5 simulation GFS global reanalysis data is used. They include information on measurements of the real Katrina through the reanalysis process. Nevertheless, the resolution of the GFS is too coarse to resolve a realistic TC. For this reason many TC prediction systems utilize an initialization scheme which can increase skill in track forecasting significantly, as for example TCLAPS (Davidson and Weber 2000). In particular in TCLAPS a nudging method helps the model to spin up the circulation with a bogus, an artificial cyclone. In other words, the coarse input data from the global model is pushed artificially towards a more realistic scenario in the regional TC model which incorporates a higher resolution. Although the fact that no bogus vortex is used in the WRF model, the smoothed pressure and wind fields are sufficiently strong to generate a TC.

The so called 'best track' is the 'best' estimation of TC evolution and intensity with available data (land stations, ships, buoys, geostationary and polar-orbiting satellites,
Figure 4.1: (a) The minimum sea level pressure in red and the maximum of total wind speed in dark-blue. Solid lines are best track and dashed-dotted lines are of WRF run GM5, respectively. The light-blue solid line with crosses shows the maximum of mean tangential wind at 850 hPa. (b) sea surface temperature (colour shaded) at initial time and tracks of central pressure. Blue solid line with plus signs indicates observational ‘best track’ data in 6 h intervals from Knabb et al. (2006) from 00UTC 26 Aug 2005 to 00UTC 29 Aug 2005. The hourly location of the first guess center of the WRF tracks of run GM5 are indicated with red dots and the TPF center by a black solid line with asterisks every 6 h. The TPF center algorithm could not determine a center at 0 - 10 h.
4.1 The GM5 simulation

aircraft, dropsonde and radar). The term ‘best track’ track is commonly used for maximum 1-min sustained surface wind speeds as well as for minimum central pressure estimates. Observations of best track for Katrina are available from the National Hurricane Center (NHC 2008). Results of simulation GM5 with the intensity and track evolution compared to ‘best track’ data are shown in Figure 4.1a and b, respectively. The smoothed data in the global model results in an initial minimum pressure of 1004 hPa and maximum wind of \( \sim 20 \, \text{m s}^{-1} \) instead of the measured 983 hPa and 36 m s\(^{-1}\) (Katrina was a category 1 TC on the Saffir-Simpson scale at the initial time). Note that the observations in best track data are an average of multiple measurements and show the maximum in the 1-min sustained surface wind. In contrast, the maximum wind speed of the model (dark blue dash-dotted line) is the one that occurs anywhere in the model domain. The maximum of the mean tangential wind at 850 hPa is more comparable as it is averaged over a circle around the center, although it really should be interpolated to the surface. However, these variables are shown here to illustrate the intensification of the vortex with a focus on the intensification rate in particular on the 850 hPa level. Despite the weakened intensity at the initial time, the model’s circulation rapidly intensifies in about 50 to 60 hours to a category 5 TC (1-min sustained winds are higher than 70 m s\(^{-1}\)). Finally GM5 predicts the intensity of Katrina after 72 h at 00 UTC 29 Aug 2005 accurately with a minimum pressure of 901 hPa compared to the measured 903 hPa at that time. Although GM5 accurately predicts the final minimum pressure the evolution of the minimum pressure is different from the observed one. There are 2 phases of constant pressure in the best track data whereas in the GM5 simulation there is a continuous decrease of central pressure with a variation in the pressure tendency.

The track of Katrina in GM5 is opposed to the observed track in Fig. 4.1a. The shape of the track is similar in both the observations and model, but the endpoints are different. The track of GM5 has a distance to the observed track of 46, 126 and 158 km for +24, +48 and +72 h, respectively. In the first nine hours the translating pressure fit (TPF) code was not able to determine a TPF center. Therefore the TPF center starts at 10 h forecast time at 10 UTC 26 Aug 2005 and ends at the maximum intensity of the real TC after +72 h at 00 UTC 29 Aug 2005. Again as shown with CM5 in section 3.2 the first guess center undulates around the much smoother TPF center and finally converges with the first guess center (the first guess is calculated as the mean location of the pressure minimum and the wind speed minimum at the surface).

The main interest of this work is the occurrence of VCs in the intensification phase. A typical feature of VCs is a pressure decrease in the asymmetric intensification phase as indicated by vertical black dashed lines for four asymmetric phases A1 - A4 and a constant minimum surface pressure or weaker deepening in symmetric phases as indicated by black solid lines for three symmetric phases S1 - S3. This stepwise intensification indicated by the deepening of minimum surface pressure gives suspicion for VCs. It is not clearly identifiable in this illustration but in the following section 4.1.2 the pressure
Figure 4.2: Outgoing longwave radiation (OLR) [W m$^{-2}$] at the top of the atmosphere and horizontal wind vectors at 850 hPa at (a) +23 h and (b) +42 h simulation time
4.1 The GM5 simulation

tendency shown in Fig. 4.7 will better illustrate this.

To compare model results to observations in chapter 5, Fig. 4.2a and b show the outgoing longwave radiation (OLR) at the top of the atmosphere of GM5 and the horizontal wind vectors at 850 hPa at +23 and +42 h, respectively. The strongly asymmetric shape of the vortex at +23 h is obvious in the color shaded outgoing longwave radiation as well as in the horizontal wind, indicated by black arrows. The region of highest brightness temperature indicates a region of deep convection caused by an intense VHT at the eastern side of the vortex at +23 h. Later, the outgoing longwave radiation shows a more symmetric shape with a well established eyewall convection at +42 h. Around the inner vortex, most notably at +42 h, legs of high brightness temperature, known as spiral rain bands, indicate outward propagating VRWs as described in section 1.2.2.

Figure 4.3 shows azimuthally averaged variables plotted against the height as defined by equation 3.5 in section 3.3. The shaded intervals are kept similar at both times to illustrate the intensification. As indicated by the tangential wind speeds (color shaded) as well as by the vertical wind (contours) in 4.3a and 4.4a the RMW contracts from 80 km at 23 h to 60 km at 42 h. Similarly the eyewall region with maximum updraft is contracting and is located just inside the RMW. The eyewall is tilted centerwards at 23 h. Later at 42 h the updraft is deflected outwards by the well established secondary circulation. The typical warm core of the TC can be recognized by the bowl like shape of the isentropes in Fig. 4.3b and 4.4b, in the core region inside the RMW they are tilted towards the surface as if a cold front would approach the center from all sides. The potential temperature gradient \( \nabla \theta \) illustrates the influence of \( \nabla \theta \) on the PV. First, it shows that Ertel’s isentropic PV in equation 2.21 is a close approximation to equation 2.33 as \( \nabla \theta \) is dominantly vertical. Second, as the PV is the product of the absolute vorticity \( \eta \) (contours in Fig. 4.3c and c) and \( \nabla \theta \) times a constant one can see the separate contributions to the PV. For instance in Fig. 4.3b between 6 and 8 km at the center there is an only slight maximum in \( \eta \), so the PV is high due to the dense layering of the isentropes. Contrarily in Fig. 4.4b below 1 km in the center \( \eta \) is relatively larger than at 2 km height but the PV is low due to a smaller potential temperature gradient.

Moreover, the temporal evolution of the azimuthal mean fields of run GM5 is displayed as Hovmöller diagrams in Fig. 4.5. The acceleration of the circulation that is shown in Fig. 4.5a has three phases around +30, +36 and +47 h, each exceeding 7 m/s\(^{-1}\) h\(^{-1}\), as indicated with orange color. These accelerations have an effect on the whole vortex; the RMW contracts by about 20 km from +18-24 h then relaxes again before contracting by another 20 km from +27-38 h. At the same time the PV and its radial gradient, shown in Fig. 4.5b, change congruently. The PV gradient at the center is positive at +20 h before A1 and turns negative after the strongest acceleration. Similarly in the second acceleration at +31 h in A2 h the PV gradient turns repeatedly negative in the
Results of WRF simulations

Figure 4.3: Azimuthal mean of tangential wind [m s$^{-1}$] (shaded with increments of 10 m s$^{-1}$) and vertical wind (contours with increments of 0.3 m s$^{-1}$) in (a). Azimuthal mean of equivalent potential temperature $\theta_e$ [K] (shaded with increments of 5K) and potential temperature $\theta$ (contours with increments of 2.5K) in (b). Azimuthal mean of potential vorticity PV [PVU] (shaded with increments of 5 PVU) and absolute vorticity $\eta$ [$10^{-5}$ s$^{-1}$] (contours with increments of $5*10^{-5}$ s$^{-1}$) in (b). All subplots are at 23 UTC 26 Aug 2005, hence +23 h from initial time.
4.1 The GM5 simulation

Figure 4.4: As figure 4.3 but at 18 UTC 27 Aug 2005, hence +42 h from initial time.
Results of WRF simulations

Figure 4.5: Radius-time plots at 850 hPa for model run GM5, azimuthally averaged: (a) tendency of azimuthal mean tangential wind $\frac{\partial v}{\partial t}$ shaded with $v$ as blue contours (in [m s$^{-1}$] with increments of 10), (b) radial gradient of azimuthal mean PV ($\frac{\partial PV}{\partial r}$) shaded with $PV$ as blue contours (in PVU with increments of 10) and (c) azimuthal mean of equivalent potential temperature $\bar{\theta}$ with $\bar{\theta}$ as blue contours (in [K] with increments of 1). Solid black line indicates the radius of maximum tangential wind, respectively.

center, again after A3 at +41 h and also after A4 at +49 h. In conjunction with Fig. 4.6 the blue contours in Fig. 4.5b illustrate the temporal evolution of PV around the first acceleration at +23 h with high variability in the period from +20-26 h. Note that $\bar{\theta}$, shown in Fig. 4.5c, increases in the center in the first three asymmetric phases A1 - A3 at +23, +31 and +39 h, respectively. This is the similar pattern as observed above in Fig. 4.6, the PV vacillates in the center as a result of variations in potential temperature.

Figure 4.6 displays the temporal variability of the mean azimuthal wind and PV at 850 hPa at +23 h and +42 h forecast time and three hours before and after that. For both periods there is a very small variability in PV at radii greater that 75 km and largest variability in the center but comparatively small variability in the mean tangential wind. The maximum PV is located at the steepest radial increase of the tangential wind and is typically located just inside the RMW. The variability of PV can be partially explained by the variability in the tangential wind, although in the center there is little variation in the wind. Hence the vertical potential temperature gradient has to change its amplitude i.e. the isentropic surfaces in the center of the vortex vacillate. The remaining symmetric and asymmetric phases are shown in Appendix A.4 in Fig. A.1. In the mean there is a PV ring structure already from A2 at +31 h on, also indicated by the blue contour in Fig. 4.5b. This contradicts with the idealized separation of symmetric and asymmetric phases into a ring and a monopolar low level PV. In GM5 it is rather a fluent transfer between a more symmetric and a more asymmetric state with larger variations of central PV in asymmetric phases than in symmetric phases.
4.1 The GM5 simulation

Figure 4.6: Azimuthal mean of PV and tangential wind speed at 850 hPa in (a) and (b) at +23 h (±3 h) and at +42 h (±3 h) in (c) and (d), respectively. The red dashes show the mean over 7 hours, the wide blue bar shows the 25 and 75 percentile and the blue line the most extreme data points, circles show not considered outliers.
4.1.2 Symmetric and asymmetric phases

Now, features of the symmetric and asymmetric phases and the transition between the two are presented. These typical features, first observed by N11, are further investigated. Symmetric phases are typically characterized by an azimuthally symmetric ring of PV at 850 hPa with small PV variations in the center and a constant or weaker deepening of the central pressure. Contrarily asymmetric phases show a monopole-like structure in low level PV or larger variations of PV in the center (in case the PV ring structure cannot be broken down to a PV monopole) which comes along with a rapid deepening of the minimum surface pressure.

A reasonable measure for the symmetry is the maximum of standard deviation of PV (SDPV$_{\text{max}}$, as defined in section 3.3). The temporal evolution of SDPV$_{\text{max}}$ on 850 hPa is plotted in Fig. 4.7a in conjunction with the pressure tendency, or vortex deepening/intensification index $\frac{dp}{dt}$. The negative correlation coefficient of $-0.56$ between those two parameters reveals the connection between the degree of symmetry and the respective intensification (stepwise pressure decrease in Fig. 4.1a). The time derivative of the maximal tangential velocity on 850 hPa $\frac{dv_{\text{max}}}{dt}$ shown in Fig. 4.7b is correlated with SDPV$_{\text{max}}$ with a very weak positive correlation coefficient of 0.16; this result contradicts with N11 who found a negative correlation between those parameters. The

![Figure 4.7:](image)

third and fourth asymmetric phases A3 and A4 differ from the first two as the vortex has established a strong PV ring with a positive PV gradient that cannot be broken down any more, as one can see by $\frac{\partial PV}{\partial r}$ in Fig. 4.5b. The reasons for nonetheless defining it as an asymmetric phase will be given in section 4.3. Additionally, horizontal cross sections of PV on isentropic surfaces will further complement the information on vacillations between symmetric and asymmetric phases in GM5 and also clarify the definition of the phases.
4.1 The GM5 simulation

Figure 4.8: Amplitudes of azimuthal wavenumbers from 0-6 of PV [PVU] at 20, 30 and 40 km radius in (a), (b) and (c), respectively. The horizontal axis shows simulation time elapsed from 00 UTC 26 Aug 2005 on.
The wavenumber spectrum in Fig. 4.8 obtained by an FFT of PV at 20, 30, and 40 km radius at 850 hPa complements the information on the degree of symmetry (see FFT definition in section 3.4). The y-axis shows the number of waves that fit into a circle around the center, and thus how many local PV maxima lie along this circle. The shaded values of intensity show how much power each wavenumber contains. Note that errors in the center finding introduce errors in this analysis, although the improved center finding with the TPF method considerably decreased the noise in the spectrum (not shown). High amplitudes for wavenumbers higher than two indicate asymmetric phases at all radii (durations for phases are listed in table 4.1). For symmetric phases S1-S3 there is an increase in wavenumber zero at 40 km in Fig. 4.8c, similar to N11’s results as shown in Fig. 1.5c. At 20 and 30 km the FFT shows low amplitudes in symmetric phases for all wavenumbers visible in Fig. 4.8a and b. The third and fourth asymmetric phases A3 and A4 are in a transitional stage to the mature vortex, high PV enters the center region but the wavenumber zero signal at 40 km stays dominant with an amplitude of above 10 after A2 at 30 h. At +51, +58 and +65 h high values of PV enter the center, indicated by a signal in wavenumbers higher than two at 20 and 30 km radii.

Table 4.1: Definition of time periods for symmetric (S) and asymmetric (A) phases.

<table>
<thead>
<tr>
<th>Run</th>
<th>WRF GM5</th>
<th>TCLAPS E5C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phases</td>
<td>time</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18-26 h</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>27-29 h</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>30-33 h</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>34-37 h</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>38-40 h</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>41-44 h</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>45-48 h</td>
</tr>
<tr>
<td></td>
<td>mature</td>
<td>55+</td>
</tr>
</tbody>
</table>

Times for the symmetric and asymmetric phases of GM5 are opposed to run E5C of N11 in table 4.1. The peak times are chosen by means of relative maxima in SDPV\textsubscript{max} and a coincident deepening of the pressure (Fig. 4.7). The time duration of the single phases for each phase includes the subjective impression of the time evolution of horizontal cross sections of PV and vertical velocity (not shown for all time steps, single examples are shown in section 4.3). It was not possible to obtain an objective way of identifying symmetric and asymmetric phases. In any case, compared to N11’s E5C the time duration of all phases in GM5 is shorter and peak times of symmetry and asymmetry differ. Although the exact timing and length of the single phases are different, the patterns that characterize the single phases agree. There is a certain stochastic part to the timing and extend of the single phases as also shown in the next section 4.2. For this reason, consistent with the results here, the runs of WRF and TCLAPS are not expected to be identical. The internal variability of VCs should change due to the stochastic nature of this convectively driven process.
4.2 Sensitivity tests

Several runs were performed with differences in the initial and boundary conditions. This is conducted to investigate the variety of realizations that are calculated by the WRF model under such conditions. First in section 4.2.1 simulation CM5 is presented for which the SST is held constant in space and time. In run RM5 (section 4.2.2) the radiation parameterization is switched off and in simulation EN3 (section 4.2.3) the horizontal grid size is decreased to 3.3 km and ECMWF data is used instead of GFS data. Lastly seven simulations are shown collectively in section 4.2.4.

4.2.1 Constant SST

To test whether the VCs originate from variability in SST, an additional run CM5 was conducted with constant SST. All settings are similar to GM5 described above, except from the SST is held constant at 303.5K in space and time. Despite constant SST run CM5 shows patterns of VCs, there is a negative correlation of the deepening and the degree of asymmetry again, but the time duration and the exact timing of the single phases is different to GM5.

Figure 4.9: Subfigure (a) is similar to 4.1a and (b) similar to 4.7a but for run CM5.

The track of CM5 has been shown in section 3.2 Fig. 3.1 already. In Fig. 4.9a (similar to 4.1a) and Fig. 4.9b (similar to 4.7a) several features indicate VCs. First, the pressure of CM5 decreases with varying tendency reaching its minimum of 906.6 hPa at +69 h, thereafter rising slightly to 908 hPa at +72 h. The correlation of the pressure tendency with SDPV\(_{\text{max}}\) in Fig. 4.9b is negative with a negative correlation coefficient of −0.64 between +12 and +54 h. Second, the temporal evolution of the azimuthally averaged wind tendency and radial PV in Fig. 4.10 show the characteristic patterns of VCs. The typical patterns of acceleration towards the center in conjunction with a contraction of the RMW are clearly identifiable. Although the vortex ends up at a similar intensity to GM5, it evolves differently towards this final state. The accelerations in A1 and A2 are weaker than in GM5 whereas A3 lasts longer and is intenser as all asymmetric phases in GM5. Also the PV ring that characterizes the mature stage is established later than in
GM5 it is established within the asymmetric phase A3 after about 38 h whereas in GM5 this happens at 30 h already. (see blue contours in 4.5b and 4.10b).

### 4.2.2 Run RM5: no radiation parameterization

This run is similar to GM5 but the parameterization for radiation is switched off. It incorporates the strongest and most distinct VCs as there might be more energy kept in the system. RM5 is the simulation where the stepwise decrease in pressure is most obvious and therefore also exhibits a high correlation to the degree of asymmetry. This simulation is unrealistic, but it shows that VCs are generated by the ocean heat and not by a diurnal variation in radiative energy input.

In comparison to run GM5 the potential temperature \( \theta \) in the center in run RM5 is much warmer above the 500 hPa level (e.g. \( \approx 5 \) K difference between 500 and 200 hPa and even 14 K at 100 hPa at +72 h). Contrarily the latent and sensible surface heat flux is generally higher in GM5 due to the solar radiation. There is a large number of interactions of radiation and the circulation that are not examined in detail here. It is only vaguely suspected that as a net effect the capped cooling at the top of the atmosphere keeps a higher amount of energy in the system. This might be the reason that the TC spins up to extremely high wind speeds up to 112 m s\(^{-1}\) and deepens to an unrealistically low minimum pressure of 887 hPa. Katrina was with an observed central pressure of 903 hPa one of the most intense TCs in the Atlantic basin compared to the
4.2 Sensitivity tests

Figure 4.11: As Figure 4.9 but for run RM5

period from 1989 until 2004 investigated by Knaff and Zehr (2007).

Figure 4.12: Radius-time plots as in Fig. 4.5 but for model run RM5

Figure 4.11 indicates VCs immediately. The negative correlation coefficient of $-0.74$ between the pressure tendency and SDPV$_{\text{max}}$ in Fig. 4.11b underpins the distinct stepwise pressure decrease in Fig. 4.11a. There is a strong acceleration in A1 and A2 indicated with orange color for the acceleration of the tangential wind exceeding 7 m s$^{-1}$ in Fig. 4.12a and a congruent PV monopole shown with blue contours in Fig. 4.12b. Later from +42-60 h there is a constant acceleration of the vortex while the establishment of a PV ring and the constant RMW after +44 h indicate the maturity of the vortex. Hence, in RM5 the PV ring structure establishes comparatively late, 6 h later than in CM5 and 14 h later than in GM5. Finally the mean structure of the azimuthal profiles of PV and tangential
Results of WRF simulations

wind are shown in Appendix A in Fig. A.2. This is shown to illustrate that a centerward movement of mesovortices in the transition from symmetric to asymmetric phases of VCs are possible even though there is no PV ring structure in the symmetric phases S1 and S2, which is necessary for barotropic instability. This will be further discussed in chapter 5.

4.2.3 Run EN3: 3.3 km grid size

With $\Delta x = 3.3$ km run EN3 has a slightly higher horizontal resolution than the other WRF runs. At this resolution the explicit calculation of convection should theoretically be more reliable and model the fine scale structures of Katrina more realistically. In the duration of the VCs run EN3 is quite similar to run CM5: there are two shorter and weaker intensifications A1 (3 h and $\approx 1$ hPa h$^{-1}$) and A2 (3 h and $\approx 1$ hPa h$^{-1}$) and a long lasting deep intensification at A3 (8 h and $\approx 4$ hPa h$^{-1}$).

![Graphs showing wind speed and central pressure over time for runs S1, A1, S2, and A2] 

Figure 4.13: As Figure 4.9 but for run EN3

Note that the forecast times referred to in this section start twelve hours later at 26.08.05 12 UTC but also end at the 29.08.05 00 UTC as the other runs but after +60 h. EN3 is initialized with ECMWF data with 1011 hPa at its initial time. By then GM5, RM5 and CM5 all have 995 hPa minimum pressure already. Nevertheless, the intensification phase shows a stepwise decrease of minimum pressure in 4.13a and a coincident vacillation of the pressure tendency in b with the highest negative correlation coefficient of $-0.76$ between the pressure tendency and $SDPV_{\text{max}}$. There is no distinct peak in $SDPV_{\text{max}}$ at A1 but at A2 and A3. A similar picture appears in Fig. 4.14 where there is a weak acceleration of $\vec{v}$ before A1. A very strong and continuous acceleration occurs in A2 and A3 with a coincident contraction of the RMW. In contrast, to the expectation of a more noisy distribution at higher resolution the mean fields are surprisingly smooth, clearly showing the acceleration towards the center. Also the PV and its radial gradient complement the patterns of VCs. The PV maxima in the center incorporate less intensity as in GM5 but show a monopolar structure with 10 PVU in the center at A1 and A2. A3 is again in the transition towards the mature stage, the vortex constantly accelerates from +36-44 h while the RMW contracts. Finally there is a negative PV gradient at the center at +43-45 h that indicates a monopolar structure in the center but embedded in the PV ring inside
4.2 Sensitivity tests

In EN3 the RMW has an initially larger radius than for GM5, RM5 and CM5 but contracts to 50 km at +54 h, similar to the latter runs. Even though EN3 does not reach the observed intensity of Katrina, it clearly shows patterns of VCs.

![Figure 4.14: Radius-time plots as in Fig. 4.5 but for model run EN3](image)

4.2.4 Ensemble simulations

This section illustrates the stochastic nature of VCs. There are only slight modifications of the single runs (two data sets for boundary and initial conditions started at two initial times and sensitivity experiments with constant SST and no radiation) but the timing and extend of the single VCs differs considerably. Evolution of central pressure and maximum wind speed show a separation of the ensemble members into two groups; one group initialized with GFS has higher intensity of maximum wind and lower central pressure as the other group with ECMWF initial data.

Altogether seven simulations are presented herein: GM5, GN5, CM5 and RM5 using GFS and EM5, EN5 and EN3 using ECMWF data for initial and boundary conditions. GM5, CM5, RM5 and EM5 start at 00 UTC 26 Aug 2005 while GN5, EN5 and EN3 start later at 12 UTC. The evolution of minimum pressure in Fig. 4.15 appears in two groups; the initial data obviously influences the resulting intensity, hence the minimum pressure and maximum wind. The runs initialized with ECMWF data end up with a higher pressure (918 hPa in EN5, 919 hPa in EM5 and 924 hPa in EN3) compared to GFS runs (901 hPa in GM5, 906 hPa in GN5 and 908 hPa in CM5). Also in the
Results of WRF simulations

maximum wind the GFS runs and the ECMWF runs remain in those two groups. An exception is GFS run RM5, it moves towards the middle of minimum pressure of the two groups up to +42 h and later on intensifies rapidly down to 887 hPa at +72 h (congruently in wind maximum). Compared to best track data the maximum wind in

(a)

(b)

Figure 4.15: Minimum surface pressure and maximum velocity of all model runs

Fig. 4.15b seems to be over predicted in all simulations. But, as mentioned above, the best track maximum, the black line in Fig. 4.15b, shows the maximum of 1-min sustained winds at the surface, whereas the model output shows the maximum wind anywhere in the model. These maximum winds only illustrate the intensification but cannot be compared to the surface winds of the best track. The intensification rate of the mean tangential wind does not show a negative correlation with $SDPV_{\text{max}}$. Instead there is a positive correlation of $R = (0.16, 0.31, 0.70, 0.58)$ for the the runs GM5, CM5, RM5 and EN3, respectively. This positive correlation contradicts with the negative correlation of these variables found by N11 (see Fig. 4.7 and A.3 in Appendix A).
4.2 Sensitivity tests

Table 4.1: Initial conditions of the ensemble model runs.

<table>
<thead>
<tr>
<th>Longitude [deg]</th>
<th>Latitude [deg]</th>
<th>SST [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>268</td>
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<td>276</td>
<td>278</td>
<td>304</td>
</tr>
<tr>
<td>278</td>
<td>280</td>
<td>304.5</td>
</tr>
<tr>
<td>280</td>
<td>282</td>
<td>305</td>
</tr>
</tbody>
</table>

Figure 4.16: Track of all model runs with color shaded GFS SST at initial time 00 UTC 26 Aug 2005.

The first guess tracks of the ensemble are shown in Fig. 4.16. The picture here is different to the groups above: EN3 with the higher resolution gets closest to the real track despite it has the weakest circulation and the highest minimum pressure at 72 h (60 h forecast time in EN3). In contrast, the final locations of the runs driven by GFS reanalysis is closer to the best track than the two other runs EM5 and EN5 driven by ECMWF data, whereas the latter are closer to the best track in the first 24 h.

To sum up, the minimum surface pressure decreases in a stepwise manner indicating VCs in all simulations. Nevertheless the occurrence of the stepwise decrease has a different set of characteristics. The pressure tendency ($\frac{dp}{dt}$) of the single phases in particular the respective inner core structure of wind and PV differ in timing and magnitude (only shown for GM5, CM5, RM5 and L3).
4.3 Vortex Rossby wave breaking

This section presents isentropic PV maps for run GM5 wherein anticyclonic and cyclonic VRW breaking is hypothesized to dominate the PV patterns. Isentropic maps of PV have proved to be very useful for illustration of planetary scale dynamical fields and Rossby wave breaking (RWB) (Hoskins et al. 1985). Despite the strong diabatic and frictional processes, isentropic maps prove very useful in understanding the PV mixing processes in the intensification of TCs. A short review of PV fundamentals was given in section 2 to introduce into the 'PV thinking' perspective, which will help now to interpret the model output. The PV of run GM5 interpolated in isentropic surfaces in 5 min resolution was studied to investigate the temporal evolution of PV. In all simulations there was a clear coincident occurrence of asymmetric phases and cyclonic VRW breaking as defined below. Only single plots could be shown herein to illustrate this cyclonic VRW breaking. At last, another illustration shown in Fig. 4.20 shall further clarify the PV evolution. It reinforces the statement that the vacillation of PV in the center occur in the vertical stability, not in the absolute vorticity part of the PV.

Looking at the PV gradients at low levels (900-800 hPa), it is found that cyclonic and anticyclonic VRW breaking events accompany the intensification of Katrina. As stated by equation 2.35, PV can only increase its value if there is a source of $\theta$ or a frictional force on its boundary. PV can therefore be viewed as a tracer that cannot cross isentropic surfaces. For this reason PV is interpolated on isentropic surfaces for the analysis of RWB in this section. As McIntyre (1993) recalls, there is a nonlocal influence of the PV of one layer on the entire core region. He states that a PV layer interacts with the surrounding volume in the same way as a given pressure distribution would induce a static displacement on a stretched membrane. So a TC can be compared with a drum with several membranes of constant potential temperature $\theta$ with a certain force on every single layer induced by every point source of PV. He further describes how on a horizontal surface a velocity field induced by a vorticity field 'can roll itself up into a nearly circular vortex'. On a planetary scale this occurs around the polar vortex as cyclonic and anticyclonic waves break along strong negative PV gradients just outside the polar jet (McIntyre 1993, his Fig. 2). In contour plots of PV on isentropic surfaces this well known occurrence of RWB is characterized by an irreversible deformation of overturning PV contours. The main difference between a TCs symmetric phase to the polar vortex with a monopole structure, defined by a PV maximum in its center, is the TCs ring structure with a relative PV minimum in the center of the vortex at low levels. Along this ring VRWs break on the inner and outer side along the PV gradients as in breaking waves at the tropopause (for wave breaking at the tropopause see, e.g., McIntyre and Palmer 1983; Thorncroft et al. 1993; Martius et al. 2007).

The ideas of VRW breaking from a planetary scale applied to a TC might explain the breakdown of symmetric phases: The isentropic PV fields of the model output show
similar patterns in every asymmetric phase. Before asymmetric phases there is one or several VHTs that shoots through the whole troposphere up to the tropopause, thereby it locally releases latent energy and generates high PV anomalies in the eyewall. These anomalies detach from their parent VHTs, break cyclonically and propagate towards or mix into the center. To clarify the expressions cyclonic and anticyclonic VRW, it is hypothesized here that two different types of breaking occur:

**anticyclonic VRW:** On a negative radial PV gradient contours of PV overturn in an anticyclonic shape and deform irreversibly. These anticyclonic VRWs are Rossby-like waves that occur dominantly in the shear flow environment at the outside of the eyewall at all times of the simulation. While propagating radially outwards they are elongated and coincide with spiral rain bands.

**cyclonic VRW:** On a positive radial PV gradient contours of PV overturn in a cyclonic shape and deform irreversibly. These cyclonic VRWs are Rossby-like waves that occur dominantly in the shear flow environment at the inside of the eyewall. Strong events prior to the mature stage lead into asymmetric phases by wrapping themselves up to strong PV anomalies (mesovortices). In the mature stage cyclonic VRWs frequently inject PV to low levels into the center.

Anticyclonic wave breaking occurs at all times and is most pronounced in symmetric phases and the mature stage when a circular PV gradient outside of the eyewall and just inside of the RMW provides a favorable base for wave propagation. Cyclonic wave breaking is identified subjectively before every asymmetric phase. It can also be described as mesovortex that enters the center and dominates the flow with a deepening of the pressure at the location of the mesovortex and a congruent maximum asymmetry of the mean flow. It is able to breakdown the vortex in early stages, and is still visible in the mature stage, but cannot dominate the whole vortex any more.

These patterns just described above were apparent in all conducted WRF simulations and are shown for run GM5 now. The thin solid PV isoline in Fig. 4.17a - d represents 3 PVU and illustrates the anticyclonic overturning of PV contours in the symmetric phases S1-S3. The spiral outward movement of vertical updrafts and coexistent rain bands is indicated by thick black solid contours in Fig. 4.17 and also in Fig. 4.19a for A3. This happens at all times outside the RMW, as well in the asymmetric phases A1-A3 as imaginable in Fig. 4.18 and 4.19. Basically, PV is generated in the updraft regions, while it propagates outwards it decelerates and spirally elongates. These are the VRW described in section 1.2.2, named anticyclonic VRW in here, which are distinguished in this thesis from cyclonic VRW. The three symmetric phases are characterized by relatively lower PV values in the center region as well as in the eyewall region compared to asymmetric phases. The symmetrisizing effect of VRW smoothes the vortex structure, VRW propagate outwards on a negative PV gradient and lower the PV in the center.
Results of WRF simulations

Figure 4.17: Contours of PV at the 310K isentropic surface, dotted lines have the increments 3, 6, 9, 12, 15, 18 and 21 PVU, thin solid line shows the longest 3 PVU isoline in S1 and S2 at forecast hour 28:45 h and 36:15 h in (a) and (b), respectively. For symmetric phase S3 subfigures (c) and (d) show the PV at forecast hours 45:00 h and 45:30 h, respectively. Vertical velocity is shaded with intervals of 0.5, 1.5 and 3 m s$^{-1}$. Thick solid lines indicate anticyclonic VRW breaking. X and Y labels indicate distance from the center in km.
4.3 Vortex Rossby wave breaking

Figure 4.18: As Fig. 4.17 but thin solid line is longest 9 PVU isoline in A1 for forecast hour 23:00 h and 23:40 h for (a) and (b), and longest 12 PVU isoline for A2 at 30:50 h and 31:25 h in (c) and (d), respectively. Thick dashed lines indicates cyclonic VRW breaking.
Results of WRF simulations

t again (see undulations at 60 km radius in Fig. 4.6a).

Cyclonic VRW breaking events are shown in Fig. 4.18. In the first asymmetric phase A1, the 9 PVU contour wraps itself in at two locations at +23:00 h and merges to one cyclonic mesovortex with one dominating maximum of PV in the center at +23:40 h. Such breaking events occur repeatedly just before asymmetric phases as evident in A1-A4 (see Fig. 4.18 and 4.19). The breaking event in A3 and A4 does again intensify the vortex but it is different to A1 and A2 as the well established eyewall shows a circular structure in the vertical velocity (see grey shaded rings in Fig. 4.19). There is a region of weak updraft that disconnects the circular eyewalls but on azimuthal average the PV shows a ring structure (see Fig. A.1g and i). Note that the center shown here is not the TPF center. The TPF code is implemented for pressure surfaces hence it is not used for isentropic surfaces here. The center for all plots on isentropic surfaces herein is defined as the minimum of geopotential height which is located at intense PV gradients. This coincidental occurrence of minimum geopotential height and PV maxima again shows the vacillation of $\theta$ and PV observed in Fig. 4.6 and 4.5 that is associated with asymmetric phases. The isentropic surface is lowered locally in conjunction with positive PV anomalies and hence higher values of $\theta$ occur on pressure surfaces (see Fig. 4.5).

In the light of this illustration the parameter SDPV$_{\text{max}}$ becomes clearer: an intense mesovortex and its associated PV anomaly entering the center region with typically lower PV results in high magnitudes of SDPV$_{\text{max}}$. Similarly the FFT of PV shown in Fig. 4.8 becomes clearer now. The wavenumber 2 and 3 maxima occur frequently when cyclonic VRW break towards the center in the asymmetric phases A1-A4. In contrast, there is low energy in higher wavenumbers in symmetric phases. The symmetric outer ring starts to dominate after S2 (see increasing wavenumber 0 at +44 h in Fig. 4.8c) but there are several PV anomalies that enter the center frequently indicated by amplitudes in wavenumber 2 or higher in Fig 4.8a and b. Throughout the whole simulation PV is accumulated in the center while wave breaking causes vacillations continuously.

Finally, Fig. 4.20 further clarifies the contribution of the vertical stability towards the PV evolution in the inner core region. The PV is calculated by multiplying the absolute vorticity by the gradient of the potential temperature, hence the vertical stability (see equation 2.33). It is shown here that the vertical stability part of the PV vacillates in the center. For this purpose the PV on the 310K isentropic surface is averaged over three areas. First, the PV is averaged over the center region R1 within radii less than 15 km from the TPF center of the 850 hPa surface. Here an error is introduced as the 310K surface is lowering throughout the simulation, but the error is less compared to the error of the first guess center used above. The second area is the ring R2, averaging the PV between 15 and 30 km from the TPF center and the third area R3 is the eyewall region between 30 and 70 km from the center. The evolution of PV on the three areas R1, R2 and R3 are indicated with a solid, a dotted and a dashed line, respectively. Similarly
Figure 4.19: As Fig. 4.17 but solid line is longest 12 PVU isoline in A3 for forecast hour 38:05 h and 38:35 h for (a) and (b) and for A4 at 47:35 h and 47:55 h in (c) and (d), respectively. Thick dashed lines indicate cyclonic VRW breaking and thick solid lines in (a) anticyclonic VRW breaking.
the the vertical gradient of the potential temperature $\frac{\Delta \theta}{\Delta z}$ is averaged, whereas $\Delta \theta$ the difference of 315K minus 310K, hence 5K, is divided by $\Delta z$ the height of the respective surfaces. High values of $\frac{\Delta \theta}{\Delta z}$ in R1 and R2 indicate a thinning of the volume between the 310 and 315K surface and coincide with maxima in the PV in R1 and R2 at A1 and A2 (solid and dotted line in Fig. 4.20a, respectively). After A2 the PV in R2 keeps higher values than in R1 while the layer above 310K gets gradually thicker and even thicker than in the eyewall region after S3. Also the PV in the center is after A2 lower than at outer radii which illustrates the continuous PV ring structure of the mean field. This happens while the 310K layer subsides into the PBL and gets closer to the surface (see Fig. 4.3b and 4.4b). Fig. 4.20c and d show the same information as 4.20a and b but for the PV on the 315K surface and $\frac{\Delta \theta}{\Delta z}$ above it. The 315 and 320K isentropic surfaces keep constant distances on all three areas R1-R3 until A2 while the PV increases slightly in R1 and R2 before and after A1 already. At A2 the PV on R1 and R2 starts to increase and rises most in the circle R1 while rising also in R2. In contrast, PV enters the center frequently at the same time but stronger in R2 than in R1. Although the single phases are not clearly identifiable on the basis of the vacillation of isentropic surfaces it hints onto the contribution of the potential temperature gradient on the PV in the core region.

In summary it is shown here that PV which is generated in regions of updraft does not stay at the location of their parent convective entities. The PV maxima start to propagate upstream and break cyclonically towards the center or wrap themselves inward. As stated above contours of PV overturn and deform irreversibly in an cyclonic shape and occur dominantly in the shear flow environment at the inside of the eyewall. Strong events prior to the mature stage as in A1 and A2 lead into asymmetric phases by wrapping themselves up to strong PV anomalies (mesovortices). In the mature stage when the eyewall is well established mesovortices in asymmetric phases, as in A3 and A4 of GM5, still enter the center and frequently inject PV into the center but cannot break down the PV ring to a monopolar PV any more (see also azimuthal averages of A2 - A4 in Fig. A.1). In conclusion the cyclonic and anticyclonic VRW breaking described here has been observed in all the simulated runs that are presented in this thesis. In particular, cyclonic VRW breaking prior to asymmetric phases is a pattern that occurs in all asymmetric phases with differing extend (just shown for GM5).
4.3 Vortex Rossby wave breaking

Figure 4.20: Evolution of average PV at the 310K isentropic surface in (a) and average of vertical derivative of potential temperature $\frac{\partial \theta}{\partial z}$ in (b) ($\Delta \theta = 10 K$, $\Delta z = z(320K) - z(310K)$), averaged over a circle region R1 with radius less than 15 km (0.15°) from the center as solid line and two adjacent rings R2 and R3. R2 in the intersection zone from 15-30 km (0.15 – 0.3°) indicated by a dotted line and R3 covering the eyewall region from 30-70 km (0.3 – 0.7°), dashed line. The same for (c) and (d) on 315K, respectively.
Results of WRF simulations
Chapter 5

Discussion

The main question addressed in this thesis is: Do vacillation cycles (VCs) occur in the intensification phase of Hurricane Katrina in the non-hydrostatic WRF model? This can clearly be answered with yes. The results of all WRF simulations presented in chapter 4 show patterns of VCs as described by Nguyen et al. (2011) (N11). Now, the significance of these results is discussed, involving first of all the comparison of the results with previous literature as well as with observations of Katrina. Also, strengths and limitations of the methods used herein and their significance for TC research in general are discussed.

One of the main conclusions of N11 is the distinction between VCs and eyewall replacement cycles (ERCs). In the Tropical Cyclone Report of Knabb et al. (2006) passive microwave imagery of Hurricane Katrina from the NASA TRMM satellite are presented. They argue that an ERC causes the break down of the inner eyewall as visible in Fig. 5.1a. Knabb et al. (2006) find that this temporary deterioration, which is also visible in the constant pressure observed from 12 UTC 27 Aug to 00 UTC 28 Aug 2005 (see evolution from +36 h to +48 h, e.g. in Fig. 4.1), prevents further intensification. In the morning of 28 Aug 2005 Katrina deepened again with a newly established vortex (see 5.1b). In contrast, N11 clearly identify this stepwise pressure decrease in their model simulations as VCs instead of ERCs. In an ERC the circulation and deepening of the vortex is weakened as the energy is consumed by a secondary eyewall which develops at a greater radius in a symmetric manner. The inner eyewall thereby constantly weakens while the outer eyewall starts to contract again whereafter the pressure continues to decrease. The central pressure evolution alone does not differentiate ERCs from VCs, but as a ERC is a symmetric process VCs can be clearly distinguished from ERCs by means of the characteristic vacillation of inner core PV due to vortical hot towers (VHTs) and the associated asymmetries of VCs. Also do VCs occur in the transitional stage towards a mature TC whereas ERCs usually occur in mature TCs. ERCs do not occur in any of the WRF runs, hence, the stepwise decrease in minimum pressure with coincident maximum asymmetry can be explained by VCs but not by ERCs. N11 reason that the breakdown of the vortex in the model as well as in the observations is not caused by the formation of an outer eyewall but the inner core dynamics of VCs. The developing
Discussion

Figure 5.1: Passive microwave imagery from the NASA TRMM satellite of Hurricane Katrina, at (a) 0420 UTC 27 Aug and (b) 0324 UTC 28 Aug 2005. All images are from the 85GHz channel in which ice scattering reveals areas of deep convection displayed in the red shades. Images courtesy of the Naval Research Laboratory (NRL). Adapted from Fig. 5 of Knabb et al. (2006)

asymmetries in the eyewall itself in form of VHTs are thought to break down the vortex and are hypothesized to be the dominant mechanism for the inner core intensification. The respective outgoing longwave radiation of GM5 in Fig. 4.2 shows a similar structure to the areas of deep convection shown in Fig. 5.1. By taking account of the observations of minimum surface pressure the first pressure decrease from +6-12 h in Fig. 4.1a could be classified to asymmetric phase A1 and the following weakened pressure decrease to S1. Furthermore, the two microwave images indicate areas of deep convection and can be classified in conjunction with the pressure decrease from +24-36 h to asymmetric phase A2 for Fig. 5.1a and with constant pressure from +36-48 h to symmetric phase S2. Fig. 5.1b is then in the transitional stage towards A3 of the real Katrina, which lasts from +49-72 h. Additionally, Fig. 5.1b reminds one of the intersected ring in GM5s A3, as is shown in Fig. 4.19c and d. Probably the unrealistically high surface pressure and weak circulation in the initial conditions cause the delayed intensification in the model runs herein(compared to the timing of VCs in Katrina observations). The focus of this work is the understanding of the inner core processes, and not the perfect reconstruction of reality. As shown in chapter 4 patterns of VCs are apparent in all WRF simulations presented herein. Even though there are only small changes in initial and boundary conditions, the variety of realizations in the model is large. Hence, in awareness of the incompleteness of reconstructing reality it is found here that a range of possibilities for the timing and extent of the VCs illustrates the probabilistic nature of predicting inner core development.

As foreshadowed in section 1.2.5, N11’s basic point that the breakdown from the
symmetric to the asymmetric phase is caused by convective-barotropic instability is questioned now. First approaches to explain mesovortices in literature (see section 1.2.3) were formulated in a barotropic framework, i.e. a breakdown of vorticity rings initiated by barotropic instability (Schubert et al. 1999; Kossin and Schubert 2001). Recently N11 suggested convective-barotropic instability to be the cause for mesovortices in their simulations and simultaneously the initiating mechanism for asymmetric phases of VCs. Only the barotropic instability part of the argument is in question and it will be shown that several facts contradict with N11’s explanations. Firstly, there is no symmetric PV ring structure in the mean fields of GM5’s A1, and hence, there is no change in sign of the radial PV gradient. In other words, there is no inflection point in the radial PV profile, which is a necessary condition for barotropic instability (compare Fig. 1.2 and 4.6). Following the argument of N11 there must be a ring like structure of PV that breaks down to a monopolar structure of PV, but before the first asymmetric phase of all runs there is no PV ring in the mean azimuthal PV. The run RM5 is conducted without radiation parameterization and has hence the most intense VCs. Nonetheless there is no PV ring in the mean fields up to A3 at +48 h (see Fig. A.2). The PV ring is apparent in the mean field of N11’s simulations (see Fig. 1.5a and wavenumber 0 at S1 and S2 at the lower graph of Fig. 1.5c) but not in the mean fields of S1 and S2 of RM5 (see Fig. A.2c, A.2g). The fact that the vortex breaks down even though there is no PV ring shows that VCs are possible without barotropic instability. On the other hand N11 show that asymmetric phases are characterized by a monopolar structure in azimuthal mean PV but for instance in GM5 a PV ring in the mean field establishes between +28-30 h that does not show a monopolar structure in A2-A4 (see Fig. 4.5, 4.6 and A.1). VHTs with their inherent convective instability play a major role, but it is shown herein that the mechanism that wraps a mesovortex towards the center can be different from convective-barotropic instability described by N11. Barotropic instability and Rossby wave propagation are very closely related ideas. In fact, barotropic instability can be thought of as the interaction of two counter propagating Rossby waves, one propagating on the negative PV gradient and the other along the positive PV gradient. Moreover there may be little distinction between the two in environments that are only weakly unstable or weakly stable, or when the waves are large amplitude. It’s plausible that the vacillation could be driven by either large amplitude stable Rossby waves or large amplitude barotropic instability - in either case, the important point is that the PV is irreversibly stirred inward.

Another point to criticize is: N11 use the $\beta$ plane explanation to be the reason for the centerward movement of cyclonic PV. N11 argue that VHTs travel analogous to a vortex on a $\beta$ plane towards higher PV, and hence towards the center on a negative PV gradient. On one hand the ring structure of PV, which presumes both an inner positive PV gradient and an outer negative one, are a necessary condition for barotropic instability, but at the same time a PV anomaly should move inwards to the center on a negative PV gradient. In the symmetric phase, the PV gradient is positive inside the radius of maximum winds and negative outside of it. During the transition from symmetric to asymmetric states,
the positive PV gradient quickly becomes negative (in the mean) because cyclonic PV is advected inward. It is this gradient that guides isolated VHT towards the centre. This condition is apparent in S1 and S2 of RM5, but the β plane explanation still fails in accounting for the centerward movement of mesovortices along a positive PV gradient in the transition to the mature phase (see Fig. 4.19a and b). Also in the simulation GM5 the above condition is apparent in S1 only but not in S2 and S3. This thesis focuses on the PV perspective and it is argued with PV thinking to account for the centerward movement of cyclonic PV. The argumentation herein cannot prove what is relevant, but it can locate gaps in the ability to explain inner core intensification. The review on PV fundamentals in chapter 2 recalls the capabilities of using PV as a diagnostic tool. Specifically, the derivation of the simple rotation symmetric example of Pichler (1997) in section 2.3 illustrates the role of PV in the inner core PV redistribution. In the center of the vortex up to the RMW the radial wind increases approximately linear with less variations in the symmetric phases compared to asymmetric phases (see Fig. 4.6 and A.1), hence for the moment we assume the balance condition 2.32 to be valid for such a solid body rotation in the inner core inside the radius of maximum wind (RMW) for symmetric phases. Hence, the PV is in balance with the potential temperature gradient in the center region. As identified in section 4.1.1 the PV vacillates in the center as a result of variations in the vertical potential temperature gradient. The high PV generated in the eyewall region by VHTs induces an imbalance in equation 2.32. A local lowering of the isentropic surfaces in GM5 (see section 4.3) generated by VHTs in the eyewall region is forced into the center to balance the PV distribution. This redistribution of PV has to happen horizontally as by equation (2.35) the PV cannot cross isentropic surfaces but has to be advected horizontally (see Fig. 4.3, 4.6, 4.5 and 4.20). In turn this stirring of the PV into the center in asymmetric phases accumulates PV in the center while the pressure decreases rapidly. Unfortunately, time did not allow an investigation of the interaction between upper tropospheric PV anomalies and the inner core PV dynamics described herein, but could be a question for future research.

For further understanding the PV dynamics Haynes and McIntyre (1987) draw a thought experiment (their point 11 in section 2) where a localized cooling imposed on an unbounded, stably stratified rotational fluid at relative rest induces a redistribution of PV. According to theorem (i) of Haynes and McIntyre (1987) the adjustment of PV distribution must occur horizontally. Hence, a cooling in the BL forces a redistribution of PV at low levels by horizontal advection along isentropic surfaces. Furthermore, this change in low-level PV is expected to disturb the PV balance and induces a change of PV at midlevels. In other words, the PV and θ fields of the whole vortex have to react on disturbances in one layer. Haynes and McIntyre (1987) also show that PV can just be generated at the intersection of θ surfaces with the ground or the boundary layer, so the whole redistribution comes necessarily from the bottom. Now we turn the thought experiment upside down: the diabatic warming in the eyewall region causes an adiabatic adjustment in the center by displacement of the isentropic surfaces that redistribute the
PV horizontally. The picture of a drum with several impermeable layers can be used as an analogy to imagine the PV dynamics. A change of potential temperature influences the whole body of such a drum as if someone would hit the bottom of it and all layers above would start to vibrate with decreasing amplitude towards the top. Following this analogy the strong PV anomaly associated with the cyclonic VRW breaking induce a vacillation of the whole vortex where not necessarily two layers of the vortex vacillate but the whole vortex as one entity redistributes the PV that is injected after cyclonic VRW breaking in low levels. In summary, it is suspected that the centerward movement is an imbalance in the PV field of the three dimensional volume of a TCs inner core that cannot be reduced to a two dimensional problem as assumed in barotropic instability. A PV anomaly that is generated by a VHT detaches from its parent convective entity to balance the mean field of the inner core and in turn by entering the center with a so called cyclonic VRW breaking it accelerates the inner core by accumulating PV. Future research could investigate an inversion of PV like in pervious work but with a focus on VCs (Shapiro and Möller 2003; Hausman et al. 2006; Wang 2002a,b). Furthermore, it would be desirable to clearly separate the dominating processes of VCs and to formulate them into a simplified model that can estimate the range of possible intensities in a probabilistic manner.

Although VCs could be clearly identified in the WRF simulations, there are inconsistencies. The negative correlation between the tendency of mean tangential wind $\frac{\partial \mathbf{v}_{\text{max}}}{\partial t}$ and SDPV$\max$ as found by N11 is inconsistent with WRF results shown here. This correlation as shown for GM5 in Fig. 4.7b is positive in all simulations (not shown for GN5, EM5 and EN5 but in Fig. A.3 for CM5, RM5 and EN3) whereas in TCLAPS run E5C shown in the upper graph of Fig. 1.5b it is negative. This positive correlation also arises from the radius time plots of azimuthally averaged tangential wind as indicated by the maximum acceleration just before asymmetric phases in Fig. 4.5a, 4.10a, 4.12a and 4.14a for GM5, CM5, RM5 and EN3, respectively. Actually this inconsistency does not affect the results, the acceleration periods are just phase shifted. N11 state that the vortex accelerates in symmetric phases whereas it is found herein that the TC accelerates just before maximum asymmetry. Furthermore, the magnitude of SDPV$\max$ is much higher in WRF than in TCLAPS and also stays at high values in the mature stage in WRF (see ensemble simulations of Nguyen 2010). This could arise from errors in center finding that affect the higher SDPV$\max$ in the mature phase but the converging of the first guess center and the TPF center indicate an accurate center finding. A difference that is not mentioned yet is the magnitude of $\theta_e$ which is biased at all times. It is lower in WRF than in the calculations with TCLAPS of N11. Unfortunately, the reason for this is not obvious to the author. Generally there is also a difference in the mean fields of PV as the WRF runs do not show monopolar PV fields in all asymmetric phases (see Fig. 4.5a, 4.10a, 4.12a and 4.14a for GM5, CM5, RM5 and EN3, respectively, and Fig. 4.6, A.1 for GM5 and A.2 for RM5). Asymmetric phases rather need to be characterized by single intense PV anomalies (mesovortices). When they occur in early stages of the
intensification they dominate the mean PV what results in a monopolar structure. But as this occurs still later while there is a PV ring in the mean asymmetric phases cannot be characterized by a monopolar PV pattern.

To answer the question asked in the outline: the variability in SST does not cause VCs as they occur at constant SST in CM5 (see acceleration in in Fig. 4.9b and pressure deepening in Fig 4.10b). Also the diurnal variation of incoming solar radiation does not cause VCs as they occur in RM5 without radiation parameterization. Despite VCs are a robust feature in the intensification of all WRF runs herein further work in modelling other TCs in other locations is necessary to substantiate the importance of VCs.

As pointed out in section 3.1.2 higher horizontal diffusion and a divergence damping in the parameterization of TCLAPS in conjunction with the non-hydrostatic formulation of WRF causes a more complex structure of all diagnostic variables and a more noisy structure of the mean fields in WRF. With 5 and 3 km horizontal mesh size vertical up- and downdrafts are not yet well resolved. In addition, one needs to be aware that real convection as well as turbulent processes in the planetary boundary layer (PBL) are not well represented by TC simulations in general (see section 8.2 of Montgomery and Smith 2011). Although this work shows, that VCs are strongly linked to PV redistribution processes, it is not clear weather the occurrence of cyclonic wave breaking initiating an asymmetric phase is representative for real hurricanes or if it is simply a feature of the more viscous behavior of the fluid dynamics in the model world. In general, features can just be resolved when their size is in the order of multiple times the grid size (if wavelength equals four times of the mesh size; see e.g. Pielke 2002). Hence, at 5 km mesh size, features such as wave breaking with a wavelength of less than 20 km are possibly just numerical features. Therefore, a resolution of 1 km would be of interest to better resolve these processes and to judge the validity of lower resolutions used herein. In conclusion, as far as the author knows are VCs patterns are only investigated by N11 and herein so far. Hence, further investigations of the typical patterns of VCs in observations in conjunction with model studies at higher resolutions are necessary to judge the relevance of VCs for the intensification process.
Chapter 6

Conclusions

This thesis investigated vacillation cycles (VCs), that occur during the evolution of Hurricane Katrina, based on simulations with the Weather Research and Forecasting model WRF. VCs were first discovered by Nguyen et al. (2011). They are the repeated oscillations between an asymmetric and a symmetric mean azimuthal low level PV structure. The PV thereby takes the shape of approximately circular PV rings in symmetric phases which frequently break down to one or several monopolar PV anomalies that propagate centerwards. Nguyen et al. (2011) investigated VCs based on simulations with the hydrostatic model TCLAPS. Major findings of this work that confirm the results of Nguyen et al. (2011) are:

- **VCs occur also in a non-hydrostatic model.** The patterns of PV structures (PV rings and monopoles) that characterize the single phases agree with the results of Nguyen et al. (2011), although the structures are less smooth due to the non-hydrostatic. Sensitivity simulations with constant sea surface temperature (SST) or without radiation parameterization show that VCs are not caused by inhomogeneities in SST or radiation in space or time.

- **The intensification process possesses a highly asymmetric and probabilistic manner.** Although the exact timing and length of the individual symmetric and asymmetric phases for WRF simulations with varying initial and boundary conditions are different, the patterns of VCs agree among one another.

- **VCs are totally different from eyewall replacement cycles (ERCs).** An ERC is a dominantly symmetric process in the mature stage of a tropical cyclone (TC), whereas VCs inhibit a high degree of asymmetry occurring in the transitional stage towards maturity.

On top of this unprecedented key findings of this thesis are:

- **Maximum asymmetry occurs shortly after cyclonic vortex Rossby waves (VRWs) break towards the center.** VRW breaking is characterized by overturning of a PV contour on a low-level isentropic surface.
Conclusions

- As shown by Nguyen et al. (2011) VCs are characterized by a centerward propagation of low level PV generated in vortical hot towers (VHTs). The increased circulation in VHTs accelerates the mean flow and, hence, increases its kinetic energy. Inner core oscillations in vertical stability show that this increased kinetic energy is transported towards the center and is converted into potential energy.

- Theoretical considerations motivated to explain the centerward transport of PV to be caused by an imbalance in the PV field of the three dimensional volume of a TCs inner core. The remote interaction of PV in the TCs inner core cannot be reduced to a two dimensional problem as assumed in barotropic instability.

Questions that may be relevant but could not be answered herein are the role of the topography of Florida in disturbing or promoting VCs and the impact of an increased, cloud-resolving model resolution of about 1 km on the structure of VCs. Hence, an aim of further research could be to test the robustness of VCs by simulating other TCs at other locations, higher resolutions and flat topography.
Appendix A

Appendix

A.1 WRF GM5 input namelist

```
&time_control
  run_days = 3,
  run_hours = 12,
  run_minutes = 0,
  run_seconds = 0,
  start_year = 2005, 2005,
  start_month = 08, 08,
  start_day = 26, 26,
  start_hour = 00, 00,
  start_minute = 00, 00,
  start_second = 00, 00,
  end_year = 2005, 2005,
  end_month = 08, 08,
  end_day = 29, 29,
  end_hour = 12, 12,
  end_minute = 00, 00,
  end_second = 00, 00,
  interval_seconds = 21600,
  input_from_file = .true.,.true.,
  history_interval = 720,60,
  frames_per_outfile = 1,1,
  restart = .false.,
  restart_interval = 720,
  io_form_history = 2,
  io_form_restart = 2,
  io_form_input = 2,
  io_form_boundary = 2,
  debug_level = 0,
/
```
&domains
  time_step = 30,
  time_step_fract_num = 0,
  time_step_fract_den = 1,
  max_dom = 2,
  s_we = 1, 1,
  e_we = 280, 301,
  s_sn = 1, 1,
  e_sn = 240, 301,
  s_vert = 1, 1,
  e_vert = 45, 45,
  num_metgrid_levels = 27,
  dx = 16676.62,5558.87,
  dy = 16676.62,5558.87,
  grid_id = 1, 2,
  parent_id = 0, 1,
  i_parent_start = 1, 104,
  j_parent_start = 1, 71,
  parent_grid_ratio = 1, 3,
  parent_time_step_ratio = 1, 3,
  feedback = 1,
  smooth_option = 0,

&physics
  mp.physics = 2, 2,
  ra_lw.physics = 1, 1,
  ra_sn.physics = 1, 1,
  radt = 10, 3,
  sf_sfcclay.physics = 1, 1,
  sf_surface.physics = 1, 1,
  bl_pbl.physics = 1, 1,
  bldt = 0, 0,
  cu.physics = 1, 0,
  cudt = 5, 5,
  isfflx = 2,
  ifsnow = 0,
  icloud = 1,
  surface_input_source = 1,
  num_soil_layers = 4,
  sf_urban.physics = 0, 0,
  maxiens = 1,
  maxens = 3,
  maxens2 = 3,
  maxens3 = 16,
  ensdim = 144,
&dynamics
v_damping = 0,
diff_opt = 1,
km_opt = 4,
diff_6th_opt = 0, 0,
diff_6th_factor = 0.12, 0.12,
base_temp = 290.
damp_opt = 0,
zdamp = 5000., 5000.,
dampcoef = 0.2, 0.2,
khdif = 0, 0,
kvdif = 0, 0,
non_hydrostatic = .true., .true.,
moist_adv_opt = 1, 1,
scalar_adv_opt = 1, 1,

&bdy_control
spec_bdy_width = 5,
spec_zone = 1,
relax_zone = 4,
specified = .true., .false., .false.,
nested = .false., .true., .true.,
/
&grib2
/
&namelist_quilt
nio_tasks_per_group = 0,
nio_groups = 1,
A.2 WRF GM5 WPS namelist

```
&share
wrf_core = 'ARW',
max_dom = 2,
start_date = '2005-08-26 00:00:00', '2005-08-26 00:00:00',
end_date = '2005-08-26 00:00:00', '2005-08-26 00:00:00',
interval_seconds = 21600,
io_form_geogrid = 2,
/

&geogrid
parent_id = 1, 1,
parent_grid_ratio = 1, 3,
i_parent_start = 1,104,
j_parent_start = 1,71,
e_we = 280, 301,
e_sn = 240, 301,
geog_data_res = '10m','2m',
dx = 16676.62,
dy = 16676.62,
map_proj = 'mercator',
ref_lat = 25.0,
ref_lon = -87.0,
truelat1 = 26.0,
stand_lon = -87.0,
geog_data_path = '/scratch/c707222/WRFinput/geog'
/

&ungrib
out_format = 'WPS',
prefix = 'FILE',
/

&metgrid
fg_name = 'FILE',
io_form_metgrid = 2,
/
```
A.3 Calculation of diagnostic variables

The hydrostatic form of Ertel’s PV is calculated with the NCL routine `wrf_user_getvar` by equation 3.4:

\[
PV_\sigma = -g \left[ -\frac{\partial v}{\partial p}\frac{\partial \theta}{\partial x}_\sigma + \frac{\partial u}{\partial p}\frac{\partial \theta}{\partial y}_\sigma + \left( \frac{\partial v}{\partial x}_\sigma - \frac{\partial u}{\partial y}_\sigma + f \right) \frac{\partial \theta}{\partial p}_\sigma \right].
\]

Equation 3.4 can be derived by using (2.21)

\[
Q = \frac{1}{\rho} \eta \cdot \nabla \theta.
\]

For this case this equation can be expressed with respect to WRF’s \( \sigma \) surfaces in the vertical. This is denoted in the derivatives by the subscripts: \( \nabla_\sigma = \left( \frac{\partial}{\partial x}_\sigma, \frac{\partial}{\partial y}_\sigma, \frac{\partial}{\partial z}_\sigma \right) \).

So

\[
Q = \frac{1}{\rho} \eta_\sigma \cdot \nabla_\sigma \theta.
\]

which can with \( \eta = 2\Omega + \nabla_\sigma \times \mathbf{v} \) be written as

\[
Q = \frac{1}{\rho} (2\Omega + \nabla_\sigma \times \mathbf{v}) \cdot \nabla_\sigma \theta
\]

is equal to the expanded notation

\[
\rho Q = \begin{bmatrix}
2 \left( \begin{array}{c}
\frac{\partial \Omega \cos \phi}{\partial y} \\
\frac{\partial \Omega \sin \phi}{\partial z}
\end{array} \right) + \left( \begin{array}{c}
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial z} \\
\frac{\partial u}{\partial z} - \frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial x} - \frac{\partial w}{\partial y}
\end{array} \right) \cdot \left( \begin{array}{c}
\frac{\partial \theta}{\partial x}_\sigma \\
\frac{\partial \theta}{\partial y}_\sigma \\
\frac{\partial \theta}{\partial p}_\sigma
\end{array} \right) & .
\end{bmatrix}
\]

The horizontal derivatives of the vertical velocity \( \frac{\partial w}{\partial x} \) and \( \frac{\partial w}{\partial y} \) as well as the horizontal component of the Coriolis force can be neglected \( 2\Omega = \mathbf{k} \cdot f \) (\( \phi \) is the latitude).

\[
\rho Q = \begin{bmatrix}
0 \\
0 \\
f
\end{bmatrix} + \left( \begin{array}{c}
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial z} \\
\frac{\partial u}{\partial z} - \frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial x} - \frac{\partial w}{\partial y}
\end{array} \right) \cdot \left( \begin{array}{c}
\frac{\partial \theta}{\partial x}_\sigma \\
\frac{\partial \theta}{\partial y}_\sigma \\
\frac{\partial \theta}{\partial p}_\sigma
\end{array} \right).& .
\]

With the chain rule one can extend this by \( \frac{\partial}{\partial z}_\sigma = \frac{\partial}{\partial p}_\sigma \frac{\partial}{\partial z}_\sigma \) to

\[
\rho Q = \frac{\partial p}{\partial z}_\sigma \left[ \left( \begin{array}{c}
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial z} \\
\frac{\partial u}{\partial z} - \frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial x} - \frac{\partial w}{\partial y} + f
\end{array} \right) \cdot \left( \begin{array}{c}
\frac{\partial \theta}{\partial x}_\sigma \\
\frac{\partial \theta}{\partial y}_\sigma \\
\frac{\partial \theta}{\partial p}_\sigma
\end{array} \right) \right].
\]

which reduces to 3.4 again by using the hydrostatic approximation \( \frac{\partial p}{\partial z} = -\rho g \). A coordinate transformation from \( \sigma \) levels to pressure levels is avoided by NCL by directly calculating the finite pressure differences on the respective \( \sigma \) levels to obtain \( \frac{\partial}{\partial p}_\sigma \).

Throughout this thesis plots showing the absolute vorticity are calculated by:

\[
\eta = \left( \frac{\partial v}{\partial x}_\sigma - \frac{\partial u}{\partial y}_\sigma + f \right)
\]

which is the vertical component of \( \eta = 2\Omega + \nabla_\sigma \times \mathbf{v} \) only.
A.4 Additional plots

Figure A.1: Azimuthal mean of PV and tangential wind on 850 hPa at S1 at 28 h in a and b, A2 at 31 h in c and d, S2 at 36 h in e and f, A3 at 39 h in g and h and A4 at 46 h in i and j, respectively.
Figure A.2: As Fig. A.1 but for PV and mean tangential wind for A1 in (a) and (b), for S1 in (c) and (d) for A2 in (e) and (f), for S2 in (g) and (h) and for A3 in (i) and (j), respectively.
Figure A.3: The time evolution of $SDPV_{\text{max}}$ in red with circles and in blue with triangles $\frac{dSJPV_{\text{max}}}{dt}$ on 850 hPa (a) for CM5, (b) for RM5 and (c) for EN3.
Bibliography


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